

Chapter 1 - Working With Surds

Ex 1A

1. a) $\sqrt{8}$ surd

b) $\sqrt{81} = 9$

c) $\sqrt[3]{27} = 3$

d) $\sqrt{5}$ surd

e) $\sqrt[3]{1} = 1$

f) $\sqrt{900} = 30$

g) $\sqrt{2.5}$ surd

h) $\sqrt{0.25} = 0.5$

i) $\sqrt[3]{52}$ surd

j) $\sqrt{0.04} = 0.2$

k) $\sqrt{63}$ surd

l) $\sqrt{10}$ surd

2. a) $3\sqrt{5} + 7\sqrt{5}$
 $= 10\sqrt{5}$

b) $6\sqrt{2} - 5\sqrt{2}$
 $= \sqrt{2}$

c) $9\sqrt{7} - 4\sqrt{7}$
 $= 5\sqrt{7}$

d) $\sqrt{3} + 8\sqrt{3}$
 $= 9\sqrt{3}$

e) $3\sqrt{11} - 5\sqrt{11}$
 $= -2\sqrt{11}$

f) $\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$
 $= 4\sqrt{3} - 4\sqrt{2}$

g) $16\sqrt{5} - 3\sqrt{10} - 7\sqrt{5}$
 $= 9\sqrt{5} - 3\sqrt{10}$

h) $4\sqrt{3} + \sqrt{3} - 6\sqrt{3}$
 $= -\sqrt{3}$

i) $5\sqrt{2} + 3\sqrt{3} - 3\sqrt{2} + 8\sqrt{3}$
 $= 2\sqrt{2} + 11\sqrt{3}$

3. a) $\sqrt{24}$
 $= \sqrt{4}\sqrt{6}$
 $= 2\sqrt{6}$

b) $\sqrt{500}$
 $= \sqrt{100}\sqrt{5}$
 $= 10\sqrt{5}$

c) $\sqrt{32}$
 $= \sqrt{16}\sqrt{2}$
 $= 4\sqrt{2}$

d) $\sqrt{75}$
 $= \sqrt{25}\sqrt{3}$
 $= 5\sqrt{3}$

e) $\sqrt{1000}$
 $= \sqrt{100}\sqrt{10}$
 $= 10\sqrt{10}$

f) $3\sqrt{8}$
 $= 3\sqrt{4}\sqrt{2}$
 $= 3 \times 2\sqrt{2}$
 $= 6\sqrt{2}$

g) $6\sqrt{12}$
 $= 6\sqrt{4}\sqrt{3}$
 $= 6 \times 2\sqrt{3}$
 $= 12\sqrt{3}$

h) $5\sqrt{50}$
 $= 5\sqrt{25}\sqrt{2}$
 $= 5 \times 5\sqrt{2}$
 $= 25\sqrt{2}$

a) $5\sqrt{2} + \sqrt{12}$
 $= 5\sqrt{2} + \sqrt{4}\sqrt{3}$
 $= 5\sqrt{2} + 2\sqrt{3}$

b) $\sqrt{50} - 6\sqrt{2}$
 $= \sqrt{25}\sqrt{2} - 6\sqrt{2}$
 $= 5\sqrt{2} - 6\sqrt{2}$
 $= -\sqrt{2}$

c) $3\sqrt{7} + \sqrt{48}$
 $= 3\sqrt{7} + \sqrt{16}\sqrt{3}$
 $= 3\sqrt{7} + 4\sqrt{3}$

d) $\sqrt{27} - 4\sqrt{3}$
 $= \sqrt{9}\sqrt{3} - 4\sqrt{3}$
 $= 3\sqrt{3} - 4\sqrt{3}$
 $= -\sqrt{3}$

e) $\sqrt{125} + 3\sqrt{5}$
 $= \sqrt{25}\sqrt{5} + 3\sqrt{5}$
 $= 5\sqrt{5} + 3\sqrt{5}$
 $= 8\sqrt{5}$

f) $\sqrt{112} - \sqrt{28}$
 $= \sqrt{16}\sqrt{7} - \sqrt{4}\sqrt{7}$
 $= 4\sqrt{7} - 2\sqrt{7}$
 $= 2\sqrt{7}$

g) $\sqrt{8} - 3\sqrt{32}$
 $= \sqrt{4}\sqrt{2} - 3\sqrt{16}\sqrt{2}$
 $= 2\sqrt{2} - 3 \times 4\sqrt{2}$
 $= 2\sqrt{2} - 12\sqrt{2}$
 $= -10\sqrt{2}$

h) $3\sqrt{48} + 2\sqrt{75}$
 $= 3\sqrt{16}\sqrt{3} + 2\sqrt{25}\sqrt{3}$
 $= 3 \times 4\sqrt{3} + 2 \times 5\sqrt{3}$
 $= 12\sqrt{3} + 10\sqrt{3}$
 $= 22\sqrt{3}$

i) $6\sqrt{4} - 4\sqrt{9}$
 $= 6 \times 2 - 4 \times 3$
 $= 12 - 12$
 $= 0$

Ex 1B

1. a) $\sqrt{3} \times \sqrt{2}$
 $= \sqrt{6}$

b) $\sqrt{5} \times \sqrt{5}$
 $= 5$

c) $\sqrt{16} \times \sqrt{9}$
 $= 4 \times 3$
 $= 12$

d) $\sqrt{6} \times \sqrt{3}$
 $= \sqrt{18}$
 $= \sqrt{9} \sqrt{2}$
 $= 3\sqrt{2}$

e) $\sqrt{10} \times \sqrt{40}$
 $= \sqrt{400}$
 $= 20$

f) $3\sqrt{18} \times 4\sqrt{2}$
 $= 12\sqrt{36}$
 $= 12 \times 6$
 $= 72$

g) $5\sqrt{6} \times 3\sqrt{2}$
 $= 15\sqrt{12}$
 $= 15\sqrt{4}\sqrt{3}$
 $= 15 \times 2\sqrt{3}$
 $= 30\sqrt{3}$

h) $4\sqrt{32} \times \sqrt{2}$
 $= 4\sqrt{64}$
 $= 4 \times 8$
 $= 32$

i) $7\sqrt{5} \times 7\sqrt{5}$
 $= 49 \times 5$
 $= 245$

2. a) $\sqrt{8} \div \sqrt{2}$
 $= \sqrt{4}$
 $= 2$

b) $\sqrt{32} \times \sqrt{\frac{9}{16}}$
 $= \sqrt{18}$
 $= \sqrt{9}\sqrt{2}$
 $= 3\sqrt{2}$

c) $\frac{\sqrt{30}}{\sqrt{10}}$
 $= \sqrt{3}$

d) $\sqrt{5} \div \sqrt{5}$
 $= 1$

e) $\sqrt{48} \div \sqrt{3}$
 $= \sqrt{16}$
 $= 4$

f) $\frac{10\sqrt{50}}{2\sqrt{5}}$
 $= 5\sqrt{10}$

g) $\frac{6\sqrt{28}}{3\sqrt{7}}$
 $= 2\sqrt{4}$
 $= 2 \times 2$
 $= 4$

h) $16\sqrt{20} \div 2\sqrt{2}$
 $= 8\sqrt{10}$

i) $9\sqrt{7} \div 3\sqrt{7}$
 $= 3$

3. a) $8\sqrt{5} \times 2\sqrt{6} \div 4\sqrt{10}$
 $= 4\sqrt{3}$

b) $12\sqrt{21} \div 2\sqrt{3} \times 3\sqrt{2}$
 $= 18\sqrt{14}$

c) $4\sqrt{15} \div 2\sqrt{5} \times 3\sqrt{3}$
 $= 6\sqrt{9}$
 $= 6 \times 3$
 $= 18$

d) $\frac{10\sqrt{2} \times 3\sqrt{8}}{5\sqrt{2}}$
 $= 2\sqrt{2} \times 3\sqrt{8}$
 $= 6\sqrt{16}$
 $= 6 \times 4$
 $= 24$

e) $\left(\frac{2}{\sqrt{3}}\right)^2$
 $= \frac{4}{3}$

f) $\left(\frac{\sqrt{7}}{5}\right)^2$
 $= \frac{7}{25}$

1. a) $\sqrt{125} = \sqrt{25} \sqrt{5} = 5\sqrt{5}$ b) $\sqrt{54} = \sqrt{9} \sqrt{6} = 3\sqrt{6}$ c) $\sqrt{288} = \sqrt{144} \sqrt{2} = 12\sqrt{2}$ d) $6\sqrt{3} + \sqrt{27} = 6\sqrt{3} + \sqrt{9} \sqrt{3} = 6\sqrt{3} + 3\sqrt{3} = 9\sqrt{3}$

e) $10\sqrt{7} - \sqrt{98} = 10\sqrt{7} - \sqrt{49} \sqrt{2} = 10\sqrt{7} - 7\sqrt{2}$ f) $\sqrt{7} \times \sqrt{8} = \sqrt{56} = \sqrt{4} \sqrt{14} = 2\sqrt{14}$ g) $\sqrt{20} \times \sqrt{10} = \sqrt{200} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$ h) $\sqrt{56} \div \sqrt{8} = \sqrt{7}$

i) $\frac{\sqrt{54}}{\sqrt{18}} = \sqrt{3}$ j) $3\sqrt{6} \times 5\sqrt{2} \times 4\sqrt{3} = 60\sqrt{36} = 60 \times 6 = 360$ k) $7\sqrt{6} \times 6\sqrt{2} \div 2\sqrt{8} = 21\sqrt{9} = 21 \times 3 = 63$

3. a) $\sqrt{6} \times \sqrt{x} = \sqrt{30}$ b) $3\sqrt{x} \times \sqrt{10} = 30$
 $x=5$ $x=10$

c) $4\sqrt{x} \times \sqrt{x} = 20$ d) $6\sqrt{x} \times 2\sqrt{x} \div 3\sqrt{x} = \sqrt{32}$
 $x=5$ $4\sqrt{x} = \sqrt{16} \sqrt{2}$
 $4\sqrt{x} = 4\sqrt{2}$
 $x=2$

Ex 1C

1. a) i) $1000 \div \sqrt{7} = 1000 \div 2.64575 = 377.964473 = 378.0$ (to 1 dp)
ii) $1000 \div 3 = 333.3$ (to 1 dp)

b) i) $1000 \div \sqrt{7} = 1000 \div 2.64575 = 377.964473 = 378.0$ (to 1 dp)
ii) $1000 \div 2.6 = 384.6$ (to 1 dp)

c) $1000 \div \sqrt{7} = 1000 \div 2.64575 = 377.964473 = 377.4$ (to 1 dp)

b) large variation in final answer. Best to leave rounding to end of calculation

a) i) $\sqrt{77} = 8.8$ ii) $\sqrt{3} = 1.7$ iii) $8.8 \div 1.7 = 5.2$ (to 1 dp)

c) $\sqrt{77} \div \sqrt{3} = 5.1$ (to 1 dp)

d) rounding best left to end of calculation

$$3. a) A = lb \\ = 3 \times \sqrt{5} \\ = 3\sqrt{5} \text{ cm}^2$$

$$b) A = lb \\ = \sqrt{2} \times \sqrt{18} \\ = \sqrt{36} \\ = 6 \text{ mm}^2$$

$$c) A = lb \\ = \sqrt{24} \times \sqrt{3} \\ = \sqrt{72} \\ = \sqrt{36 \times 2} \\ = 6\sqrt{2} \text{ m}^2$$

$$d) A = lb \\ = 2\sqrt{50} \times 3\sqrt{5} \\ = 6\sqrt{250} \\ = 6\sqrt{25 \times 10} \\ = 6 \times 5\sqrt{10} \\ = 30\sqrt{10} \text{ cm}^2$$

$$1. a) A = lb \\ = \sqrt{7} \times \sqrt{7} \\ = 7 \text{ cm}^2$$

$$b) A = lb \\ = 2\sqrt{5} \times 2\sqrt{5} \\ = 4 \times 5 \\ = 20 \text{ m}^2$$

$$c) A = lb \\ = 4\sqrt{11} \times 4\sqrt{11} \\ = 16 \times 11 \\ = 176 \text{ mm}^2$$

$$1. A = 7 \times 6\sqrt{5} - 4 \times 2\sqrt{5} \\ = 42\sqrt{5} - 8\sqrt{5} \\ = 34\sqrt{5} \text{ cm}^2$$

$$2. a) (8\sqrt{3} - 2) \text{ m by } 3 \text{ m}$$

$$b) A = 3 \times (8\sqrt{3} - 2) \\ = (24\sqrt{3} - 6) \text{ m}^2$$

$$c) \text{ Grass} = 5 \times 8\sqrt{3} - (24\sqrt{3} - 6) \\ = 40\sqrt{3} - 24\sqrt{3} + 6 \\ = (16\sqrt{3} + 6) \text{ m}^2$$

$$7. a) x^2 = 1^2 + 1^2 \\ = 2 \\ x = \sqrt{2} \text{ cm}$$

$$b) x^2 = 5^2 + 3^2 \\ = 25 + 9 \\ = 34 \\ x = \sqrt{34} \text{ cm}$$

$$c) x^2 = (\sqrt{10})^2 - 2^2 \\ = 10 - 4 \\ = 6 \\ x = \sqrt{6} \text{ cm}$$

$$d) x^2 = (\sqrt{26})^2 - (\sqrt{10})^2 \\ = 26 - 10 \\ = 16 \\ x = \sqrt{16} \\ = 4 \text{ cm}$$

$$8. PR^2 = 2^2 + 1^2 \\ = 4 + 1 \\ = 5$$

$$PS^2 = 5 + 1^2 \\ = 6 \\ PS = \sqrt{6} \text{ cm}$$

$$9. a) x^2 = 2^2 + 2^2 + 2^2 \\ = 12 \\ x = \frac{\sqrt{12}}{\sqrt{4}\sqrt{3}} \\ = 2\sqrt{3} \text{ cm}$$

$$b) x^2 = 5^2 + 3^2 + 4^2 \\ = 25 + 9 + 16 \\ = 50 \\ x = \frac{\sqrt{50}}{\sqrt{25}\sqrt{2}} \\ = 5\sqrt{2} \text{ cm}$$

$$c) x^2 = 4^2 + (\sqrt{8})^2 + (\sqrt{3})^2 \\ = 16 + 8 + 3 \\ = 27 \\ x = \frac{\sqrt{27}}{\sqrt{9}\sqrt{3}} \\ = 3\sqrt{3} \text{ cm}$$

$$10. a) x^2 = 3^2 + 2^2 \\ = 9 + 4 \\ = 13 \\ x = \sqrt{13} \text{ cm}$$

$$b) x^2 = (2\sqrt{10})^2 - (\sqrt{5})^2 \\ = 40 - 5 \\ = 35 \\ x = \sqrt{35} \text{ cm}$$

$$1. a) x^2 = 4^2 + 4^2 + 4^2 \\ = 48 \\ x = \frac{\sqrt{48}}{\sqrt{16}\sqrt{3}} \\ = 4\sqrt{3} \text{ cm}$$

$$b) x^2 = (2\sqrt{3})^2 + 3^2 + (2\sqrt{5})^2 \\ = 12 + 9 + 20 \\ = 41 \\ x = \sqrt{41} \text{ cm}$$

