Differentiation

Reminder
Laws of Indices

We need to be able to use these rules to rewrite expressions so that they contain no $\sqrt{}$ signs, no x's on the bottom of fractions and no brackets.

- $a^m \times a^n =$ a^{m+n}
- $\dfrac{a^m}{a^n} =$ a^{m-n}
- $(a^m)^n =$ a^{mn}
- $(ab)^m =$ $a^m b^m$
- $a^{-m} =$ $\dfrac{1}{a^m}$
- $a^{m/n} =$ $\sqrt[n]{a^m}$
- $a^0 =$ 1
- $a^1 =$ a

eg ① $\sqrt{x} = x^{\frac{1}{2}}$ ② $\sqrt[3]{x^4} = x^{\frac{4}{3}}$

③ $\sqrt{y^5} = y^{\frac{5}{2}}$ ④ $\frac{1}{x^3} = x^{-3}$

⑤ $\frac{4}{y^7} = 4y^{-7}$ ⑥ $\frac{1}{5x^2} = \frac{1}{5}x^{-2}$

⑦ $\frac{2}{3\sqrt{x}} = \frac{2}{3}x^{-\frac{1}{2}}$ ⑧ $\frac{3}{4x^2} + 5x = \frac{3}{4}x^{-2} + 5x$

⑨ $\frac{x^3+4x-1}{x^2} = \frac{x^3}{x^2} + \frac{4x}{x^2} - \frac{1}{x^2} = x + 4x^{-1} - x^{-2}$

uls.

For a function f (x), the derived function or derivative of f (x) is denoted by f ′(x) or $\dfrac{dy}{dx}$ and given as

$$\mathbf{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Rules for Differentiation

"Multiply by the power and reduce the power by 1"

* If $\mathbf{f(x) = x^n}$ then $\mathbf{f'(x) = nx^{n-1}}$

take one off the power

power goes to front

* If $\mathbf{f(x) = ax^n}$ then $\mathbf{f'(x) = nax^{n-1}}$

* If $\mathbf{f(x) = g(x) + h(x)}$ then $\mathbf{f'(x) = g'(x) + h'(x)}$

AS 1.3 Applying Calculus Skills of Differentiation

Note

- f(x) or y needs to be in the form of a sum of powers of x before we can differentiate it.
- return the derivatives to a sensible form
- derivative of a constant = 0

Examples

1) $f(x) = x^3$

$f'(x) = 3x^2$

2) $y = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$

$= -\frac{1}{2x^{\frac{3}{2}}}$

answers should be left with positive powers of x

3) $g(x) = -\frac{3}{4}x^6$

$g'(x) = 6 \times -\frac{3}{4}x^5$

$= -\frac{9}{2}x^5$

4) $k(x) = 3x$

$k'(x) = 3$

5) $m(x) = 4$

$m'(x) = 0$

6) $f(x) = \frac{3}{4}x$

$f'(x) = \frac{3}{4}$

7) $y = -x^{-\frac{1}{2}}$

$\frac{dy}{dx} = +\frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2x^{\frac{3}{2}}}$

8) differentiate $-5x^{-\frac{4}{3}}$

$y = -5x^{-\frac{4}{3}}$

$\frac{dy}{dx} = \frac{20}{3}x^{-\frac{7}{3}}$

$= \frac{20}{3x^{\frac{7}{3}}}$

p. 210 Ex 9A

Examples (cont)

9) $f(x) = x^4 - 3x^{-2} + 9x - 6$

$f'(x) = 4x^3 + 6x^{-3} + 9$

$= 4x^3 + \frac{6}{x^3} + 9$

10) $g(x) = 3x^2 - \frac{x}{5} - \frac{3}{2}x^{-5}$

$g'(x) = 6x - \frac{1}{5} + \frac{15}{2}x^{-6}$

$= 6x - \frac{1}{5} + \frac{15}{2x^6}$

continue p. 210 Ex 9A

Remember :-

Deal with brackets and fractions **before** attempting to differentiate.

