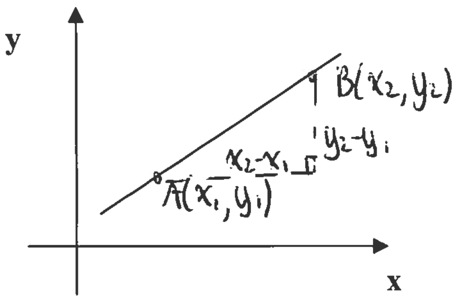
Distance Formula

For any two points **A** (x_1 , y_1) and **B**(x_2 , y_2)



Pythagoras tells us that

$$(AB)^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

and so

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

This is known as the **Distance Formula**

In three dimensions

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Examples

1) Calculate the distance between the points C(-2,5) and D(3,8)

$$CD = \sqrt{(3-(-2))^2 + (8-5)^2}$$
$$= \sqrt{25+9}$$
$$= \sqrt{34}$$

2) Calculate the distance between C(-2, 4, 3) and D(2, -3, 4)

$$CD = \sqrt{(2-(-2))^2 + (-3-4)^2 + (4-3)^2}$$
$$= \sqrt{16+49+1}$$
$$= \sqrt{66}$$

2) A(-1,1) B(1,3) C(3,1)
Show that triangle ABC is isosceles.

To show that the triangle is isosceles we must show that two sides are the same length.

$AB = \sqrt{(1-(-1))^2 + (3-1)^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$

$BC = \sqrt{(3-1)^2 + (1-3)^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$

$AC = \sqrt{(3-(-1))^2 + (1-1)^2}$
$= \sqrt{16}$
$= 4$

Since $AB = BC$ the triangle is isosceles.

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N5 Page 214 Ex20D

Gradient

Gradient	$m > 0$	line slopes like so
Gradient	$m < 0$	line slopes like so
Gradient	$m = 0$	horizontal line
Gradient	undefined	vertical line

Parallel Lines

Parallel lines never meet and have the same gradient.

$$\text{AB and CD are parallel} \Leftrightarrow \quad m_{AB} = m_{CD}$$

Examples

1) Consider the points S(2,5), T(5,9), U(-3,-4) and V(3,4).
   Show that ST and UV are parallel.

$$m_{ST} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{5-2} = \frac{4}{3}$$

$$m_{UV} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-4)}{3-(-3)} = \frac{8}{6} = \frac{4}{3}$$

$m_{ST} = m_{UV}$ so ST is parallel to UV

2) Find the equation of the line GH which passes through the point (4, -2) and is parallel to the line RS with equation 3x + 2y = 7.

Gradient

$$3x + 2y = 7$$
$$2y = -3x + 7$$
$$y = -\frac{3}{2}x + \frac{7}{2} \qquad m = -\frac{3}{2}$$

Equation

$$y - b = m(x - a)$$
$$y - (-2) = -\frac{3}{2}(x - 4)$$
$$2y + 4 = -3(x - 4)$$
$$2y + 4 = -3x + 12, \qquad 2y + 3x = 8$$

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$\times 3$

Collinearity

For any points A, B, C if $m_{AB} = m_{BC}$ (i.e. AB and BC are parallel), since B is a common point all three points must be on the same line i.e. collinear.

Example

Show that A(-6,-5), B(0,-1) and C(3,1) are collinear.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-5)}{0-(-6)} = \frac{4}{6} = \frac{2}{3}$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{3-0} = \frac{2}{3}$$

So $m_{AB} = m_{BC}$.

So AB is parallel to BC and since B is a common point, A, B and C are collinear.

Extra Note

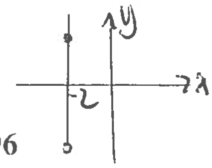
Always use $y - b = m(x-a)$ when finding equation of a line. except when gradient $= \frac{\bullet}{0}$ = undefined.

Then equation is $x =$ number ← get number from point.

eg Find the equation of the line through A(-2, 4) and B(-2, -5)

$$m_{AB} = \frac{-5-4}{-2-(-2)} = \frac{-9}{0} = \text{undefined}$$

Equation $\underline{x = -2.}$



Ex 13B, pg 296

Perpendicular Lines

Two lines with gradients m_1 and m_2 are perpendicular $\Leftrightarrow m_1 \times m_2 = -1$

Examples

1) P(2,-3) and Q(-1,6). Find the gradient of the line perpendicular to PQ

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{6-(-3)}{-1-2}$$
$$= \frac{9}{-3}$$
$$= -3$$

$$m_{perp} = \frac{1}{3}$$

turn gradient upside down and change sign

2) Are the lines y = 2x + 6 and 2y = -x + 4 perpendicular ?