Ex 10

$$1. a) \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ = \frac{\sqrt{5}}{5}$$

$$b) \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{\sqrt{2}}{2}$$

$$c) \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = \frac{6\sqrt{3}}{3} \\ = 2\sqrt{3}$$

$$d) \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{8\sqrt{2}}{2} \\ = 4\sqrt{2}$$

$$e) \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{\sqrt{2}}{6}$$

$$f) \frac{5}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ = \frac{5\sqrt{7}}{14}$$

$$g) \frac{\sqrt{12}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ = \frac{\sqrt{84}}{7} \\ = \frac{\sqrt{4}\sqrt{21}}{7} \\ = \frac{2\sqrt{21}}{7}$$

$$h) \frac{6}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = \frac{6\sqrt{3}}{15} \\ = \frac{2\sqrt{3}}{5}$$

$$2. a) \frac{\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = \frac{\sqrt{15}}{3}$$

$$b) \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{\sqrt{2}}{8}$$

$$c) \frac{4}{5\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ = \frac{4\sqrt{5}}{25}$$

$$d) \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ = \frac{\sqrt{7}}{7}$$

$$e) \sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{\sqrt{10}}{2}$$

$$f) \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = \frac{\sqrt{3}}{3}$$

$$g) \frac{6}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ = \frac{6\sqrt{5}}{5}$$

$$h) \frac{2}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ = \frac{2\sqrt{7}}{21}$$

$$l) \frac{4}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{4\sqrt{2}}{10} \\ = \frac{2\sqrt{2}}{5}$$

$$\begin{aligned}
 j) \quad & \frac{10}{\sqrt{40}} \times \frac{\sqrt{40}}{\sqrt{40}} \\
 & = \frac{10\sqrt{40}}{\sqrt{40}} \\
 & = \frac{40}{\sqrt{4}\sqrt{10}} \\
 & = \frac{4}{2\sqrt{10}} \\
 & = \frac{\sqrt{10}}{2}
 \end{aligned}$$

$$\begin{aligned}
 k) \quad & \frac{3\sqrt{5}}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} \\
 & = \frac{3\sqrt{40}}{8} \\
 & = \frac{3\sqrt{4}\sqrt{10}}{8} \\
 & = \frac{6\sqrt{10}}{8} \\
 & = \frac{3\sqrt{10}}{4}
 \end{aligned}$$

$$\begin{aligned}
 l) \quad & \frac{4}{\sqrt{18}} \times \frac{\sqrt{18}}{\sqrt{18}} \\
 & = \frac{4\sqrt{18}}{18} \\
 & = \frac{2\sqrt{9}\sqrt{2}}{9} \\
 & = \frac{2\sqrt{2}}{3}
 \end{aligned}$$

Ex 1E

$$1. a) \sqrt{2}(3 + \sqrt{2}) = 3\sqrt{2} + 2$$

$$b) \sqrt{5}(\sqrt{5} - 1) = 5 - \sqrt{5}$$

$$c) \sqrt{7}(5 - \sqrt{7}) = 5\sqrt{7} - 7$$

$$d) 2\sqrt{3}(6 - \sqrt{3}) = 12\sqrt{3} - 6$$

$$e) 3(\sqrt{2} - \sqrt{5}) = 3\sqrt{2} - 3\sqrt{5}$$

$$f) 7(\sqrt{3} - 8) = 7\sqrt{3} - 56$$

$$\begin{aligned}
 g) \quad & \sqrt{8}(\sqrt{2} - \sqrt{3}) \\
 & = \sqrt{16} - \sqrt{24} \\
 & = 4 - \sqrt{4}\sqrt{6} \\
 & = 4 - 2\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad & \sqrt{6}(3 - 2\sqrt{2}) \\
 & = 3\sqrt{6} - 2\sqrt{12} \\
 & = 3\sqrt{6} - 2\sqrt{4}\sqrt{3} \\
 & = 3\sqrt{6} - 4\sqrt{3}
 \end{aligned}$$

$$2. a) (\sqrt{2} + 1)(\sqrt{2} - 3) = 2 - 3\sqrt{2} + \sqrt{2} - 3 = -1 - \sqrt{2}$$

$$\begin{aligned}
 b) \quad & (\sqrt{3} - 2)(\sqrt{3} + 4) \\
 & = 3 + 4\sqrt{3} - 2\sqrt{3} - 8 \\
 & = -5 + 2\sqrt{3}
 \end{aligned}$$

$$c) (\sqrt{3} - 5)(\sqrt{3} + 5) = 3 + 5\sqrt{3} - 5\sqrt{3} - 25 = -22$$

$$\begin{aligned}
 d) \quad & (\sqrt{2} - \sqrt{7})(\sqrt{2} + \sqrt{7}) \\
 & = 2 + \sqrt{14} - \sqrt{14} - 7 \\
 & = -5
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & (\sqrt{5} + 1)^2 \\
 & = 5 + 2\sqrt{5} + 1 \\
 & = 6 + 2\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & (\sqrt{3} - \sqrt{2})^2 \\
 & = 3 - 2\sqrt{6} + 2 \\
 & = 5 - 2\sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 g) \quad & (\sqrt{7} - 5)^2 \\
 & = 7 - 10\sqrt{7} + 25 \\
 & = 32 - 10\sqrt{7}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad & 3(\sqrt{5} + 1)(\sqrt{5} + 4) \\
 & = 3(5 + 4\sqrt{5} + \sqrt{5} + 4) \\
 & = 3(9 + 5\sqrt{5}) \\
 & = 27 + 15\sqrt{5}
 \end{aligned}$$

$$3. a) A = (b) = (3 + \sqrt{2})(3 - \sqrt{2}) = 9 - 3\sqrt{2} + 3\sqrt{2} - 2 = 7 \text{ cm}^2$$

$$\begin{aligned}
 b) \quad & x^2 = (3 + \sqrt{2})^2 + (3 - \sqrt{2})^2 \\
 & = 9 + 6\sqrt{2} + 2 + 9 - 6\sqrt{2} + 2 \\
 & = 22 \\
 & x = \sqrt{22} \text{ cm.}
 \end{aligned}$$

Chapter 2 - Simplifying Expressions Using The Laws of Indices

Ex 2A

a) 2^4
 $= 16$

b) 3^6
 $= 729$

c) 4^3
 $= 64$

d) 10^6
 $= 1000000$

e) 23^1
 $= 23$

f) 0.3^3
 $= 0.027$

g) $\left(\frac{1}{4}\right)^4$
 $= \frac{1}{256}$

h) 3^0
 $= 1$

✓

a) 7×7
 $= 7^2$

b) $30 \times 30 \times 30$
 $= 30^3$

c) $0.3 \times 0.3 \times 0.3 \times 0.3$
 $= 0.3^4$

d) $10 \times 10 \times 10 \times 10$
 $= 10^4$

e) $g \times g \times g$
 $= g^3$

f) $a \times a \times a \times b \times b$
 $= a^3 b^2$

a) 7^2
 $= 49$

b) 30^3
 $= 27000$

c) 0.3^4
 $= 0.0081$

d) 10^4
 $= 10000$

Ex 2B

a) 6^{-3}
 $= \frac{1}{6^3}$
 $= \frac{1}{216}$

b) 2^{-5}
 $= \frac{1}{2^5}$
 $= \frac{1}{32}$

c) 3^{-4}
 $= \frac{1}{3^4}$
 $= \frac{1}{81}$

d) $(3a)^{-2}$
 $= \frac{1}{(3a)^2}$
 $= \frac{1}{9a^2}$

a) x^{-3}
 $= \frac{1}{x^3}$

b) y^{-8}
 $= \frac{1}{y^8}$

c) $3t^{-4}$
 $= \frac{3}{t^4}$

d) $7y^{-6}$
 $= \frac{7}{y^6}$

e) $\frac{2}{7}t^{-5}$
 $= \frac{2}{7t^5}$

f) $\frac{1}{2}y^{-3}$
 $= \frac{1}{2y^3}$

a) $\frac{6}{x^3}$
 $= 6x^{-3}$

b) $\frac{9}{t^5}$
 $= 9t^{-5}$

c) $\frac{3}{m^4}$
 $= 3m^{-4}$

d) $\frac{10}{a^8}$
 $= 10a^{-8}$

a) i) m^3
 $= 4^3$
 $= 64$

ii) m^{-2}
 $= \frac{1}{m^2}$
 $= \frac{1}{16}$

iii) $5m^{-1}$
 $= \frac{5}{m}$
 $= \frac{5}{4}$

b) i) a^5
 $= 32$

ii) a^{-3}
 $= \frac{1}{a^3}$
 $= \frac{1}{8}$

iii) $8a^{-4}$
 $= \frac{8}{a^4}$
 $= \frac{8}{16}$
 $= \frac{1}{2}$

5. a) m is a positive whole number greater than 1

$$m^{-2}, m^0, m^3$$

b) m is a negative whole number less than -1

$$m^3, m^0, m^{-2}$$

Ex 2C

1. a) $4^5 \times 4^3 = 4^8$ b) $7^4 \times 7 = 7^5$ c) $x^{10} \times x^2 = x^{12}$ d) $t^2 \times t^3 \times t^4 = t^9$

e) $3^2 \times 3^{-7} = 3^{-5} = \frac{1}{3^5}$ f) $c^3 \times c^{-9} = c^{-6} = \frac{1}{c^6}$ g) $a^8 \times a^{-8} = a^0 = 1$ h) $4y^3 \times 5y^6 = 20y^9$

i) $c \times 4c^2 \times 2c^3 = 8c^6$ j) $8c^2 \times 3c^{-7} = 24c^{-5} = \frac{24}{c^5}$ k) $10a^7 \times 3a^{-20} = 30a^{-13} = \frac{30}{a^{13}}$ l) $4t^3 \times 3t^{-8} \times 2t^2 = 24t^{-3} = \frac{24}{t^3}$

2. a) $3^7 \div 3^2 = 3^5$ b) $6 \div 6^3 = 6^{-2} = \frac{1}{6^2}$ c) $x^8 \div x^5 = x^3$ d) $t^3 \div t = t^2$ e) $p^3 \div p^{-2} = p^5$

f) $y^{-3} \div y^{-3} = y^0 = 1$ g) $12y^{10} \div 3y^3 = 4y^7$ h) $24y^3 \div 12y^8 = \frac{2y^{-5}}{y^5}$ i) $15x^2 \div 3x^{-4} = 5x^6$

j) $42p^6 \div (-7p)^{-2} = 42p^6 \div \left(\frac{1}{49p^2}\right) = 42p^6 \times 49p^2 = 2058p^8$

k) $\frac{4t^5 \times (-7t^3)}{14t^{-4}} = \frac{-28t^8}{14t^{-4}} = -2t^{12}$

l) $\frac{5y^2 \times 4y^{-6}}{2y^3} = \frac{20y^{-4}}{2y^3} = \frac{10}{y^7}$

3. a) $3x^2y \times 5x^3y^2 = 15x^5y^3$

b) $3a^2b^3 \times 7ab^4 = 21a^3b^7$

c) $30x^3y \div 6x^2y^4 = 5xy^{-3} = \frac{5x}{y^3}$

Ex 2D

a) $(3^4)^5 = 3^{20}$ b) $(2^3)^4 = 2^{12}$ c) $(10^9)^3 = 10^{27}$ d) $(t^3)^{-4} = \frac{1}{t^{12}}$ e) $(a^7)^3 = a^{21}$

a) $(3y)^2 = 9y^2$ b) $(x^3y^4)^5 = x^{15}y^{20}$ c) $(ab^3)^4 = a^4b^{12}$ d) $(3p^4q^2)^3 = 27p^{12}q^6$ e) $(2t^3u^{-2})^4 = \frac{16t^{12}u^{-8}}{u^8}$

f) $(10u^{-5}v^{-2})^3 = 1000u^{-15}v^{-6} = \frac{1000}{u^{15}v^6}$

a) $(6^4)^3 = 6^{12}$ b) $(2^7)^4 = 2^{28}$ c) $(a^9)^6 = a^{54}$ d) $(t^{-3})^7 = \frac{1}{t^{21}}$ e) $(x^{-2})^{-5} = x^{10}$

f) $(6a^3b^4)^2 = 36a^6b^8$ g) $(2x^{-3}y^5)^4 = \frac{16y^{20}}{x^{12}}$ h) $(3a^6b^{-3})^5 = \frac{243a^{30}b^{-15}}{b^{15}}$ i) $(x^4y^{-2}z^3)^3 = \frac{x^{12}y^{-6}z^9}{y^6}$

a) $3^3 \times 3^4 = 3^7$ FALSE
 b) $\frac{2^3}{3^2} = \frac{8}{9}$ FALSE
 c) $8^9 \div 8^5 = 8^4$ TRUE

d) $\frac{(4^2)^3}{(4^3)^2} = \frac{4^6}{4^6}$ TRUE
 e) $12^3 \div 6^3 = \frac{6^3 \times 2^3}{6^3} = 2^3$ TRUE
 f) $5^5 \times 5^5 = 5^{10}$ TRUE

g) $7^6 \div 7^6 = 7^0$ TRUE
 h) $(3a^5)^3 = 27a^{15}$ FALSE
 i) $\frac{2x^6 \times 3x^2}{6x^{-2}} = \frac{6x^8}{6x^{-2}} = x^{10}$ FALSE