Examples (cont)

11) $f(x) = (x^2 + 2)(x - 4)$

$f(x) = x^3 - 4x^2 + 2x - 8$

$f'(x) = 3x^2 - 8x + 2$

12) $y = \frac{1}{x^2}$

$y = x^{-2}$

$\frac{dy}{dx} = -2x^{-3}$

$= -\frac{2}{x^3}$

AS 1.3 Applying Calculus Skills of Differentiation

13) $h(x) = \sqrt{x}$

$= x^{\frac{1}{2}}$

$h'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2x^{\frac{1}{2}}}$

14) $h(x) = \sqrt{x}\,(x^3 + \sqrt{x})$

$= x^{\frac{1}{2}}(x^3 + x^{\frac{1}{2}})$

$= x^{\frac{7}{2}} + x$

$h'(x) = \frac{7}{2}x^{\frac{5}{2}} + 1$

15) $g(x) = \dfrac{x^2 - 2x + 1}{\sqrt{x}}$

$= \frac{x^2}{x^{\frac{1}{2}}} - \frac{2x}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}$

$= x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$g'(x) = \frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}} - \frac{1}{2x^{\frac{3}{2}}}$

16) $h(x) = \sqrt[3]{x^2} - \dfrac{2}{3\sqrt{x}}$

$h(x) = x^{\frac{2}{3}} - \frac{2}{3x^{\frac{1}{2}}}$

$= x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{2}}$

$h'(x) = \frac{2}{3}x^{-\frac{1}{3}} + \frac{1}{3}x^{-\frac{3}{2}}$

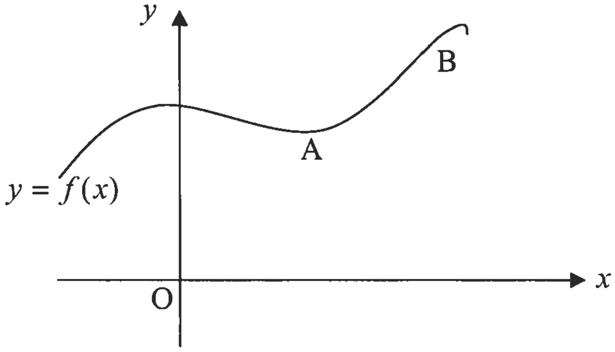
$= \frac{2}{3x^{\frac{1}{3}}} + \frac{1}{3x^{\frac{3}{2}}}$

p. 215 Ex 9B

AS 1.3 Applying Calculus Skills of Differentiation

APPLICATIONS OF DIFFERENTIATION

Consider the graph of a function f (x)



Suppose we wish to know the gradient of the tangent to the curve at point A (x, f(x)). We look at a point close by, namely B (x+h, f(x+h)). As we move the point B closer to the point A the gradient of the line AB becomes the gradient of the tangent at point A.

$$m_{AB} = \frac{f(x+h)-f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

So the gradient of the tangent at A is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f'(x)$$

Conclusion

The **derivative of a function** represents

- The **rate of change** of the function at x
- The **gradient of the tangent** to the graph of the function at x

Unit 2 AS 1.3 Applying Calculus Skills of Differentiation

Examples

1) Find the gradient of the curve $y = 6x - 2x^3$ at the point when $x = 3$

differentiate

$\frac{dy}{dx} = 6 - 6x^2.$

When $x = 3$ $\quad \frac{dy}{dx} = 6 - 6 \times 3^2$

$= -48$

gradient is -48 when $x = 3$.