$$y = 2x + 6$$
$$m_1 = 2$$

$$2y = -x + 4$$
$$y = -\frac{1}{2}x + 2$$
$$m_2 = -\frac{1}{2}$$

$$m_1 m_2 = 2 \times \left(-\frac{1}{2}\right)$$
$$= -1$$

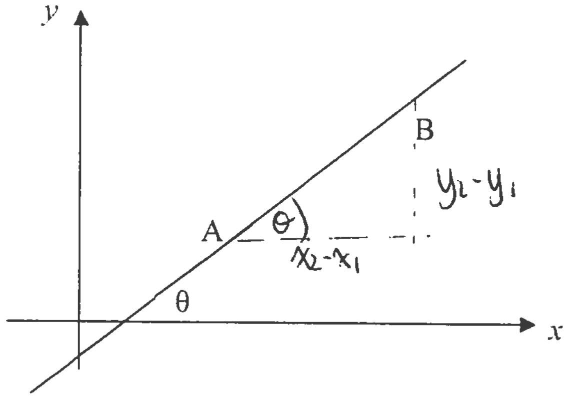
so lines are perpendicular.

Ex 13C, pg 298 - 299

AS 1.1 Applying Algebraic Skills To Rectilinear Shapes

Gradient = tanθ

For a line AB the gradient **m** = **tan θ** where θ is the angle the line makes with the positive direction of the x – axis measured anti-clockwise.



$\tan\theta = \frac{opp}{adj}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= m$.

Examples

1) Calculate the angle the line joining C(2,5) and D(4,7) makes with positive direction of the x-axis.

$m_{CD} = \frac{7-5}{4-2}$

$= \frac{2}{2}$

$= 1$

$m = \tan\theta$

$\tan\theta = 1$

$\theta = 45°$.

2) Calculate the angle the line joining P(-5,6) and Q(1,3) makes with the positive direction of the x-axis.

$m_{PQ} = \frac{3-6}{1-(-5)}$

$= \frac{-3}{6}$

$= \frac{-1}{2}$

$m = \tan\theta$

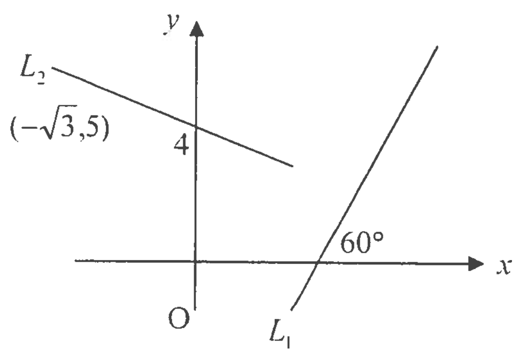
$\tan\theta = -\frac{1}{2}$

ra. 26.6°

$\theta = 180 - 26.6°$

$= 153.4°$

3) Will the lines shown meet at 90° ?



L_1 $m_1 = \tan\theta$

$= \tan 60$

$= \sqrt{3}$

L_2 $m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-5}{0+\sqrt{3}}$

$= -\frac{1}{\sqrt{3}}$

$m_1 m_2 = \sqrt{3} \times -\frac{1}{\sqrt{3}}$

$= -1$

Ex 13D, pg 300 - 301

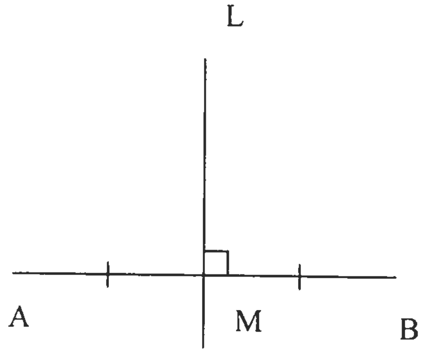
So L_1 and L_2 are perpendicular ie lines meet at 90°

AS 1.1 Applying Algebraic Skills To Rectilinear Shapes

Finding the Equations of Perpendicular Bisectors, Altitudes and Medians

Perpendicular Bisectors

A perpendicular bisector of a line AB is a line **perpendicular to AB and passing through its midpoint.**



If LM is a **perpendicular bisector** then AM = MB and AML = BML = 90°

Example
A(1,3) and B(5,-7). Find the equation of the perpendicular bisector of AB.

NB midpoint of $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

'average' co-ords.

* To find equation need to know a point, the gradient.