Ex 2E

a) $a^{1/3} = \sqrt[3]{a}$ b) $a^{1/5} = \sqrt[5]{a}$ c) $t^{1/2} = \sqrt{t}$ d) $a^{2/3} = \sqrt[3]{a^2}$ e) $a^{3/5} = \sqrt[5]{a^3}$

f) $\frac{t^{5/2}}{\sqrt{t^5}} = \frac{t^{5/2}}{t^{5/2}} = 1$ g) $\frac{x^{4/3}}{\sqrt[3]{x^4}} = \frac{x^{4/3}}{x^{4/3}} = 1$ h) $\frac{y^{2/5}}{\sqrt[5]{y^2}} = \frac{y^{2/5}}{y^{2/5}} = 1$ i) $\frac{p^{1/4}}{\sqrt[4]{p}} = \frac{p^{1/4}}{p^{1/4}} = 1$ j) $\frac{m^{3/4}}{\sqrt[4]{m^3}} = \frac{m^{3/4}}{m^{3/4}} = 1$

a) $\sqrt{t^5} = t^{5/2}$ b) $\sqrt[4]{a^3} = a^{3/4}$ c) $\sqrt[5]{x^3} = x^{3/5}$ d) $\sqrt[7]{m^4} = m^{4/7}$ e) $\sqrt[3]{a^{12}} = a^{12/3} = a^4$

$$3. a) a^{1/2} = \sqrt{a} = 3$$

$$b) 16^{1/4} = \sqrt[4]{16} = 2$$

$$c) 8^{2/3} = \sqrt[3]{8^2} = 2^2 = 4$$

$$d) 49^{1/2} = \sqrt{49} = 7^3 = 343$$

$$e) 25^{-1/2} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$

$$f) 81^{-3/4} = \frac{1}{\sqrt[4]{81^3}} = \frac{1}{3^3} = \frac{1}{27}$$

$$g) 100^{-3/2} = \frac{1}{\sqrt{100^3}} = \frac{1}{1000}$$

$$h) \left(\frac{1}{27}\right)^{2/3} = \frac{1}{\sqrt[3]{27^2}} = \frac{1}{3^2} = \frac{1}{9}$$

$$i) \left(\frac{49}{81}\right)^{1/2} = \frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$$

$$j) \left(\frac{16}{25}\right)^{3/2} = \frac{4^3}{5^3} = \frac{64}{125}$$

$$4. a) (x^6)^{1/2} = x^3$$

$$b) (y^3)^{2/3} = y^2$$

$$c) (a^{-12})^{1/4} = a^{-3}$$

$$d) (t^{4/3})^0 = t^0 = 1$$

$$e) 3t^{1/2} \times 6t^{-1/2} = 18t^0 = 18$$

$$f) 5a^{2/3} \times 3a^{1/3} = 15a$$

$$g) 12x^{4/3} \div 6x^{-2/3} = 2x^2$$

$$h) 5y^{2/5} \times (-3y^{7/5}) = -15y^{9/5}$$

$$i) 10t^{3/2} \div 5t^{5/2} = 2t^{-1} = \frac{2}{t}$$

$$5. 3^0 = 1 \quad 20^{-1} = \frac{1}{20} = 0.05$$

$$8^{2/3} = \sqrt[3]{8^2} = 2^2 = 4$$

$$4^{1/2} = \sqrt{4} = 2$$

$$20^{-1}, 5^0, 4^{1/2}, 8^{2/3}$$

$$6. a) i) 81^{1/2} = \sqrt{81} = 9$$

$$ii) 27^{2/3} = \sqrt[3]{27^2} = 3^2 = 9$$

$$iii) 36^{-1/2} = \frac{1}{\sqrt{36}} = \frac{1}{6}$$

$$iv) 100^{-3/2} = \frac{1}{\sqrt{100^3}} = \frac{1}{10^3} = \frac{1}{1000}$$

$$v) \left(\frac{25}{49}\right)^{1/2} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}$$

$$vi) \left(\frac{8}{27}\right)^{-1/3} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

$$b) \quad i) (y^3)^3 = y^9$$

$$ii) (x^{1/2})^{-5} = x^{-5/2} = \frac{1}{\sqrt{x^5}} = \frac{1}{x^{5/2}}$$

$$iii) \sqrt[7]{a^7} = a^{7/7} = a$$

$$iv) \sqrt[4]{y^3} = y^{3/4}$$

$$v) 4t^{1/2} \cdot 3t^{-3/2} = 12t^{-1} = \frac{12}{t}$$

$$vi) 6t^{1/3} \cdot 2t^{-2/3} = 12t^{-1/3} = \frac{12}{\sqrt[3]{t}}$$

Ex 2F

$$1. a) a^2(a^3+1) = a^5 + a^2$$

$$b) x^{-4}(x^2+x^{-1}) = x^{-2} + x^{-5}$$

$$c) y^3(y^{-2}+y^{-3}) = y + 1$$

$$d) m^3(m^{-5}-4) = m^{-2} - 4m^3$$

$$e) 5a^2(2a^{-2}-7a^3) = 10 - 35a^5$$

$$2. a) p^{1/2}(p+3) = p^{3/2} + 3p^{1/2} = \sqrt{p^3} + 3\sqrt{p}$$

$$b) t^{1/2}(t+t^{1/2}) = t^{3/2} + t = \sqrt{t^3} + t$$

$$c) 3z^{1/3}(z^{5/3}+2) = 3z^2 + 6z^{1/3} = 3z^2 + 6\sqrt[3]{z}$$

$$d) b^{-1/4}(b^{3/4}-b^{-1/2}) = b^{1/2} - b^{-3/4} = \sqrt{b} - \frac{1}{\sqrt[4]{b^3}}$$

$$e) y^{4/5}(y^{-4/5}+3y^{1/5}) = y^0 + 3y = 1 + 3y$$

$$f) c^{-1/2}(c^{1/2}-c^{1/3}) = c^0 - c^{-1/6} = 1 - \frac{1}{\sqrt[6]{c}}$$

$$1. a) a^5 + a^2 = 3^5 + 3^2 = 243 + 9 = 252$$

$$b) \frac{1}{x^2} + \frac{1}{x^5} = \frac{1}{2^2} + \frac{1}{2^5} = \frac{1}{4} + \frac{1}{32} = \frac{9}{32}$$

$$c) y+1 = 10+1 = 11$$

$$d) \frac{1}{m^2} - 4m^3 = \frac{1}{5^2} - 4 \times 5^3 = \frac{1}{25} - 4 \times 125 = -499 \frac{24}{25}$$

$$e) 10 - 35a^5 = 10 - 35 \times 3^5 = 10 - 35 \times 243 = 10 - 8505 = -8495$$

$$2. a) \sqrt{p^3} + 3\sqrt{p} = \sqrt{4^3} + 3\sqrt{4} = 2^3 + 3 \times 2 = 8 + 6 = 14$$

$$b) \sqrt{t^3} + 1 = \sqrt{9^3} + 1 = 3^3 + 1 = 27 + 1 = 28$$

$$c) 3z^2 + 6\sqrt[3]{z} = 3 \times 27^2 + 6\sqrt[3]{27} = 3 \times 729 + 6 \times 3 = 2187 + 18 = 2205$$

$$\begin{aligned}
 d) \sqrt{16} - \frac{1}{\sqrt[4]{16^3}} \\
 = \sqrt{16} - \frac{1}{\sqrt[4]{16^3}} \\
 = 4 - \frac{1}{2^3} \\
 = 4 - \frac{1}{8} \\
 = 3\frac{7}{8}
 \end{aligned}$$

$$\begin{aligned}
 e) 1 + 3y \\
 = 1 + 3 \times 10 \\
 = 1 + 30 \\
 = 31
 \end{aligned}$$

$$\begin{aligned}
 f) 1 - \frac{1}{\sqrt[4]{16}} \\
 = 1 - \frac{1}{\sqrt[4]{16}} \\
 = 1 - \frac{1}{2} \\
 = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 4. a) (t^2 - 4)^2 \\
 = t^4 - 8t^2 + 16
 \end{aligned}$$

$$\begin{aligned}
 b) (y^5 + 2)(y^3 - 1) \\
 = y^2 - y^3 + 2y^3 - 2 \\
 = -y^5 + y^2 - 2 + \frac{2}{y^3}
 \end{aligned}$$

$$\begin{aligned}
 c) (x^{1/2} + 3)^2 \\
 = x + 6x^{1/2} + 9 \\
 = x + 6\sqrt{x} + 9
 \end{aligned}$$

$$\begin{aligned}
 d) (m^3 + 1)(m^3 - 1) \\
 = m^6 - m^3 + m^3 - 1 \\
 = m^6 - 1
 \end{aligned}$$

$$\begin{aligned}
 e) (c^{2/3} + 3)(c^{2/3} - 3) \\
 = c^{4/3} - 3c^{2/3} + 3c^{2/3} - 9 \\
 = \sqrt[3]{c^4} - 9
 \end{aligned}$$

$$\begin{aligned}
 f) (5 - m^{1/4})(m^{1/2} + 2) \\
 = 5m^{1/2} + 10 - m^{3/4} - 2m^{1/4} \\
 = 5\sqrt{m} + 10 - \sqrt[4]{m^3} - 2\sqrt[4]{m}
 \end{aligned}$$

Ex 20.

$$\begin{aligned}
 1. a) 2\,340\,000 \\
 = 2.34 \times 10^6
 \end{aligned}$$

$$\begin{aligned}
 b) 1070 \\
 = 1.07 \times 10^3
 \end{aligned}$$

$$\begin{aligned}
 c) 35\,000\,000 \\
 = 3.5 \times 10^7
 \end{aligned}$$

$$\begin{aligned}
 d) 27 \\
 = 2.7 \times 10
 \end{aligned}$$

$$\begin{aligned}
 e) 3\frac{1}{2} \text{ million} \\
 = 3.5 \times 10^6
 \end{aligned}$$

$$\begin{aligned}
 f) 712\,000\,000\,000 \\
 = 7.12 \times 10^{11}
 \end{aligned}$$

$$\begin{aligned}
 g) 0.00056 \\
 = 5.6 \times 10^{-4}
 \end{aligned}$$

$$\begin{aligned}
 h) 0.0312 \\
 = 3.12 \times 10^{-2}
 \end{aligned}$$

$$\begin{aligned}
 l) 0.000\,000\,408 \\
 = 4.08 \times 10^{-7}
 \end{aligned}$$

$$\begin{aligned}
 j) 0.78 \\
 = 7.8 \times 10^{-1}
 \end{aligned}$$

$$\begin{aligned}
 k) 0.00604 \\
 = 6.04 \times 10^{-3}
 \end{aligned}$$

$$\begin{aligned}
 n) 0.000\,005\,100 \\
 = 5.100 \times 10^{-6}
 \end{aligned}$$

$$\begin{aligned}
 2. a) 5 \times 10^6 \\
 = 5\,000\,000
 \end{aligned}$$

$$\begin{aligned}
 b) 6.32 \times 10^4 \\
 = 63\,200
 \end{aligned}$$

$$\begin{aligned}
 c) 7.01 \times 10^8 \\
 = 701\,000\,000
 \end{aligned}$$

$$\begin{aligned}
 d) 47 \times 10^{-5} \\
 = 0.000\,047
 \end{aligned}$$

$$\begin{aligned}
 e) 8.04 \times 10^{-2} \\
 = 0.008\,040\,804
 \end{aligned}$$

$$\begin{aligned}
 f) 8.89 \times 10^{-8} \\
 = 0.000\,000\,0889
 \end{aligned}$$

2H

$$1. d) (4.2 \times 10^7) \times (2 \times 10^5) \\ = 8.4 \times 10^{12}$$

$$c) (8.4 \times 10^7) \div (4 \times 10^5) \\ = 2.1 \times 10^2$$

$$e) \frac{(8.4 \times 10^4) \times (4 \times 10^9)}{2 \times 10^5} \\ = \frac{33.6 \times 10^{11}}{2 \times 10^5} \\ = 16.8 \times 10^6 \\ = 1.68 \times 10^7$$

$$b) (6.34 \times 10^8) \times (3 \times 10^{-3}) \\ = 19.02 \times 10^5 \\ = 1.902 \times 10^6 \\ d) (4.2 \times 10^7) \div (6 \times 10^{-1}) \\ = 0.7 \times 10^9 \\ = 7 \times 10^8$$

$$f) \frac{(9.4 \times 10^9) \times (4 \times 10^{-3})}{8 \times 10^{-6}} \\ = \frac{37.6 \times 10^2}{8 \times 10^{-6}} \\ = 4.7 \times 10^8$$

$$2. (3 \times 10^8) \times 60 \times 60 \\ = 3 \times 10^8 \times 3600 \\ = 10800 \times 10^8 \\ = 1.08 \times 10^{12} \text{ m}$$

$$3. (5 \times 10^{30}) \times (2.7 \times 10^{-23}) \\ = 13.5 \times 10^7 \\ = 1.35 \times 10^8 \text{ g}$$

$$4. 558 \ 000 \ 000 \times 75 \\ = 41 \ 850 \ 000 \ 000 \\ = 4.185 \times 10^{10} \text{ miles}$$

$$\begin{array}{r} 558 \\ \times 75 \\ \hline 2790 \\ 39060 \\ \hline 41850 \end{array}$$

$$5. \frac{225 \ 000 \ 000}{3 \times 10^5} \\ = \frac{225 \times 10^6}{3 \times 10^5} \\ = 75 \times 10^1 \\ = 7.5 \times 10^2 \text{ seconds}$$

$$6. a) 5.88 \times 10^{12} \times 1000 \\ = 5.88 \times 10^{15} \text{ miles} \\ b) \frac{5.88 \times 10^{12}}{365 \times 24 \times 60} \\ = \frac{5.88 \times 10^{12}}{525600} \\ = 1.12 \times 10^7 \text{ miles}$$

$$7. \frac{2.75 \times 10^{35}}{1 \times 10^{21}} \\ = 2.75 \times 10^{14} \text{ eertins}$$

$$8. (2 \times 10^{-2}) \times (7 \times 10^{-3}) \\ = 14 \times 10^{-5} \\ = 1.4 \times 10^{-4} \text{ cm}^2$$

$$9. 2 \times 10^{13} \times 10^{-10} \\ = 2 \times 10^3 \text{ g}$$

Ex 2 I

- 1.
- a) m/s
 - b) m/s^2
 - c) $1/m$
 - d) $1/s$
 - e) $\frac{m \cdot kg}{s^2}$
 - f) $\frac{kg}{ms^2}$
 - g) $\frac{m^2 kg}{s^2}$
 - h) $\frac{m^2 kg}{s^3}$
 - i) $\frac{m^2 kg}{s^3 A}$
 - j) $\frac{m^2 kg}{s^3 A^2}$
 - k) $\frac{kg}{s^2 A}$
 - l) $\frac{m^2 kg}{s^4 A^2}$