2) A curve has equation $y = 3x(x^3 - 2)$. Find $\frac{dy}{dx}$ when $x = -1$

$y = 3x^4 - 6x$

$\frac{dy}{dx} = 12x^3 - 6$

When $x = -1$ $\quad \frac{dy}{dx} = 12(-1)^3 - 6$

$= -18$

3) Find the gradient of the tangent to the curve $y = 3x^2 - \frac{1}{x}$ at $x = -1$

differentiate

$y = 3x^2 - x^{-1}$

$\frac{dy}{dx} = 6x + x^{-2}$

$= 6x + \frac{1}{x^2}.$

When $x = -1$ $\quad \frac{dy}{dx} = -6 + 1$

$= -5$

4. The pulse rate in beats per minute of a runner t minutes after starting to run is given by

$$p(t) = 60 + \frac{1}{2}t^2 - t \qquad \text{for } t < 10$$

Find the rate of change of the pulse of the runner 6 minutes after starting.

differentiate

$p'(t) = t - 1$
$p'(6) = 6 - 1$
$= 5$ beats per minute.

5. A ball is thrown vertically upwards. The height (in metres) of the ball above the ground, t seconds after it is thrown, can be represented by the formula

$$h(t) = 30t - 6t^2.$$

The velocity, v ms^{-1}, of the ball at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the cricket ball one second after it is thrown.

$v = \frac{dh}{dt} = 30 - 12t^2$

When $t = 1$ $\quad \frac{dh}{dt} = 30 - 12 \times 1^2$

$v = 18$ m/s.

p. 222 Ex 9D

AS 1.3 Applying Calculus Skills of Differentiation

The Derivatives of sinx and cosx

$y = \sin x \Rightarrow \dfrac{dy}{dx} = \cos x$ or $f(x) = \sin x \Rightarrow f'(x) = \cos x$ or $\dfrac{d(\sin x)}{dx} = \cos x$

$y = \cos x \Rightarrow \dfrac{dy}{dx} = -\sin x$ or $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ or $\dfrac{d(\cos x)}{dx} = -\sin x$

Note :- **Any calculus involving trigonometric functions** MUST **be done in radians.**

Examples :- **Determine the derivatives of :-**

1) 5 sin x

$y = 5\sin x$

$\dfrac{dy}{dx} = 5\cos x$

2) -4 cos x

$y = -4\cos x$

$\dfrac{dy}{dx} = 4\sin x$

3) cos x – sin x

$y = \cos x - \sin x$

$\dfrac{dy}{dx} = -\sin x - \cos x$

4) $y = \dfrac{1}{2}x^2 - \cos x$

$\dfrac{dy}{dx} = x + \sin x$

5) $h(x) = \dfrac{5}{\sqrt{x}} - 4\sin x + \dfrac{1}{\sqrt[3]{x}}$

$h(x) = \dfrac{5}{x^{\frac{1}{2}}} - 4\sin x + \dfrac{1}{x^{\frac{1}{3}}}$

$h(x) = 5x^{-\frac{1}{2}} - 4\sin x + x^{-\frac{1}{3}}$

$h'(x) = -\dfrac{5}{2}x^{-\frac{3}{2}} - 4\cos x - \dfrac{1}{3}x^{-\frac{4}{3}}$

$= \dfrac{-5}{2x^{\frac{3}{2}}} - 4\cos x - \dfrac{1}{3x^{\frac{4}{3}}}$

p. 227 Ex 9E Q1

AS 1.3 Applying Calculus Skills of Differentiation

Further application

6) Find the gradient of the tangent to the curve $f(x) = -4\cos x$ when $x = \frac{4\pi}{3}$

↑ differentiate.