Point: midpoint of AB $M = \left(\frac{1+5}{2}, \frac{3+(-7)}{2}\right)$

$= (3, -2)$

Gradient → find m_{AB} first then perpendicular gradient

$m_{AB} = \frac{-7-3}{5-1}$

$= \frac{-10}{4}$

$= \frac{-5}{2}$ $\quad m_{perp} = \frac{2}{5}$

Old book Exercise 11

Equation $y - b = m(x-a)$

$y + 2 = \frac{2}{5}(x-3)$

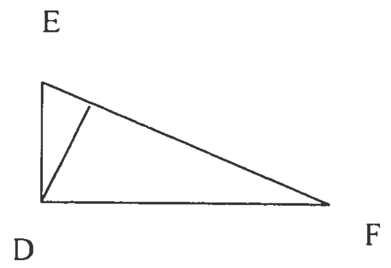
$5y + 10 = 2x - 6$

$5y = 2x - 16.$

AS 1.1 Applying Algebraic Skills To Rectilinear Shapes

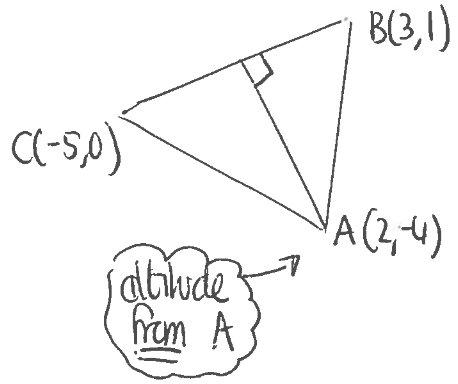
Altitudes of a Triangle

An altitude of a triangle is a straight line from a vertex and **perpendicular to the opposite side of the triangle.**



Example
Triangle ABC has vertices A(2,-4) B(3,1) C(-5,0).
Find the equation of the altitude from A.

Make a rough sketch.



Point $(2,-4)$

Gradient $m_{BC} = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{0-1}{-5-3}$

$= \frac{-1}{-8}$

$= \frac{1}{8}$

$m_{perp} = -8$

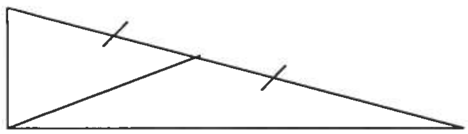
Equation

$y-b = m(x-a)$
$y-(-4) = -8(x-2)$
$y+4 = -8x+16$
$y+8x = 12.$

Old Book. p15 Ex1K

Medians of a Triangle

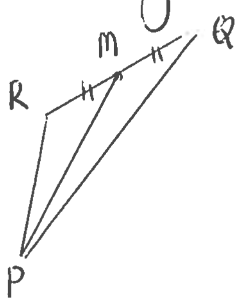
A median of a triangle is a straight line from a vertex to the midpoint of the opposite side.



Example

Triangle PQR has vertices P(3,-7) Q(8,1) R(4,-3).
Find the equation of the median from P.
Find the point of intersection of this median with the line x-y-5=0

Make a rough sketch.



Find midpoint first. **

Midpoint RQ

$$M = \left(\frac{4+8}{2}, \frac{-3+1}{2}\right)$$
$$= (6, -1)$$

Use co-ordinates to find gradient

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1-(-7)}{6-3}$$
$$= \frac{6}{3}$$
$$= 2$$

Equation

$$y - b = m(x - a)$$
$$y - (-1) = 2(x - 6)$$
$$y + 1 = 2x - 12$$
$$y = 2x - 13.$$

Point of intersection

To find where two lines meet / cross / intersect } find equations of lines and solve simultaneously

Solve $y = 2x - 13 \quad \ldots (1)$

$x - y - 5 = 0 \quad \ldots (2)$

Substitute (1) in (2) $\quad x - (2x - 13) - 5 = 0$

$-x + 13 - 5 = 0$

$x = 8$

In (1) $\quad y = 16 - 13$

$= 3$

point $(8, 3)$

Ex 13F, pg 304 - 305

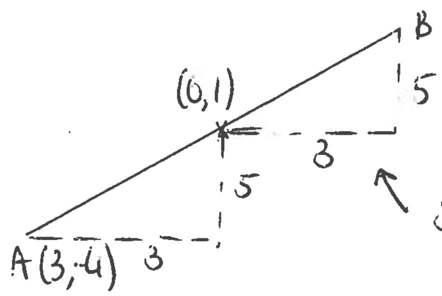
Note

In a triangle:

Orthocentre – where altitudes meet
Centroid – where medians meet
Circumcentre – where perpendicular bisectors meet.

Extra Example

A is the point (3 , -4). The midpoint of AB is (6 , 1). Find B.



same 'step' in x and y direction

B(9,6)