- 2.
- a) $kg/m^3 = kg \cdot m^{-3}$
 - b) $m^3/kg = m^3 \cdot kg^{-1}$
 - c) $A/m = A \cdot m^{-1}$
 - d) $m^2/s = m^2 \cdot s^{-1}$
 - e) $mol/s = mol \cdot s^{-1}$
 - f) $m^2 \cdot kg/s^2 A = m^2 kg s^{-2} A^{-1}$
 - g) $s^3 A^2 / m^2 kg = s^3 A^2 m^{-2} kg^{-1}$
 - h) $s^4 A^2 / m^2 kg = s^4 A^2 m^{-2} kg^{-1}$

Chapter 3 Working With Algebraic Expressions Involving Expansion of Brackets

Ex 3A

- a) $3t + 4t = 7t$ b) $6y + 2y - 10y = -2y$ c) $5x^2 - 3x^2 = 2x^2$
- d) $4ab + 7ab - ab = 10ab$ e) $8x + 9y - 3x + 7y = 5x + 16y$ f) $4a - 6a^2 + 7a^2 + 2a = 6a + a^2$
- a) $8p + 2p - 5p = 5p$ b) $9t - 4t = 5t$ c) $6c + 7c - 8 = 13c - 8$
- d) $-3pq + 7qp = 4pq$ e) $4rt + 3rt - 7rt = 0$ f) $5a^2 - 6a - 8a^2 = -3a^2 - 6a$
- g) $-2 + 3a - 4a^2$ h) $4m - 7n + 3m + 2n = 7m - 5n$ i) $8rs + 7s - 12sr - 2s = -4rs + 5s$

Ex 3B

- a) $2(t+4) = 2t+8$ b) $5(m-3) = 5m-15$ c) $-6(2a+1) = -12a-6$ d) $-10(11-9y) = -110+90y$
- e) $8(2t+3y+1) = 16t+24y+8$ f) $-5(-4m+2n-7r) = 20m-10n+35r$ g) $a(4+c) = 4a+ac$
- h) $2a(8-c) = 16a-2ac$ i) $5x(3y-4) = 15xy-20x$ j) $y(y-4) = y^2-4y$
- k) $-b(b-c) = -b^2+bc$ l) $a(b-c+a) = ab-ac+a^2$

- a) $3(2x+7)-12 = 6x+21-12 = 6x+9$ b) $10(3y-7)+8y = 30y-70+8y = 38y-70$ c) $6+3(2+y) = 6+6+3y = 12+3y$
- d) $12+4(2t-3) = 12+8t-12 = 8t$ e) $8p-5(4-p) = 8p-20+5p = 13p-20$ f) $7+6(-3+2y) = 7-18+12y = -11+12y$
- g) $7-(2p+3) = 7-2p-3 = 4-2p$ h) $2t-(9+2t) = 2t-9-2t = -9$

3. a) $5x(2x+3) = 10x^2+15x$ b) $3y(4y-5) = 12y^2-15y$ c) $6t(-5t+1) = -30t^2+6t$
- d) $-4c(2c-7) = -8c^2+28c$ e) $9m(5m+4) = 45m^2+36m$ f) $8w(2m-3w) = 16mw-24w^2$

$$g) x(-x+7y) \\ = -x^2 + 7xy$$

$$h) -9s(-4u+3s) \\ = 36us - 27s^2$$

$$4. a) 5x^2 + 3x(x+2) \\ = 5x^2 + 3x^2 + 6x \\ = 8x^2 + 6x$$

$$b) 2y - y(5y-4) \\ = 2y - 5y^2 + 4y \\ = 6y - 5y^2$$

$$c) 8 - 5x(2x+3) \\ = 8 - 10x^2 - 15x$$

$$d) 11t^2 - t(t+3) \\ = 11t^2 - t^2 - 3t \\ = 10t^2 - 3t$$

$$e) 4x(x+7) + 3(2x-1) \\ = 4x^2 + 28x + 6x - 3 \\ = 4x^2 + 34x - 3$$

$$f) 6w(2w+1) - 4w(w+1) \\ = 12w^2 + 6w - 4w^2 - 4w \\ = 8w^2 + 2w$$

$$5. a) 2(5x-4) - 3(2x-1) \\ = 10x - 8 - 6x + 3 \\ = 4x - 5$$

$$b) 10(2x+3y) + 2(3x+5y) \\ = 20x + 30y + 6x + 10y \\ = 26x + 40y$$

$$c) -7(2t-3w) - 11(t-1) \\ = -14t + 21w - 11t + 11 \\ = -25t + 21w + 11$$

$$d) a(b-c) + b(c-a) + c(a-b) \\ = ab - ac + bc - ab + ac - bc \\ = 0$$

$$e) x(3+y) + y(4+x) \\ = 3x + xy + 4y + xy \\ = 3x + 2xy + 4y$$

$$f) a(2b+7) + b(5-3a) \\ = 2ab + 7a + 5b - 3ab \\ = -ab + 7a + 5b$$

$$g) 5t(2m+3) - 3m(2t-7) \\ = 10mt + 15t - 6mt + 21m \\ = 4mt + 15t + 21m$$

$$6. a) 2t(5t-1) \\ = 10t^2 - 2t$$

$$b) 4y(3y+2) \\ = 12y^2 + 8y$$

$$c) 7x(3x-4) \\ = 21x^2 - 28x$$

$$7. a) \frac{1}{2}y(y+6) \\ = \frac{1}{2}y^2 + 3y$$

$$b) \frac{1}{2} \times 3t(4t+10) \\ = 6t^2 + 15t$$

$$c) \frac{1}{2} \times 2w(5w-3) \\ = 5w^2 - 3w$$

$$8. (4x+5)(6x+3) \\ = 24x^2 + 12x + 30x + 15 \\ = (24x^2 + 42x + 15) m^2$$

$$9. \frac{1}{2} \times 2t(4t+3) \\ = 4t^2 + 3t$$

$$t(2t+3) \\ = 2t^2 + 3t$$

$$\frac{1}{2}t(5t+6) \\ = \frac{5}{2}t^2 + 3t$$

a) triangle

b) rectangle

Ex 30

$$\begin{aligned} \text{a)} \quad & (x+3)(x+2) \\ & = x^2 + 2x + 3x + 6 \\ & = x^2 + 5x + 6 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (y+7)(y+4) \\ & = y^2 + 4y + 7y + 28 \\ & = y^2 + 11y + 28 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (t+4)(t+8) \\ & = t^2 + 8t + 4t + 32 \\ & = t^2 + 12t + 32 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (a-7)(a-3) \\ & = a^2 - 3a - 7a + 21 \\ & = a^2 - 10a + 21 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & (w-2)(w-9) \\ & = w^2 - 9w - 2w + 18 \\ & = w^2 - 11w + 18 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & (z-10)(z-8) \\ & = z^2 - 8z - 10z + 80 \\ & = z^2 - 18z + 80 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad & (r-3)(r+10) \\ & = r^2 + 10r - 3r - 30 \\ & = r^2 + 7r - 30 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & (t+11)(t-4) \\ & = t^2 - 4t + 11t - 44 \\ & = t^2 + 7t - 44 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & (a-9)(a+7) \\ & = a^2 + 7a - 9a - 63 \\ & = a^2 - 2a - 63 \end{aligned}$$

$$\begin{aligned} \text{1. a)} \quad & (2x+1)(x-3) \\ & = 2x^2 - 6x + x - 3 \\ & = 2x^2 - 5x - 3 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (5y+7)(3y-4) \\ & = 15y^2 - 20y + 21y - 28 \\ & = 15y^2 + y - 28 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (8a-3)(a+6) \\ & = 8a^2 + 48a - 3a - 18 \\ & = 8a^2 + 45a - 18 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (7a-2)(a+5) \\ & = 7a^2 + 35a - 2a - 10 \\ & = 7a^2 + 33a - 10 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & (6t-5)(3t-2) \\ & = 18t^2 - 12t - 15t + 10 \\ & = 18t^2 - 27t + 10 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & (b-8)(8b-3) \\ & = 8b^2 - 3b - 64b + 24 \\ & = 8b^2 - 67b + 24 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad & (7+3w)(2w-5) \\ & = 14w - 35 + 6w^2 - 15w \\ & = 6w^2 - w - 35 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & (4+3s)(6-7s) \\ & = 24 - 28s + 18s - 21s^2 \\ & = 24 - 10s - 21s^2 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & (3m-2)(4+m) \\ & = 12m + 3m^2 - 8 - 2m \\ & = 3m^2 + 10m - 8 \end{aligned}$$

$$\begin{aligned} \text{3. a)} \quad & (x+5)^2 \\ & = x^2 + 10x + 25 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (t-2)^2 \\ & = t^2 - 4t + 4 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (4a-3)^2 \\ & = 16a^2 - 24a + 9 \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & (3x+1)^2 \\ & = 9x^2 + 6x + 1 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & (x+2)^2 + (x+6)^2 \\ & = x^2 + 4x + 4 + x^2 + 12x + 36 \\ & = 2x^2 + 16x + 40 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & (y+8)^2 - (y-3)^2 \\ & = (y^2 + 16y + 64) - (y^2 - 6y + 9) \\ & = 22y + 55 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad & (x^2+7)^2 \\ & = x^4 + 14x^2 + 49 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad & (t^2-4)^2 \\ & = t^4 - 8t^2 + 16 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad & (a^2+a)^2 + (7-a)^2 \\ & = (a^4 + 18a^2 + 8) + (49 - 14a + a^2) \\ & = a^4 + 19a^2 - 14a + 130 \end{aligned}$$

$$\begin{aligned} \text{4. a)} \quad & (4y+7)(2y+1) \\ & = 8y^2 + 4y + 14y + 7 \\ & = 8y^2 + 18y + 7 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & (8a+3)(5a+9) \\ & = 40a^2 + 72a + 15a + 27 \\ & = 40a^2 + 87a + 27 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & (5t+4)(3t-2) \\ & = 15t^2 - 10t + 12t - 8 \\ & = 15t^2 + 2t - 8 \end{aligned}$$

$$5. \quad a) \quad \frac{1}{2} (3y-1)(4y+7) \\ = \frac{1}{2} (12y^2 + 21y - 4y - 7) \\ = \frac{1}{2} (12y^2 + 17y - 7)$$

$$c) \quad \frac{1}{2} (3y+2)(5y+3) \\ = \frac{1}{2} (15y^2 + 9y + 10y + 6) \\ = \frac{1}{2} (15y^2 + 19y + 6)$$

$$b) \quad \frac{1}{2} (2t-1)(6t-3) \\ = \frac{1}{2} (12t^2 - 6t - 6t + 3) \\ = \frac{1}{2} (12t^2 - 12t + 3)$$

$$6. \quad b) \quad (5x+1)^2 \\ = 25x^2 + 10x + 1$$

a) Denise just squared the two terms and forgot about the middle term in the answer.

Keith forgot to do 5×5 .

$$7. \quad a) \quad (2y-3)(6y+5) \\ = 12y^2 + 10y - 18y - 15 \\ = 12y^2 - 8y - 15$$

$$b) \quad (12y^2 - 8y - 15) - 2y^2 \\ = 10y^2 - 8y - 15$$

$$c) \quad y=2 \rightarrow 10 \times 4 - 8 \times 2 - 15 \\ = 40 - 16 - 15 \\ = 9 \text{ sq metres}$$

$$\text{cost} = 9 \times \text{£}1.50 \\ = \text{£}13.50$$

d) 9m.