$f(x) = -4\cos x$

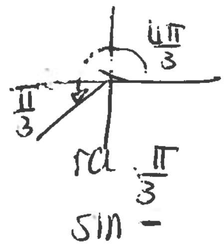
$f'(x) = 4\sin x$

When $x = \frac{4\pi}{3}$ $f'\left(\frac{4\pi}{3}\right) = 4\sin\frac{4\pi}{3}$

$= -4\sin\left(\frac{\pi}{3}\right)$

$= -4 \cdot \frac{\sqrt{3}}{2}$

$= -2\sqrt{3}$



7) For $f(x) = 2\sin x$, find $f'\left(\frac{\pi}{3}\right)$

$f'(x) = 2\cos x$

$f'\left(\frac{\pi}{3}\right) = 2\cos\frac{\pi}{3}$

$= 2 \cdot \frac{1}{2}$

$= 1$

8) $f(x) = \dfrac{2x - 3\sqrt{x}\sin x}{\sqrt{x}}$ find $f'(x)$

$f(x) = \dfrac{2x - 3x^{\frac{1}{2}}\sin x}{x^{\frac{1}{2}}}$

$= \dfrac{2x}{x^{\frac{1}{2}}} - \dfrac{3x^{\frac{1}{2}}\sin x}{x^{\frac{1}{2}}}$

$= 2x^{\frac{1}{2}} - 3\sin x$

$f'(x) = x^{-\frac{1}{2}} - 3\cos x$

$= \dfrac{1}{x^{\frac{1}{2}}} - 3\cos x.$

p. 227 Ex 9E

AS 1.3 Applying Calculus Skills of Differentiation

The "Chain" Rule for Differentiation of composite functions

The chain rule is used to find **derivatives** of functions within other functions.

You may do this by making a substitution, or remembering to **differentiate the bracket, then multiply the result by the derivative of the contents of the bracket.**

i.e for

$$h(x) = f(g(x))$$

$$h'(x) = f'(g(x)).\ g'(x)$$

derivative of bracket = derivative of whole thing × derivative of bit in brackets.

If substituting (using *u*), then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Examples :- **Differentiate the following functions**

1. $y = (3x^2 + 6)^3$

$\frac{dy}{dx} = 3(3x^2+6)^2 \times 6x$

derivative of whole thing × derivative of bit in brackets.

$= 18x(3x^2+6)^2$

2. $g(x) = 6(2x - 7)^{-2}$

$g'(x) = -12(2x-7)^{-3} \times 2$

$= -24(2x-7)^{-3}$

$= -\frac{24}{(2x-7)^3}$

3. $y = \sqrt{(2x^5 + 3x)}$

$y = (2x^5+3x)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x^5+3x)^{-\frac{1}{2}} . 10x^4$

$= 5x^4(2x^5+3x)^{-\frac{1}{2}}$

$= \frac{5x^4}{(2x^5+3x)^{+\frac{1}{2}}}$.

4) $\frac{1}{(2x-1)^4}$

$y = \frac{1}{(2x-1)^4}$

$= (2x-1)^{-4}$

$\frac{dy}{dx} = -4(2x-1)^{-5} \times 2$

$= \frac{-8}{(2x-1)^5}$

5) $y = \frac{2}{\sqrt{3x^2 - 4}}$

$y = \frac{2}{(3x^2-4)^{\frac{1}{2}}}$

$= 2(3x^2-4)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -(3x^2-4)^{-\frac{3}{2}} . 6x$

$= -\frac{6x}{(3x^2-4)^{\frac{3}{2}}}$

p. 232 Ex 9G

Application

6) Find the gradient of the tangent to the curve y = $(4x + 3)^6$ when x = -1

differentiate

$$\frac{dy}{dx} = 6(4x+3)^5 \times 4$$
$$= 24(4x+3)^5$$

When $x = -1$ $\quad \frac{dy}{dx} = 24(4\times(-1)+3)^5$
$$= 24(-1)^5$$
$$= -24$$

p. 234 Ex 9G Q. 7 onwards

The chain rule and trigonometry
Examples: differentiate

1) $y = \sin^2 x$ 2) $y = 4\cos 6x$ 3) $y = 2\cos^3 x$

① $y = \sin^2 x$
$= (\sin x)^2$ ← write with brackets

$$\frac{dy}{dx} = 2(\sin x)^1 \times \cos x$$
$$= 2\sin x \cos x$$

② $y = 4\cos(6x)$ ← put in brackets

$$\frac{dy}{dx} = -4\sin(6x) \times 6$$
$$= -24\sin 6x$$

③ $y = 2\cos^3 x$
$= 2(\cos x)^3$

$$\frac{dy}{dx} = 6(\cos x)^2 \times (-\sin x)$$
$$= -6\sin x \cos^2 x$$

4) $f(x) = \sin(3 - x)$

5) Find the gradient of the curve $y = 3\cos 3x$ at the point where $x = \frac{\pi}{3}$

④ $f(x) = \sin(3-x)$
$f'(x) = \cos(3-x) \times (-1)$
$= -\cos(3-x)$

⑤ $y = 3\cos(3x)$
$\frac{dy}{dx} = -3\sin(3x) \times 3$
$= -9\sin 3x$

When $x = \frac{\pi}{3}$ $\quad \frac{dy}{dx} = -9\sin 3\left(\frac{\pi}{3}\right)$
$= -9\sin\pi$
$= 0.$

p. 235 Ex 9H

Using Differentiation

Determining the equation of a tangent to a curve by differentiation

(you have already done questions involving finding the gradient of tangents)

Reminder: straight lines can be modelled using gradient ***m*** and any point ***(a,b)*** by

$$\mathbf{y - b = m(x - a)}$$

1) Find the equation of the tangent to the curve $y = x^2 - 4x + 7$ at the point (3,4)

$x=3$

gradient $\frac{dy}{dx} = 2x-4$

When $x=3$ $\frac{dy}{dx} = 6-4$

$= 2.$

Equation $y-b = m(x-a)$ $m=2$

$(a,b) = (3,4)$

$y-4 = 2(x-3)$

$y-4 = 2x-6$

$y = 2x-2.$

2) The gradient of a tangent to the curve $\mathbf{y = x^3 - 2x^2 + 7}$ is **4** at point P. Find the equation of the tangent at P. with $x \geq 0$

$\frac{dy}{dx} = 3x^2 - 4x$

Gradient is 4 so $3x^2 - 4x = 4$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$x = -\frac{2}{3}$ or $x = 2.$

$x = \frac{2}{3}$ $y = (\frac{-2}{3})^3 - 2(\frac{-2}{3})^2 + 7$

$= \frac{157}{27}.$

Equation $y - b = m(x-a)$

$y - \frac{157}{27} = 4(x - \frac{2}{3})$

$27y - 157 = 108x - 72$

$27y = 108x + 85.$

or with $x = 2$ $y = 8-8+7 = 7$

Equation $y-b = m(x-a)$

$y-7 = 4(x-2)$

$y-7 = 4x-8$

$y = 4x-1$

3) Find the equation of the tangent to the curve y = 6 sinx at the point where $x = \frac{\pi}{6}$