Exercise 3D

$$1. \quad a) \quad (x+1)(3x^2+2x+7) \\ = 3x^3 + 2x^2 + 7x + 3x^2 + 2x + 7 \\ = 3x^3 + 5x^2 + 9x + 7$$

$$b) \quad (y+4)(2y^2-5y+2) \\ = 2y^3 - 5y^2 + 2y + 8y^2 - 20y + 8 \\ = 2y^3 + 3y^2 - 18y + 8$$

$$c) \quad (x+3)(5x^2-x-1) \\ = 5x^3 - x^2 - x + 15x^2 - 3x - 3 \\ = 5x^3 + 14x^2 - 4x - 3$$

$$d) \quad (t-2)(3t^2+6t-1) \\ = 3t^3 + 6t^2 - t - 6t^2 - 12t + 2 \\ = 3t^3 - 13t + 2$$

$$e) \quad (w-5)(w^2-4w-2) \\ = w^3 - 4w^2 - 2w - 5w^2 + 20w + 10 \\ = w^3 - 9w^2 + 18w + 10$$

$$\begin{aligned}
 f) & (5+a)(4a^2-2a+5) \\
 & = 20a^2 - 10a + 25 + 4a^3 - 2a^2 + 5a \\
 & = 4a^3 + 18a^2 - 5a + 25
 \end{aligned}$$

$$\begin{aligned}
 2 \ a) & (3x-2)(4x^2+3x+1) \\
 & = 12x^3 + 9x^2 + 3x - 8x^2 - 6x - 2 \\
 & = 12x^3 + x^2 - 3x - 2
 \end{aligned}$$

$$\begin{aligned}
 b) & (6y+1)(2y^2-3y-2) \\
 & = 12y^3 - 18y^2 - 12y + 2y^2 - 3y - 2 \\
 & = 12y^3 - 16y^2 - 15y - 2
 \end{aligned}$$

$$\begin{aligned}
 c) & (7a+4)(2a^2-5a+3) \\
 & = 14a^3 - 35a^2 + 21a + 8a^2 - 20a + 12 \\
 & = 14a^3 - 27a^2 + a + 12
 \end{aligned}$$

$$\begin{aligned}
 d) & (4w-9)(w^2-3w+4) \\
 & = 4w^3 - 12w^2 + 16w - 9w^2 + 15w - 20 \\
 & = 4w^3 - 17w^2 + 31w - 20
 \end{aligned}$$

$$\begin{aligned}
 e) & (8b-7)(2b^2+7b+9) \\
 & = 16b^3 + 56b^2 + 72b - 14b^2 - 49b - 63 \\
 & = 16b^3 + 42b^2 + 23b - 63
 \end{aligned}$$

$$\begin{aligned}
 f) & (6x^2+2x-3)(5x-1) \\
 & = 30x^3 - 6x^2 + 10x^2 - 2x - 15x + 3 \\
 & = 30x^3 + 4x^2 - 17x + 3
 \end{aligned}$$

$$\begin{aligned}
 3. \ a) & (x+7)(2x^2+9x+5) \\
 & = 30x^3 + 90x^2 + 50x + 14x^2 + 63x + 35 \\
 & = 30x^3 + 23x^2 + 68x + 35
 \end{aligned}$$

$$\begin{aligned}
 b) & (a-2)(3a^2-7a+4) \\
 & = 3a^3 - 7a^2 + 4a - 6a^2 + 14a - 8 \\
 & = 3a^3 - 13a^2 + 18a - 8
 \end{aligned}$$

$$\begin{aligned}
 c) & (6-a)(5a^2+6a-1) \\
 & = 30a^2 + 36a - 6 - 5a^3 - 6a^2 + a \\
 & = -5a^3 + 24a^2 + 37a - 6
 \end{aligned}$$

$$\begin{aligned}
 d) & (9u+5)(3u^2-8u+7) \\
 & = 27u^3 - 72u^2 + 63u + 15u^2 - 40u + 35 \\
 & = 27u^3 - 57u^2 + 23u + 35
 \end{aligned}$$

$$\begin{aligned}
 e) & (b+5)(6b^2-2b+5) \\
 & = 6b^3 - 2b^2 + 5b + 30b^2 - 10b + 25 \\
 & = 6b^3 + 28b^2 - 5b + 25
 \end{aligned}$$

$$\begin{aligned}
 f) & (4w^2-5w+3)(8w-1) \\
 & = 32w^3 - 4w^2 - 40w^2 + 5w + 24w - 3 \\
 & = 32w^3 - 44w^2 + 29w - 3
 \end{aligned}$$

Chapter 4

Factoring An Algebraic Expression

Ex 4A

1. a) $3b+3c = 3(b+c)$ b) $2a+10b = 2(a+5b)$ c) $4x+14y = 2(2x+7y)$ d) $at+ar = a(t+r)$
e) $12x-8y = 4(3x-2y)$ f) $ab-bc = b(a-c)$ g) $cy^2-cy = cy(y-1)$ h) $24ab-12bc = 12b(2a-c)$
i) $14y-35z = 7(2y-5z)$ j) $4t^2-6at = 2t(2t-3a)$ k) $4p-5pr = p(4-5r)$ l) $20b-20b^2 = 20b(1-b)$

2. a) $pq-qr = q(p-r)$ b) $5xt-10ay = 5(xt-2ay)$ c) $2\pi r^2-6\pi rh = 2\pi r(r-3h)$
d) $8a^2b-20ab^2 = 4ab(2a-5b)$ e) $4m^2n-5t$ f) $12t^2-6u = 6(2t^2-u)$
g) $3t^2-5ty+4t = t(3t-5y+4)$ h) $24xy-16xz = 8x(3y-2z)$ i) $ab+bc-bd = b(a+c-d)$
j) $m^4+m^3+m^2 = m^2(m^2+m+1)$ k) $r(p+q)+s(p+q) = (p+q)(r+s)$ l) $6qp-8rs = 2(3qp-4rs)$

3. a) $(53 \times 48) + (53 \times 52) = 53(48+52) = 53 \times 100 = 5300$ b) $(74 \times 63) - (74 \times 53) = 74(63-53) = 74 \times 10 = 740$
c) $(2.7 \times 8.6) + (1.4 \times 2.7) = 2.7(8.6+1.4) = 2.7 \times 10 = 27$ d) $(3.9 \times 6.75) + (3.9 \times 3.25) = 3.9(6.75+3.25) = 3.9 \times 10 = 39$
e) $(63 \times 24) + (39 \times 63) + (63 \times 37) = 63(24+39+37) = 63 \times 100 = 6300$ f) $(0.17 \times 7.9) + (2.8 \times 0.17) - (0.7 \times 0.17) = 0.17(7.9+2.8-0.7) = 0.17 \times 10 = 1.7$

4. a) $9t+12r = 3(3t+4r)$ b) $ay-by = y(a-b)$ c) $24-6t = 6(4-t)$ d) $p^2-p = p(p-1)$
e) $8yz-9ut$ f) $abc+bcd = bc(a+d)$ g) $35m-14n = 7(5m-2n)$ h) $25t^2-15tx+20tz = 5t(5t-3x+4z)$
i) $5x^2-4y$ j) $15x-9y+6z = 3(5x-3y+2z)$ k) $6rt+3ts-12ty = 3t(2r+s-4y)$
l) $t^6+t^4-t^3 = t^3(t^3+t-1)$

Ex 4B

1. a) $x^2 - 7^2 = (x-7)(x+7)$ b) $a^2 - b^2 = (a-b)(a+b)$ c) $36 - a^2 = (6-a)(6+a)$
 d) $1 - t^2 = (1-t)(1+t)$ e) $25a^2 - b^2 = (5a-b)(5a+b)$ f) $9t^2 - 25s^2 = (3t-5s)(3t+5s)$
 g) $64c^2 - 49d^2 = (8c-7d)(8c+7d)$ h) $36a^2 - 1 = (6a-1)(6a+1)$ i) $c^2 - 16d^2 = (c-4d)(c+4d)$
 j) $100x^2 - 49y^2 = (10x-7y)(10x+7y)$

2. a) $t^4 - 1 = (t^2-1)(t^2+1) = (t-1)(t+1)(t^2+1)$ b) $16 - a^4 = (4-a^2)(4+a^2) = (2-a)(2+a)(4+a^2)$
 c) $25b - p^4 = (16-p^2)(16+p^2) = (4-p)(4+p)(16+p^2)$ d) $t^4 - 81 = (t^2-9)(t^2+9) = (t-3)(t+3)(t^2+9)$
 e) $(m-n)^2 - (m+n)^2 = ((m-n)-(m+n))(m-n+m+n) = -2n \times 2m = -4mn$ f) $x^2 - (y+z)^2 = (x-(y+z))(x+(y+z)) = (x-y-z)(x+y+z)$

3. a) $t^2 - 9^2 = (t-9)(t+9)$ b) $s^2 - t^2 = (s-t)(s+t)$ c) $81 - p^2 = (9-p)(9+p)$ d) $m^2 - 1 = (m-1)(m+1)$
 e) $a^2 - 4b^2 = (a-2b)(a+2b)$ f) $25p^2 - q^2 = (5p-q)(5p+q)$ g) $25a^2 - 36b^2 = (5a-6b)(5a+6b)$
 h) $100 - 49^2 = (10-49)(10+49) = -39 \times 59 = -2301$ i) $9x^2 - 25y^2 = (3x-5y)(3x+5y)$ j) $a^2 - 4b^2c^2 = (a-2bc)(a+2bc)$

4. a) $t^2 - 1^2 = (t-1)(t+1)$ b) $2\pi R^2 - 4(2\pi r^2) = 2\pi(R^2 - 4r^2) = 2\pi(R-2r)(R+2r)$

5. a) $x^2 = 10^2 - a^2 = (10-a)(10+a)$
 $x = \sqrt{(10-a)(10+a)} \text{ cm}$ b) $x^2 = t^2 - 4^2 = (t-4)(t+4)$
 $x = \sqrt{(t-4)(t+4)} \text{ cm}$

6. a) $x^2 = 15^2 - 10^2 = (15-10)(15+10) = 5 \times 25 = 125$
 $x = \sqrt{125} = \sqrt{25} \sqrt{5} = 5\sqrt{5} \text{ cm}$ b) $x^2 = 11^2 - 3^2 = (11-3)(11+3) = 8 \times 14 = 112$
 $x = \sqrt{112} = \sqrt{16} \sqrt{7} = 4\sqrt{7} \text{ cm}$

Ex 4C

$$1. a) 8p^2 - 8q^2 \\ = 8(p^2 - q^2) \\ = 8(p-q)(p+q)$$

$$d) 2a^2 - 18b^2 \\ = 2(a^2 - 9b^2) \\ = 2(a-3b)(a+3b)$$

$$g) 45m^2 - 5n^2 \\ = 5(9m^2 - n^2) \\ = 5(3m-n)(3m+n)$$

$$j) 2a^3 - 8ab^2 \\ = 2a(a^2 - 4b^2) \\ = 2a(a-2b)(a+2b)$$

$$b) 3x^2 - 27 \\ = 3(x^2 - 9) \\ = 3(x-3)(x+3)$$

$$e) 9x^2 - 36y^2 \\ = 9(x^2 - 4y^2) \\ = 9(x-2y)(x+2y)$$

$$h) 2y^2 - 8z^2 \\ = 2(y^2 - 4z^2) \\ = 2(y-2z)(y+2z)$$

$$k) 28t^2 - 175r^2 \\ = 7(4t^2 - 25r^2) \\ = 7(2t-5r)(2t+5r)$$

$$c) 4y^2 - 100 \\ = 4(y^2 - 25) \\ = 4(y-5)(y+5)$$

$$f) 5 - 5t^2 \\ = 5(1 - t^2) \\ = 5(1-t)(1+t)$$

$$i) 3y^2 - 75z^2 \\ = 3(y^2 - 25z^2) \\ = 3(y-5z)(y+5z)$$

$$l) 8c^2 - 50d^2 \\ = 2(4c^2 - 25d^2) \\ = 2(2c-5d)(2c+5d)$$

Ex 4D

$$1. a) a^2 + 12a + 11 \\ = (a+11)(a+1) \quad \begin{array}{r} 1+11 \quad +11 \\ 1+1 \quad +1 \\ \hline \quad \quad +12 \end{array}$$

$$b) x^2 - 9x + 20 \\ = (x-4)(x-5) \quad \begin{array}{r} 1-4 \quad -4 \\ 1-5 \quad -5 \\ \hline \quad \quad -9 \end{array}$$

$$c) w^2 - 11w + 28 \\ = (w-4)(w-7) \quad \begin{array}{r} 1-4 \quad -4 \\ 1-7 \quad -7 \\ \hline \quad \quad -11 \end{array}$$

$$d) b^2 - 10b + 24 \\ = (b-6)(b-4) \quad \begin{array}{r} 1-6 \quad -6 \\ 1-4 \quad -4 \\ \hline \quad \quad -10 \end{array}$$

$$e) p^2 + 24p + 63 \\ = (p+21)(p+3) \quad \begin{array}{r} 1+21 \quad +21 \\ 1+3 \quad +3 \\ \hline \quad \quad +24 \end{array}$$

$$f) x^2 - 11x + 18 \\ = (x-2)(x-9) \quad \begin{array}{r} 1-2 \quad -2 \\ 1-9 \quad -9 \\ \hline \quad \quad -11 \end{array}$$

$$g) 19 - 20t + t^2 \\ = (19-t)(20-t) \quad \begin{array}{r} 19-1 \quad -1 \\ 1-1 \quad -19 \\ \hline \quad \quad -20 \end{array}$$

$$h) 34 + 19y + y^2 \\ = (17+y)(2+y) \quad \begin{array}{r} 17+1 \quad +2 \\ 2+1 \quad +17 \\ \hline \quad \quad +19 \end{array}$$

$$i) t^2 + t - 12 \\ = (t-3)(t+4) \quad \begin{array}{r} 1-3 \quad -3 \\ 1+4 \quad +4 \\ \hline \quad \quad +1 \end{array}$$

$$j) y^2 - 3y - 18 \\ = (y-6)(y+3) \quad \begin{array}{r} 1-6 \quad -6 \\ 1+3 \quad +3 \\ \hline \quad \quad -3 \end{array}$$

$$k) x^2 - 2x - 63 \\ = (x-9)(x+7) \quad \begin{array}{r} 1-9 \quad -9 \\ 1+7 \quad +7 \\ \hline \quad \quad -2 \end{array}$$

$$l) y^2 - 5y - 36 \\ = (y-9)(y+4) \quad \begin{array}{r} 1-9 \quad -9 \\ 1+4 \quad +4 \\ \hline \quad \quad -5 \end{array}$$

$$2. a) 7x^2 - 7y^2 \\ = 7(x^2 - y^2) \\ = 7(x-y)(x+y)$$

$$b) 2t^2 + 4t + 2 \\ = 2(t^2 + 2t + 1) \\ = 2(t+1)(t+1) \quad \begin{array}{r} 1+1 \quad +1 \\ 1+1 \quad +1 \\ \hline \quad \quad +2 \end{array}$$