gradient $\frac{dy}{dx} = 6\cos x$

when $x = \frac{\pi}{6}$ $\frac{dy}{dx} = 6\cos\frac{\pi}{6}$

$= 6\frac{\sqrt{3}}{2}$

$= 3\sqrt{3}$

Point
When $x = \frac{\pi}{6}$ $y = 6\sin\frac{\pi}{6}$

$= 6 \times \frac{1}{2}$

$= 3.$

Equation
$y - b = m(x - a)$

$y - 3 = 3\sqrt{3}\left(x - \frac{\pi}{6}\right)$

$y - 3 = 3\sqrt{3}x - \frac{\sqrt{3}\pi}{2}$

$y = 3\sqrt{3}x + 3 - \frac{\sqrt{3}\pi}{2}.$

p. 240 Ex 10A

<u>Increasing and Decreasing Functions and Stationary Points</u>

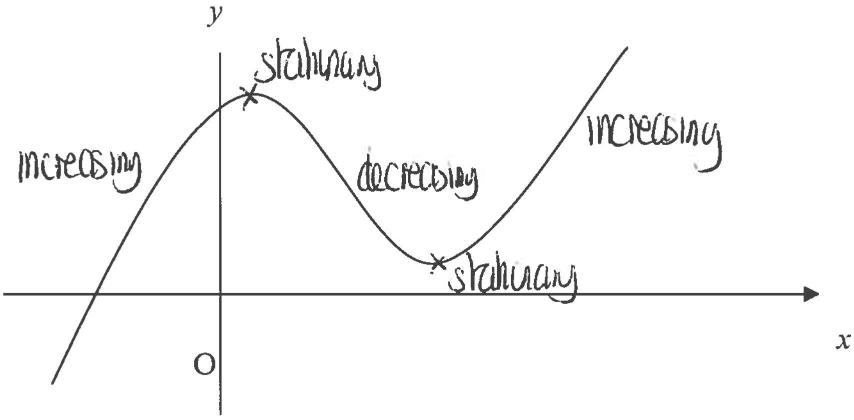
For any curve y = f (x) , at point x

If $\frac{dy}{dx}$ **> 0** (tangent has +ve gradient) the function f (x) is increasing

If $\frac{dy}{dx}$ **< 0** (tangent has –ve gradient) the function f (x) is decreasing

If $\frac{dy}{dx}$ **= 0** (tangent is horizontal) the function f (x) has a stationary point

AS 1.3 Applying Calculus Skills of Differentiation



Example

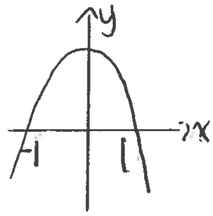
1) For the function $f(x) = 3x - x^3$, find where it is
a) stationary
b) increasing
c) decreasing

$f(x) = 3x - x^3$
$f'(x) = 3 - 3x^2$

(a) stationary $f'(x) = 0$
$3 - 3x^2 = 0$
$x = \pm 1$

(b) Increasing $f'(x) > 0$
$3 - x^2 > 0$
$-1 < x < 1$

Quadratic inequation
→ GRAPH



(c) decreasing $f'(x) < 0$
$x < -1$ or $x > 1$

2) Show that the function **f(x) = x³ -3x² +3x -10** is never decreasing.

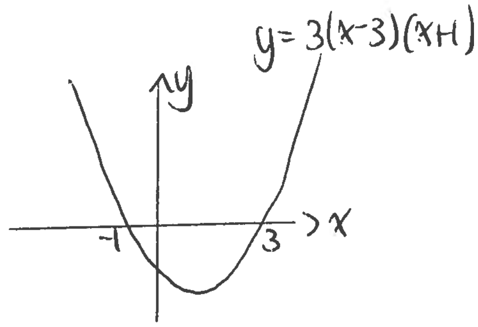
$f'(x) = 3x^2 - 6x + 3$
$= 3(x^2 - 2x + 1)$
$= 3(x-1)^2$
≥ 0 so $f(x)$ is never decreasing

AS 1.3 Applying Calculus Skills of Differentiation

3) For $g(x) = x^3 - 3x^2 - 9x + 4$, find the range of values for which g(x) is decreasing.

$$g'(x) = 3x^2 - 6x - 9$$
$$= 3(x^2 - 2x - 3)$$
$$= 3(x-3)(x+1)$$

$$g'(x) < 0$$
$$\Rightarrow \quad 3(x-3)(x+1) < 0$$
$$-1 < x < 3$$



p. 250 Ex 10B

AS 1.3 Applying Calculus Skills of Differentiation

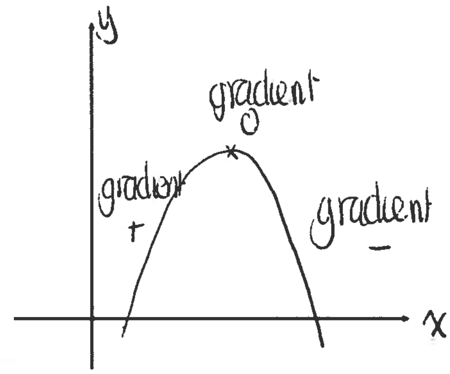
Stationary Points and Their Nature

Stationary Points occur when $\frac{dy}{dx} = 0$ (or $f'(x) = 0$)

The nature of stationary points depends on what is happening either side of them.

Nature of Stationary Points

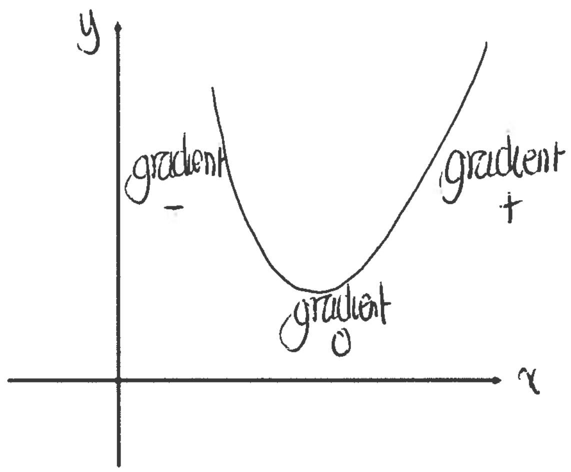
Maximum Turning Point



$\frac{dy}{dx} > 0$ before

$\frac{dy}{dx} < 0$ after

Minimum Turning Point

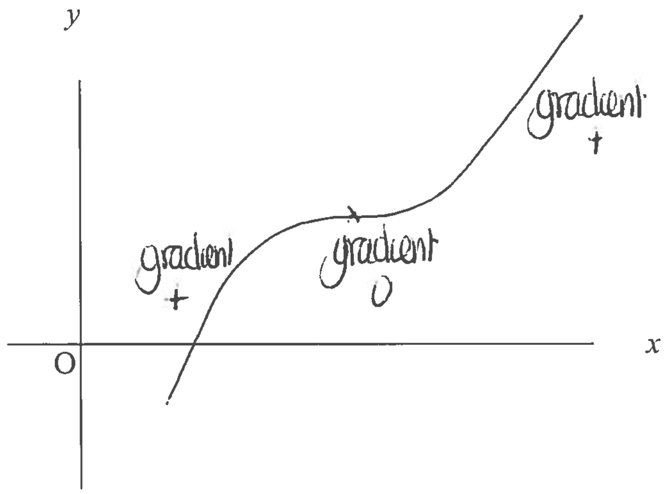


$\frac{dy}{dx} < 0$ before

$\frac{dy}{dx} > 0$ after

AS 1.3 Applying Calculus Skills of Differentiation

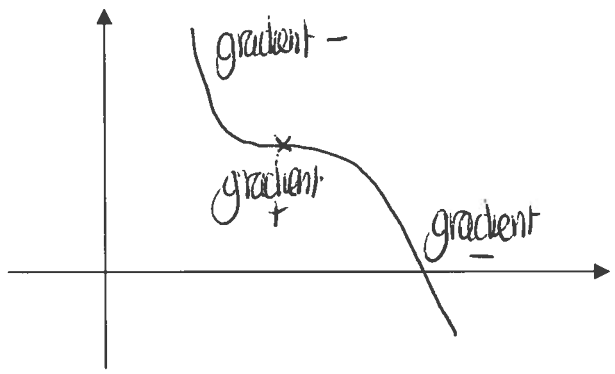
Rising Point of Inflection



$\frac{dy}{dx} > 0$ before

$\frac{dy}{dx} > 0$ after

Falling Point of Inflection



$\frac{dy}{dx} < 0$ before

$\frac{dy}{dx} < 0$ after

Examples

1) Find the co-ordinates of the stationary points on the curve **$y = 2x^3 - 9x^2 + 12x$** and determine their nature.