$$c) 3a^2 - 18a + 24 \\ = 3(a^2 - 6a + 8) \\ = 3(a-4)(a-2) \quad \begin{array}{r} 1-4 \quad -4 \\ 1-2 \quad -2 \\ \hline \quad \quad -6 \end{array}$$

$$d) 4x^2 + 20x + 24 \\ = 4(x^2 + 5x + 6) \\ = 4(x+3)(x+2) \quad \begin{array}{r} 1+3 \quad +3 \\ 1+2 \quad +2 \\ \hline \quad \quad +5 \end{array}$$

$$e) \begin{aligned} 2t^2 + 22t + 48 & \\ = 2(t^2 + 11t + 24) & \quad \begin{array}{r} 1 \quad +3 \quad +3 \\ 1 \quad +8 \quad +8 \\ \hline \quad \quad +11 \end{array} \\ = 2(t+3)(t+8) & \end{aligned}$$

$$f) \begin{aligned} 2y^2 + 30y + 100 & \\ = 2(y^2 + 15y + 50) & \quad \begin{array}{r} 1 \quad +10 \quad +10 \\ 1 \quad +5 \quad +5 \\ \hline \quad \quad +15 \end{array} \\ = 2(y+10)(y+5) & \end{aligned}$$

$$g) \begin{aligned} 5m^2 - 10m - 40 & \\ = 5(m^2 - 2m - 8) & \quad \begin{array}{r} 1 \quad -4 \quad -4 \\ 1 \quad +2 \quad +2 \\ \hline \quad \quad -2 \end{array} \\ = 5(m-4)(m+2) & \end{aligned}$$

$$h) \begin{aligned} 6t^2 + 12t - 48 & \\ = 6(t^2 + 2t - 8) & \quad \begin{array}{r} 1 \quad +4 \quad +4 \\ 1 \quad -2 \quad -2 \\ \hline \quad \quad -2 \end{array} \\ = 6(t+4)(t-2) & \end{aligned}$$

$$3. a) \begin{aligned} t^2 - t - 6 & \\ = (t-3)(t+2) & \quad \begin{array}{r} 1 \quad -3 \quad -3 \\ 1 \quad +2 \quad +2 \\ \hline \quad \quad -1 \end{array} \end{aligned}$$

$$b) \begin{aligned} m^2 + 7m - 8 & \\ = (m+8)(m-1) & \quad \begin{array}{r} 1 \quad +8 \quad +8 \\ 1 \quad -1 \quad -1 \\ \hline \quad \quad +7 \end{array} \end{aligned}$$

$$c) \begin{aligned} x^2 + 6x - 7 & \\ = (x+7)(x-1) & \quad \begin{array}{r} 1 \quad +7 \quad +7 \\ 1 \quad -1 \quad -1 \\ \hline \quad \quad +6 \end{array} \end{aligned}$$

$$d) \begin{aligned} y^2 + 4y + 4 & \\ = (y+2)(y+2) & \quad \begin{array}{r} 1 \quad +2 \quad +2 \\ 1 \quad +2 \quad +2 \\ \hline \quad \quad +4 \end{array} \end{aligned}$$

$$e) \begin{aligned} u^2 + 2u - 3 & \\ = (u+3)(u-1) & \quad \begin{array}{r} 1 \quad +3 \quad +3 \\ 1 \quad -1 \quad -1 \\ \hline \quad \quad +2 \end{array} \end{aligned}$$

$$f) \begin{aligned} c^2 - c - 20 & \\ = (c+4)(c-5) & \quad \begin{array}{r} 1 \quad +4 \quad +4 \\ 1 \quad -5 \quad -5 \\ \hline \quad \quad -1 \end{array} \end{aligned}$$

$$g) \begin{aligned} y^2 - 5y - 24 & \\ = (y-8)(y+3) & \quad \begin{array}{r} 1 \quad -8 \quad -8 \\ 1 \quad +3 \quad +3 \\ \hline \quad \quad -5 \end{array} \end{aligned}$$

$$h) \begin{aligned} m^2 - 7m - 8 & \\ = (m-8)(m+1) & \quad \begin{array}{r} 1 \quad -8 \quad -8 \\ 1 \quad +1 \quad +1 \\ \hline \quad \quad -7 \end{array} \end{aligned}$$

$$i) \begin{aligned} 2p^2 + 4p - 30 & \\ = 2(p^2 + 2p - 15) & \quad \begin{array}{r} 1 \quad -3 \quad -3 \\ 1 \quad +5 \quad +5 \\ \hline \quad \quad +2 \end{array} \\ = 2(p-3)(p+5) & \end{aligned}$$

$$j) \begin{aligned} 3y^2 + 18y + 54 & \\ = 3(y^2 + 6y + 18) & \end{aligned}$$

$$k) \begin{aligned} 2x^2 - 6x - 20 & \\ = 2(x^2 - 3x - 10) & \quad \begin{array}{r} 1 \quad -5 \quad -5 \\ 1 \quad +2 \quad +2 \\ \hline \quad \quad -3 \end{array} \\ = 2(x-5)(x+2) & \end{aligned}$$

$$l) \begin{aligned} 4a^2 - 32a + 60 & \\ = 4(a^2 - 8a + 15) & \quad \begin{array}{r} 1 \quad -3 \quad -3 \\ 1 \quad -5 \quad -5 \\ \hline \quad \quad -8 \end{array} \\ = 4(a-3)(a-5) & \end{aligned}$$

Ex 4E

$$1. a) \begin{aligned} 2x^2 + 5x + 3 & \\ = (x+1)(2x+3) & \quad \begin{array}{r} 1 \quad +1 \quad +2 \\ 2 \quad +3 \quad +3 \\ \hline \quad \quad +5 \end{array} \end{aligned}$$

$$b) \begin{aligned} 3t^2 + t - 2 & \\ = (3t-2)(t+1) & \quad \begin{array}{r} 3 \quad -2 \quad -2 \\ 1 \quad +1 \quad +3 \\ \hline \quad \quad +1 \end{array} \end{aligned}$$

$$c) \begin{aligned} 12m^2 - 8m + 1 & \\ = (6m-1)(2m-1) & \quad \begin{array}{r} 6 \quad -1 \quad -2 \\ 2 \quad -1 \quad -6 \\ \hline \quad \quad -8 \end{array} \end{aligned}$$

$$d) \begin{aligned} 4y^2 + 7y - 2 & \\ = (4y-1)(y+2) & \quad \begin{array}{r} 4 \quad -1 \quad -1 \\ 1 \quad +2 \quad +8 \\ \hline \quad \quad +7 \end{array} \end{aligned}$$

$$e) \begin{aligned} 8u^2 + 10u - 3 & \\ = (4u-1)(2u+3) & \quad \begin{array}{r} 4 \quad -1 \quad -2 \\ 2 \quad +3 \quad +12 \\ \hline \quad \quad +10 \end{array} \end{aligned}$$

$$f) \begin{aligned} 4p^2 + 3p - 7 & \\ = (p-1)(4p+7) & \quad \begin{array}{r} 1 \quad -1 \quad -4 \\ 4 \quad +7 \quad +7 \\ \hline \quad \quad +3 \end{array} \end{aligned}$$

$$g) \begin{aligned} 4t^2 + 12t + 9 & \\ = (2t+3)(2t+3) & \quad \begin{array}{r} 2 \quad +3 \quad +6 \\ 2 \quad +3 \quad +6 \\ \hline \quad \quad +12 \end{array} \end{aligned}$$

$$h) \begin{aligned} 6m^2 + 17m - 3 & \\ = (m+3)(6m-1) & \quad \begin{array}{r} 1 \quad +3 \quad +5 \\ 6 \quad -1 \quad -1 \\ \hline \quad \quad +17 \end{array} \end{aligned}$$

$$i) \begin{aligned} -8y^2 - 2y + 3 & \\ = -(8y^2 + 2y - 3) & \quad \begin{array}{r} 4 \quad +3 \quad +6 \\ 2 \quad -1 \quad -4 \\ \hline \quad \quad +5 \end{array} \\ = -(4y+3)(2y-1) & \end{aligned}$$

$$2. a) 4t^2 + 14t + 6 \quad \begin{array}{r} 1+3+6 \\ 2+1+1 \\ \hline +7 \end{array}$$

$$= 2(2t^2 + 7t + 3)$$

$$= 2(t+3)(2t+1)$$

$$b) 6m^2 - 15m + 6 \quad \begin{array}{r} 2-1 \\ 1-2 \end{array}$$

$$= 3(2m^2 - 5m + 2)$$

$$= 3(2m-1)(m-2)$$

$$c) 15x^2 - 10x - 40 \quad \begin{array}{r} 3+4+4 \\ 1-2-6 \\ \hline -2 \end{array}$$

$$= 5(3x^2 - 2x - 8)$$

$$= 5(3x+4)(x-2)$$

$$d) 45y^2 + 36y - 9 \quad \begin{array}{r} 5+1 \\ 1+1 \end{array}$$

$$= 9(5y^2 + 4y - 1)$$

$$= 9(5y-1)(y+1)$$

$$e) 8u^2 + 4u - 4 \quad \begin{array}{r} 2-1-1 \\ 1+1+2 \\ \hline +1 \end{array}$$

$$= 4(2u^2 + u - 1)$$

$$= 4(2u-1)(u+1)$$

$$f) 18c^2 - 12c - 48 \quad \begin{array}{r} 3+4 \\ 1-2 \end{array}$$

$$= 6(3c^2 - 2c - 8)$$

$$= 6(3c+4)(c-2)$$

$$g) x^2 + 8xy + 12y^2 \quad \begin{array}{r} 1+6+6 \\ 1+2+2 \\ \hline +8 \end{array}$$

$$= (x+6y)(x+2y)$$

$$h) 4m^2 - 7mn - 2n^2 \quad \begin{array}{r} 1- \\ 4+ \end{array}$$

$$= (m-2n)(4m+n)$$

$$i) 3x^2 - 5x - 2 \quad \begin{array}{r} 3+1+1 \\ 1-2-6 \\ \hline -5 \end{array}$$

$$= (3x+1)(x-2)$$

$$j) 2b^2 - 11b - 21 \quad \begin{array}{r} 2+3+3 \\ 1-7-10 \\ \hline -1 \end{array}$$

$$= (2b+3)(b-7)$$

$$3. a) 8y - 24$$

$$= 8(y-3)$$

$$b) m^2 - 36$$

$$= (m-6)(m+6)$$

$$c) x^2 + 4x + 4$$

$$= (x+2)(x+2)$$

$$d) 6y^2 + 4y$$

$$= 2y(3y+2)$$

$$e) 2u^2 + 3u - 5 \quad \begin{array}{r} 2+5+5 \\ 1-1-2 \\ \hline +3 \end{array}$$

$$= (2u+5)(u-1)$$

$$f) 14c^2 - 56cd^2$$

$$= 14(c^2 - 4cd^2)$$

$$= 14(c-2d)(c+2d)$$

$$g) 2m^2 - 7m - 15 \quad \begin{array}{r} 2+3+3 \\ 1-5-10 \\ \hline -7 \end{array}$$

$$= (2m+3)(m-5)$$

$$h) 16p^2 - 8p + 1 \quad \begin{array}{r} 4-1-4 \\ 4-1-4 \\ \hline -8 \end{array}$$

$$= (4p-1)(4p-1)$$

$$i) 10y^2 - y - 3 \quad \begin{array}{r} 5-3-6 \\ 2+1+5 \\ \hline -1 \end{array}$$

$$= (5y-3)(2y+1)$$

Chapter 5 Completing the Square in a Quadratic Expression with Unitary x^2 coefficient

Ex 5A

1. a) $x^2 + 6x = (x+3)^2 - 9$ b) $x^2 + 14x = (x+7)^2 - 49$ c) $y^2 + 20y = (y+10)^2 - 100$
- d) $m^2 - 2m = (m-1)^2 - 1$ e) $t^2 - 8t = (t-4)^2 - 16$ f) $a^2 - 12a = (a-6)^2 - 36$
- g) $y^2 - 6y = (y-3)^2 - 9$ h) $w^2 - w = (w - 1/2)^2 - 1/4$ i) $x^2 + 5x = (x + 5/2)^2 - 25/4$
- j) $y^2 + 4y = (y+2)^2 - 4$ k) $t^2 - 30t = (t-15)^2 - 225$ l) $x^2 + 7x = (x + 7/2)^2 - 49/4$
2. a) $x^2 + 10x + 3 = (x+5)^2 - 22$
 $a=5 \quad b=-22$ b) $y^2 - 4y + 6 = (y-2)^2 + 2$
 $a=-2 \quad b=2$ c) $t^2 + 14t - 9 = (t+7)^2 - 58$
 $a=7 \quad b=-58$
- d) $m^2 - 6m + 4 = (m-3)^2 - 5$
 $a=-3 \quad b=-5$ e) $w^2 - 20w + 10 = (w-10)^2 - 90$
 $a=-10 \quad b=-90$ f) $x^2 + 12x - 3 = (x+6)^2 - 39$
 $a=6 \quad b=-39$
- g) $x^2 + 8x + 1 = (x+4)^2 - 15$
 $a=4 \quad b=-15$ h) $m^2 + 7m + 3 = (m + 7/2)^2 - 37/4$
 $a=7/2 \quad b=-37/4$ i) $x^2 + 3x - 1 = (x + 3/2)^2 - 13/4$
 $a=3/2 \quad b=-13/4$
- j) $a^2 - 4a - 2 = (a-2)^2 - 6$
 $a=-2 \quad b=-6$ k) $w^2 - 18w + 5 = (w-9)^2 - 76$
 $a=-9 \quad b=-76$ l) $t^2 + 9t - 3 = (t + 9/2)^2 - 93/4$
 $a=9/2 \quad b=-93/4$
3. a) $m^2 + 2m = (m+1)^2 - 1$ b) $t^2 - 10t = (t-5)^2 - 25$ c) $x^2 + 12x = (x+6)^2 - 36$
- d) $y^2 - 8y + 4 = (y-4)^2 - 12$ e) $a^2 - 4a - 3 = (a-2)^2 - 7$ f) $t^2 + 22t - 15 = (t+11)^2 - 135$
- g) $p^2 + 16p - 7 = (p+8)^2 - 71$ h) $m^2 + 2m + 7 = (m+1)^2 + 6$ i) $y^2 + 10y - 5 = (y+5)^2 - 30$
- j) $y^2 - 5y + 3 = (y - 5/2)^2 - 13/4$ k) $a^2 - a + 4 = (a - 1/2)^2 + 15/4$ l) $x^2 + 7x - 2 = (x + 7/2)^2 - 57/4$