$y = 2x^3 - 9x^2 + 12x$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

$= 6(x^2 - 3x + 2)$

$= 6(x-2)(x-1)$ * factorise if possible

Stationary points $\frac{dy}{dx} = 0$

$6(x-2)(x-1) = 0$

$x = 2$ or $x = 1$

y co-ords $x = 2$ $y = 2\times8 - 9\times4 + 24$

$= 4$ $(2,4)$

$x = 1$ $y = 2 - 9 + 12$

$= 5$ $(1,5)$

Nature

x	$\xrightarrow{0}$	1	$\xrightarrow{1.5}$	2	$\xrightarrow{3}$
$\frac{dy}{dx} = 6(x-2)(x-1)$	12	0	-1.5	0	12.
	/	—	\	—	/

Conclusion $(1,5)$ is a max TP

$(2,4)$ is a min TP.

2) Find the stationary points on the curve $y = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + 3$ and determine their nature.

$y = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + 3.$

$\frac{dy}{dx} = x^3 - 4x^2 + 4x$

$= x(x^2 - 4x + 4)$

$= x(x-2)^2$

For stationary points $\frac{dy}{dx} = 0$

$x(x-2)^2 = 0$

$x = 0$ or $x = 2$

y co-ords

When $x = 0$ $y = 3$ $(0,3)$

$x = 2$ $y = \frac{1}{4} \times 2^4 - \frac{4}{3} \times 2 + 2 \times 2^2 + 3$

$= \frac{13}{3}$ $(2, \frac{13}{3})$

Nature

x	$\xrightarrow{(-1)}$	0	$\xrightarrow{(1)}$	2	$\xrightarrow{(3)}$
$\frac{dy}{dx} = x(x-2)^2$	-9	0	1	0	3
shape	\	—	/	—	/

$(0,3)$ is a minimum turning point

$(2, \frac{13}{3})$ is a point of inflection.

p. 256 Ex 10C

Curve Sketching

To sketch the graph of a function we need to know

- where it crosses the axes
- stationary points and nature
- what happens as x → + ∞ and as x →- ∞

Example

Sketch the curve $y = 8x^3 - 3x^4$

cuts x-axis

$y = 0$

$8x^3 - 3x^4 = 0$

$x^3(8-3x) = 0$

$x = 0 \quad x = \frac{8}{3}$ $\qquad (0,0) \quad (\frac{8}{3},0)$

cuts y-axis

$x = 0$

$y = 0$ $\qquad (0,0)$

stationary points

$\frac{dy}{dx} = 24x^2 - 12x^3$

$= 12x^2(2-x)$

For stationary points $\quad 12x^2(2-x) = 0$

$x = 0$ or $x = 2$.

y co-ords $\quad x = 0 \Rightarrow y = 0 \qquad (0,0)$

$x = 2 \Rightarrow y = 8 \times 2^3 - 3 \times 2^4$

$= 64 - 48$

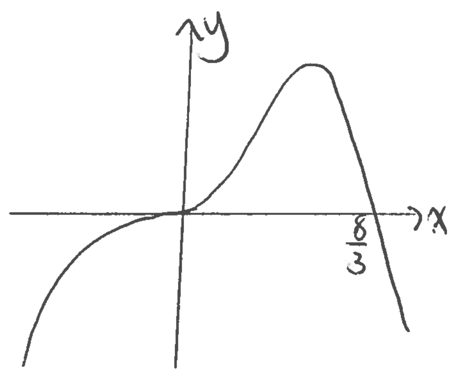
$= 16 \qquad (2,16)$

Nature

x	$\xrightarrow{(-1)}$	0	$\xrightarrow{(1)}$	2	$\xrightarrow{(3)}$
$\frac{dy}{dx} = 12x^2(2-x)$	36	0	12	0	-108
	/	—	/	—	\

(0,0) is a rising point of inflection

(2,16) is a max T.P



* Graph of derived function

→ see unit ① notes *

p. 262 Ex 10D