Ex 5B

1. a) $y^2 + 2y - 7 = 0$
 $(y+1)^2 - 8 = 0$
 $(y+1)^2 = 8$
 $y+1 = \pm\sqrt{8}$
 $y = 1 \pm \sqrt{8}$
 $= 1 \pm 2\sqrt{2}$

b) $t^2 - 6t - 11 = 0$
 $(t-3)^2 - 20 = 0$
 $(t-3)^2 = 20$
 $t-3 = \pm\sqrt{20}$
 $t = 3 \pm 2\sqrt{5}$

c) $x^2 - 10x + 4 = 0$
 $(x-5)^2 - 21 = 0$
 $(x-5)^2 = 21$
 $x-5 = \pm\sqrt{21}$
 $x = 5 \pm \sqrt{21}$

d) $a^2 + 4a + 1 = 0$
 $(a+2)^2 - 3 = 0$
 $(a+2)^2 = 3$
 $a+2 = \pm\sqrt{3}$
 $a = -2 \pm \sqrt{3}$

e) $y^2 + 8y + 13 = 0$
 $(y+4)^2 - 3 = 0$
 $(y+4)^2 = 3$
 $y+4 = \pm\sqrt{3}$
 $y = -4 \pm \sqrt{3}$

f) $t^2 - 3t - 6 = 0$
 $(t - 3/2)^2 - 33/4 = 0$
 $(t - 3/2)^2 = 33/4$
 $t - 3/2 = \pm\sqrt{33}/2$
 $t = 3/2 \pm \sqrt{33}/2$

g) $x^2 + 14x - 5 = 0$
 $(x+7)^2 - 54 = 0$
 $(x+7)^2 = 54$
 $x+7 = \pm\sqrt{54}$
 $x = -7 \pm 3\sqrt{6}$

h) $x^2 - 6x + 3 = 0$
 $(x-3)^2 - 6 = 0$
 $(x-3)^2 = 6$
 $x-3 = \pm\sqrt{6}$
 $x = 3 \pm \sqrt{6}$

l) $x^2 + 6x + 3 = 0$
 $(x+3)^2 - 6 = 0$
 $(x+3)^2 = 6$
 $x+3 = \pm\sqrt{6}$
 $x = -3 \pm \sqrt{6}$

2. a) $m^2 + 8m + 3 = 0$
 $(m+4)^2 - 13 = 0$
 $(m+4)^2 = 13$
 $m+4 = \pm\sqrt{13}$
 $m = -4 \pm \sqrt{13}$
 $= -7.6, -0.4$ (to 1 dp)

b) $x^2 - 12x + 4 = 0$
 $(x-6)^2 - 32 = 0$
 $(x-6)^2 = 32$
 $x-6 = \pm\sqrt{32}$
 $x = 6 \pm \sqrt{32}$
 $= 0.3, 11.7$ (to 1 dp)

$$\begin{aligned}
 c) \quad w^2 - 5w - 10 &= 0 \\
 (w - 5/2)^2 - 65/4 &= 0 \\
 (w - 5/2)^2 &= 65/4 \\
 w - 5/2 &= \pm \sqrt{65/4} \\
 w &= \frac{5}{2} \pm \sqrt{\frac{65}{4}} \\
 &= -1.5, 6.5 \text{ (to 1 dp)}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad t^2 - 4t + 1 &= 0 \\
 (t - 2)^2 - 3 &= 0 \\
 (t - 2)^2 &= 3 \\
 t - 2 &= \pm \sqrt{3} \\
 t &= 2 \pm \sqrt{3} \\
 &= 0.3, 3.7 \text{ (to 1 dp)}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad a^2 + 14a + 13 &= 0 \\
 (a + 7)^2 - 36 &= 0 \\
 (a + 7)^2 &= 36 \\
 a + 7 &= \pm 6 \\
 a &= 7 \pm 6 \\
 &= 1, 13
 \end{aligned}$$

$$\begin{aligned}
 f) \quad x^2 + 3x - 7 &= 0 \\
 (x + 3/2)^2 - 37/4 &= 0 \\
 (x + 3/2)^2 &= 37/4 \\
 x + 3/2 &= \pm \sqrt{37/4} \\
 x &= \frac{3}{2} \pm \sqrt{\frac{37}{4}} \\
 &= -1.5, 4.5 \text{ (to 1 dp)}
 \end{aligned}$$

$$\begin{aligned}
 3. a) \quad x^2 + 2x - 5 &= 0 \\
 (x + 1)^2 - 6 &= 0 \\
 (x + 1)^2 &= 6 \\
 x + 1 &= \pm \sqrt{6} \\
 x &= -1 \pm \sqrt{6}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad x^2 - 4x - 7 &= 0 \\
 (x - 2)^2 - 11 &= 0 \\
 (x - 2)^2 &= 11 \\
 x - 2 &= \pm \sqrt{11} \\
 x &= 2 \pm \sqrt{11}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad x^2 + 2x - 9 &= 0 \\
 (x + 1)^2 - 10 &= 0 \\
 (x + 1)^2 &= 10 \\
 x + 1 &= \pm \sqrt{10} \\
 x &= -1 \pm \sqrt{10}
 \end{aligned}$$

Ex 5C.

$$\begin{aligned}
 1. a) \quad 4x^2 + 16x + 3 & \\
 &= 4(x^2 + 4x) + 3 \\
 &= 4\{(x + 2)^2 - 4\} + 3 \\
 &= 4(x + 2)^2 - 16 + 3 \\
 &= 4(x + 2)^2 - 13
 \end{aligned}$$

$$\begin{aligned}
 b) \quad 2y^2 + 12y - 3 & \\
 &= 2(y^2 + 6y) - 3 \\
 &= 2\{(y + 3)^2 - 9\} - 3 \\
 &= 2(y + 3)^2 - 18 - 3 \\
 &= 2(y + 3)^2 - 21
 \end{aligned}$$

$$\begin{aligned}
 c) \quad 5t^2 - 30t - 8 & \\
 &= 5(t^2 - 6t) - 8 \\
 &= 5\{(t - 3)^2 - 9\} - 8 \\
 &= 5(t - 3)^2 - 45 - 8 \\
 &= 5(t - 3)^2 - 53
 \end{aligned}$$

$$\begin{aligned}
 d) \quad -m^2 + 6m + 2 & \\
 &= -\{m^2 - 6m\} + 2 \\
 &= -\{(m - 3)^2 - 9\} + 2 \\
 &= -(m - 3)^2 + 9 + 2 \\
 &= -(m - 3)^2 + 11
 \end{aligned}$$

$$\begin{aligned} e) & 6\omega^2 + 12\omega - 4 \\ &= 6(\omega^2 + 2\omega) - 4 \\ &= 6\{(\omega+1)^2 - 1\} - 4 \\ &= 6(\omega+1)^2 - 6 - 4 \\ &= 6(\omega+1)^2 - 10 \end{aligned}$$

$$\begin{aligned} f) & 3t^2 + 12t - 3 \\ &= 3(t^2 + 4t) - 3 \\ &= 3\{t(t+4) - 1\} - 3 \\ &= 3(t+2)^2 - 12 - 3 \\ &= 3(t+2)^2 - 15 \end{aligned}$$

Chapter 6 Reducing an Algebraic Fraction to its Simplest Form

Ex 6A

1. a) $\frac{5a^2}{a} = 5a$

b) $\frac{b}{3b} = \frac{1}{3}$

c) $\frac{8c^2}{2c^2} = 4$

d) $\frac{2x}{4y} = \frac{x}{2y}$

e) $\frac{6e}{3e} = 2$

f) $\frac{7xy}{14y} = \frac{x}{2}$

g) $\frac{9ab^2}{12ab} = \frac{3b}{4}$

h) $\frac{5xy^2}{15x^2y} = \frac{y}{3x}$

i) $\frac{(3p)^2}{6p} = \frac{9p^2}{6p} = \frac{3p}{2}$

j) $\frac{7y+3y^2}{3y}$
 $= \frac{y(7+3y)}{3y}$
 $= \frac{7+3y}{3}$

k) $\frac{8a+2}{2a}$
 $= \frac{2(4a+1)}{2a}$
 $= \frac{4a+1}{a}$

l) $\frac{2x+3x^2}{3x}$
 $= \frac{x(2+3x)}{3x}$
 $= \frac{2+3x}{3}$

m) $\frac{5m}{7m-m^2}$
 $= \frac{5m}{m(7-m)}$
 $= \frac{5}{7-m}$

n) $\frac{3xy}{4x+6x^2}$
 $= \frac{3xy}{3x(3+2x)}$
 $= \frac{y}{3+2x}$

o) $\frac{8x+4}{4y}$
 $= \frac{4(2x+1)}{4y}$
 $= \frac{2x+1}{y}$

p) $\frac{5p+10q}{10pq}$
 $= \frac{5(p+2q)}{10pq}$
 $= \frac{p+2q}{3pq}$

q) $\frac{12x-4xy}{8x^2}$
 $= \frac{4x(3-y)}{8x^2}$
 $= \frac{3-y}{2x}$

r) $\frac{4ab+8a^2}{5ab+10a^2}$
 $= \frac{4a(b+2a)}{5a(b+2a)}$
 $= \frac{4}{5}$

s) $\frac{(2r)^2-12r}{4r}$
 $= \frac{4r^2-12r}{4r}$
 $= \frac{4r(r-3)}{4r}$
 $= r-3$

t) $\frac{3a^2bc}{3a^3bc+3a^2b^2c}$
 $= \frac{3a^2bc}{3a^2bc(a+b)}$
 $= \frac{1}{a+b}$

2. a) $\frac{a^2-4}{3a-6}$
 $= \frac{(a-2)(a+2)}{3(a-2)}$
 $= \frac{a+2}{3}$

b) $\frac{3b+9}{b^2-9}$
 $= \frac{3(b+3)}{(b-3)(b+3)}$
 $= \frac{3}{b-3}$

c) $\frac{c^2-1}{5c-5}$
 $= \frac{(c-1)(c+1)}{5(c-1)}$
 $= \frac{c+1}{5}$

d) $\frac{d^2-3d}{d^2-2d-3}$
 $= \frac{d(d-3)}{(d-3)(d+1)}$
 $= \frac{d}{d+1}$

e) $\frac{e^2+4e}{2e^2+8e}$
 $= \frac{e(e+4)}{2e(e+4)}$
 $= \frac{1}{2}$

f) $\frac{x^2-16}{x^2+2x-8}$
 $= \frac{(x-4)(x+4)}{(x+4)(x-2)}$
 $= \frac{x-4}{x-2}$

$\frac{+4}{-2} = \frac{-2}{+2}$

$$\textcircled{9}) \frac{x^2 + 6x - 40}{x^2 - 100}$$

$$= \frac{(x-4)(x+10)}{(x-10)(x+10)}$$

$$= \frac{x-4}{x-10}$$

$$\text{n) } \frac{x^2 + 6x + 5}{x^2 - x - 2}$$

$$= \frac{(x+5)(x+1)}{(x-2)(x+1)}$$

$$= \frac{x+5}{x-2}$$

$$\text{i) } \frac{x^2 - 4x - 21}{x^2 - 5x - 14}$$

$$= \frac{(x+3)(x-7)}{(x+2)(x-7)}$$

$$= \frac{x+3}{x+2}$$

$$\frac{1+3}{1-7} \quad \frac{-3}{-4} \quad \frac{1+2}{1-7} \quad \frac{+2}{-5}$$

$$\text{j) } \frac{2x^2 + 3x - 2}{x^2 + 5x + 6}$$

$$= \frac{(2x-1)(x+2)}{(x+3)(x+2)}$$

$$= \frac{2x-1}{x+3}$$

$$\frac{2-1}{1+2} \quad \frac{-1}{+3} \quad \frac{1+3}{1+2} \quad \frac{+2}{+5}$$

$$\text{k) } \frac{4x^2 + 4x - 3}{6x^2 + x - 2}$$

$$= \frac{(2x+3)(2x-1)}{(3x+2)(2x-1)}$$

$$= \frac{2x+3}{3x+2}$$

$$\frac{2+3}{2-1} \quad \frac{+2}{+4} \quad \frac{3+2}{2-1} \quad \frac{+4}{-3}$$

$$\text{l) } \frac{6x^2 - x - 12}{6x^2 + 5x - 4}$$

$$= \frac{(3x+4)(2x-3)}{(3x+4)(2x-1)}$$

$$= \frac{2x-3}{2x-1}$$

$$\frac{3+4}{2-3} \quad \frac{-3}{-1}$$

$$\frac{3+4}{2-1} \quad \frac{+8}{-3} \quad \frac{+8}{+5}$$

$$\text{3. a) } \frac{x^2 + 3x + 2}{4x^2 - 4}$$

$$= \frac{(x+2)(x+1)}{4(x^2-1)}$$

$$= \frac{(x+2)(x+1)}{4(x-1)(x+1)}$$

$$= \frac{x+2}{4(x-1)}$$

$$\frac{1+2}{1-1} \quad \frac{+2}{+1}$$

$$\text{b) } \frac{2x^2 + 4x}{2x^2 - 8}$$

$$= \frac{2x(x+2)}{2(x^2-4)}$$

$$= \frac{2x(x+2)}{2(x-2)(x+2)}$$

$$= \frac{x}{x-2}$$

$$\text{c) } \frac{3x^2 - 3x - 6}{6x^2 - 6}$$

$$= \frac{3(x^2 - x - 2)}{6(x^2 - 1)}$$

$$= \frac{3(x-2)(x+1)}{6(x-1)(x+1)}$$

$$= \frac{x-2}{2(x-1)}$$

$$\frac{1+2}{1-1} \quad \frac{-2}{-1}$$

$$\text{d) } \frac{x^3 - 4x}{x^2 - 5x + 6}$$

$$= \frac{x(x^2 - 4)}{(x-3)(x-2)}$$

$$= \frac{x(x-2)(x+2)}{(x-3)(x-2)}$$

$$= \frac{x(x+2)}{x-3}$$

$$\frac{1-3}{1-2} \quad \frac{-3}{-1}$$

$$\text{e) } \frac{(x+2)^3}{(x^2-4)(x+2)}$$

$$= \frac{(x+2)^3}{(x-2)(x+2)(x+2)}$$

$$= \frac{x+2}{x-2}$$

$$\text{f) } \frac{12x^3 - 27x}{12x^3 + 6x^2 - 18x}$$

$$= \frac{3x(4x^2 - 9)}{3x(4x^2 - 9)}$$

$$= \frac{3x(2x-3)(2x+3)}{6x(2x+3)(x-1)}$$

$$= \frac{2x-3}{2(x-1)}$$

$$\frac{2+3}{1-1} \quad \frac{+3}{-2} \quad \frac{+3}{+1}$$

Ex 6B

$$1. a) \frac{x + \frac{1}{x}}{x} \times \frac{x}{x}$$

$$= \frac{x^2 + 1}{x^2}$$

$$b) \frac{3x + \frac{1}{4}}{\frac{1}{4}} \times \frac{4}{4}$$

$$= \frac{12x + 1}{1}$$

$$= 12x + 1$$

$$c) \frac{3x - \frac{1}{x}}{2} \times \frac{x}{x}$$

$$= \frac{3x^2 - 1}{2x}$$

$$d) \frac{x + \frac{1}{3}}{2x - \frac{1}{2}} \times \frac{6}{6}$$

$$= \frac{6x + 2}{12x - 3}$$

$$e) \frac{3x + \frac{1}{x}}{x + \frac{2}{x}} \times \frac{x}{x}$$

$$= \frac{3x^2 + 1}{x^2 + 2}$$

$$f) \frac{x^2 - \frac{1}{4}}{2x + 1} \times \frac{4}{4}$$

$$= \frac{4x^2 - 1}{8x + 4}$$

$$= \frac{(2x - 1)(2x + 1)}{4(2x + 1)}$$

$$= \frac{2x - 1}{4}$$

$$2. a) \frac{\frac{1}{x} + \frac{1}{2}}{\frac{1}{x} - \frac{1}{2}} \times \frac{2x}{2x}$$

$$= \frac{2 + x}{2 - x}$$

$$b) \frac{\frac{a}{b} + \frac{a}{c}}{\frac{a}{b} - \frac{a}{c}} \times \frac{bc}{bc}$$

$$= \frac{ac + ab}{ac - ab}$$

Ex 6C

$$a) \frac{x - 3}{3 - x}$$

$$= \frac{x - 3}{-(x - 3)}$$

$$= -1$$

$$b) \frac{9x - 3y}{2y - 6x}$$

$$= \frac{3(3x - y)}{-2(3x - y)}$$

$$= -\frac{3}{2}$$

$$c) \frac{3x^2 - 2xy}{4xy - 6x^2}$$

$$= \frac{x(3x - 2y)}{-2x(3x - 2y)}$$

$$= -\frac{1}{2}$$

$$d) \frac{x^2 - 9}{6 - 2x}$$

$$= \frac{(x - 3)(x + 3)}{-2(x - 3)}$$

$$= -\frac{(x + 3)}{2}$$

$$e) \frac{1 - x^2}{x^3 - x}$$

$$= \frac{(1 - x)(1 + x)}{x(x^2 - 1)}$$

$$= \frac{(1 - x)(1 + x)}{x(x + 1)(x - 1)}$$

$$= \frac{-(x - 1)}{x(x - 1)}$$

$$= -\frac{1}{x}$$

$$f) \frac{x^2 - a^2}{a - x}$$

$$= \frac{(x - a)(x + a)}{a - x}$$

$$= -\frac{(a - x)(x + a)}{a - x}$$

$$= -(x + a)$$

$$g) \frac{6 - x - x^2}{x^2 - 9}$$

$$= \frac{(3 + x)(2 - x)}{(x - 3)(x + 3)}$$

$$= \frac{2 - x}{x - 3}$$

$$h) \frac{x^2 - 2xy + y^2}{y^2 - x^2}$$

$$= \frac{(x - y)(x - y)}{(y - x)(y + x)}$$

$$= \frac{(x - y)(x - y)}{-(x - y)(y + x)}$$

$$= -\frac{(x - y)}{y + x}$$

Chapter 7 Applying One of The Four Operations to Algebraic Fractions

Ex 7A

$$\begin{aligned} 1. a) \quad & \frac{5x}{8} - \frac{x}{8} \\ & = \frac{4x}{8} \\ & = \frac{x}{2} \end{aligned}$$

$$\begin{aligned} b) \quad & \frac{7x}{8} + \frac{x}{4} \\ & = \frac{7x}{8} + \frac{2x}{8} \\ & = \frac{9x}{8} \end{aligned}$$

$$\begin{aligned} c) \quad & \frac{3x}{4} - \frac{x}{5} \\ & = \frac{15x}{20} - \frac{4x}{20} \\ & = \frac{11x}{20} \end{aligned}$$

$$\begin{aligned} d) \quad & \frac{x}{3} + \frac{5x}{9} \\ & = \frac{3x}{9} + \frac{5x}{9} \\ & = \frac{8x}{9} \end{aligned}$$

$$\begin{aligned} e) \quad & \frac{x}{2} + \frac{x}{3} + \frac{x}{6} \\ & = \frac{3x}{6} + \frac{2x}{6} + \frac{x}{6} \\ & = \frac{6x}{6} \\ & = x \end{aligned}$$

$$\begin{aligned} f) \quad & \frac{x}{3} - \frac{x}{4} + \frac{5x}{6} \\ & = \frac{4x}{12} - \frac{3x}{12} + \frac{10x}{12} \\ & = \frac{11x}{12} \end{aligned}$$

$$\begin{aligned} g) \quad & \frac{x}{2} + \frac{2x}{3} - \frac{3x}{4} \\ & = \frac{6x}{12} + \frac{8x}{12} - \frac{9x}{12} \\ & = \frac{5x}{12} \end{aligned}$$

$$\begin{aligned} h) \quad & \frac{5x}{3} - \frac{x}{4} - \frac{x}{5} \\ & = \frac{100x}{60} - \frac{15x}{60} - \frac{12x}{60} \\ & = \frac{73x}{60} \end{aligned}$$

$$\begin{aligned} 2. a) \quad & \frac{5}{x} - \frac{2}{y} \\ & = \frac{5y - 2x}{xy} \end{aligned}$$

$$\begin{aligned} b) \quad & \frac{a}{c} + \frac{b}{d} \\ & = \frac{ad + bc}{cd} \end{aligned}$$

$$\begin{aligned} c) \quad & \frac{3}{x} + \frac{2}{3x} \\ & = \frac{4x}{x+8} \\ & = \frac{17}{12x} \end{aligned}$$

$$\begin{aligned} d) \quad & \frac{7}{4x} - \frac{3}{5x^2} \\ & = \frac{35x}{20x^2} - \frac{12}{20x^2} \\ & = \frac{35x - 12}{20x^2} \end{aligned}$$

$$\begin{aligned} e) \quad & \frac{1}{x} + \frac{2}{y} + \frac{3}{z} \\ & = \frac{yz}{xyz} + \frac{2xz}{xyz} + \frac{3xy}{xyz} \\ & = \frac{yz + 2xz + 3xy}{xyz} \end{aligned}$$

$$\begin{aligned} f) \quad & \frac{1}{pr} - \frac{3}{qr} \\ & = \frac{q}{pqr} - \frac{3p}{pqr} \\ & = \frac{q - 3p}{pqr} \end{aligned}$$

$$\begin{aligned} g) \quad & \frac{3}{s^2t} - \frac{3}{st^2} \\ & = \frac{3t}{s^2t^2} - \frac{3s}{s^2t^2} \\ & = \frac{3t - 3s}{s^2t^2} \end{aligned}$$

$$\begin{aligned} h) \quad & \frac{a}{d} - \frac{b}{e} + \frac{c}{f} \\ & = \frac{aef - bdf + cde}{def} \end{aligned}$$

$$\begin{aligned} \text{a) } 7 + \frac{3}{x} \\ &= \frac{7x}{x} + \frac{3}{x} \\ &= \frac{7x+3}{x} \end{aligned}$$

$$\begin{aligned} \text{b) } 5 - \frac{2}{3x} \\ &= \frac{15x}{3x} - \frac{2}{3x} \\ &= \frac{15x-2}{3x} \end{aligned}$$

$$\begin{aligned} \text{c) } x - \frac{2}{3} \\ &= \frac{3x}{3} - \frac{2}{3} \\ &= \frac{3x-2}{3} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{2}{3x} + 1 \\ &= \frac{2}{3x} + \frac{3x}{3x} \\ &= \frac{2+3x}{3x} \end{aligned}$$

$$\begin{aligned} \text{e) } x + \frac{5}{x} \\ &= \frac{x^2}{x} + \frac{5}{x} \\ &= \frac{x^2+5}{x} \end{aligned}$$

$$\begin{aligned} \text{f) } 7 + \frac{x-y}{x} \\ &= \frac{7x}{x} + \frac{x-y}{x} \\ &= \frac{8x-y}{x} \end{aligned}$$

$$\begin{aligned} \text{g) } 3 - \frac{x-y}{x} \\ &= \frac{3x}{x} - \frac{x-y}{x} \\ &= \frac{2x+y}{x} \end{aligned}$$

$$\begin{aligned} \text{h) } x^2 - \frac{5}{x} \\ &= \frac{x^3}{x} - \frac{5}{x} \\ &= \frac{x^3-5}{x} \end{aligned}$$

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{R_2+R_1}{R_1R_2} \end{aligned}$$

$$\text{w. } R = \frac{R_1R_2}{R_1+R_2}.$$

Ex 7B

$$\begin{aligned} \text{a) } \frac{x}{3} + \frac{x-2}{6} \\ &= \frac{2x}{6} + \frac{x-2}{6} \\ &= \frac{3x-2}{6} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{x+1}{4} + \frac{x-3}{8} \\ &= \frac{2x+2}{8} + \frac{x-3}{8} \\ &= \frac{3x-1}{8} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{2x-1}{3} - \frac{x}{4} \\ &= \frac{8x-4}{12} - \frac{3x}{12} \\ &= \frac{5x-4}{12} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{x-3}{3} + \frac{x-2}{5} \\ &= \frac{5x-15}{15} + \frac{3x-6}{15} \\ &= \frac{8x-21}{15} \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{2x-2}{3} - \frac{x+1}{2} \\ &= \frac{4x-4}{6} - \frac{3x+3}{6} \\ &= \frac{-x-7}{6} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{2x-1}{3} - \frac{x-3}{4} \\ &= \frac{8x-4}{12} - \frac{3x-9}{12} \\ &= \frac{5x+5}{12} \end{aligned}$$

$$\begin{aligned} \text{g) } \frac{2x-1}{3} + \frac{x-3}{4} - \frac{2x-3}{6} \\ &= \frac{8x-4}{12} + \frac{3x-9}{12} - \frac{4x-6}{12} \\ &= \frac{7x-7}{12} \end{aligned}$$

$$\begin{aligned} \text{2. a) } \frac{3}{x+1} - \frac{2}{x} \\ &= \frac{3x}{x(x+1)} - \frac{2x+2}{x(x+1)} \\ &= \frac{x-2}{x(x+1)} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{4}{x-2} + \frac{3}{x} \\ &= \frac{4x}{x(x-2)} + \frac{3x-6}{x(x-2)} \\ &= \frac{7x-6}{x(x-2)} \end{aligned}$$

$$c) \frac{5}{x-2} + \frac{3}{x+3}$$

$$= \frac{5x+15+3x-6}{(x-2)(x+3)}$$

$$= \frac{8x+9}{(x-2)(x+3)}$$

$$d) \frac{3}{x+1} - \frac{2}{1-x}$$

$$= \frac{3-3x-2x-2}{(x+1)(1-x)}$$

$$= \frac{1-5x}{(x+1)(1-x)}$$

$$e) \frac{2}{2x+1} + \frac{3}{x-1}$$

$$= \frac{2x-2+6x+3}{(2x+1)(x-1)}$$

$$= \frac{8x+1}{(2x+1)(x-1)}$$

$$f) \frac{7}{3x-1} - \frac{2}{x+1}$$

$$= \frac{7x+7-6x+2}{(3x-1)(x+1)}$$

$$= \frac{x+9}{(3x-1)(x+1)}$$

$$g) \frac{x+1}{x-2} + \frac{3}{x-1}$$

$$= \frac{x^2-1+3x-6}{(x-2)(x-1)}$$

$$= \frac{x^2+3x-7}{(x-2)(x-1)}$$

$$h) \frac{x^2+1}{x+3} - \frac{x-1}{x+1}$$

$$= \frac{(x^3+x^2+x+1)-(x^2+3x-x-3)}{(x+3)(x+1)}$$

$$= \frac{x^3-x+4}{(x+3)(x+1)}$$

$$i) \frac{x+3}{2x+1} - \frac{1-2x^2}{x-1}$$

$$= \frac{(x^2-x+3x-3)-(2x-4x^3+1-2x^2)}{(2x+1)(x-1)}$$

$$= \frac{4x^3-x^2-4}{(2x+1)(x-1)}$$

$$3. a) \frac{1}{x^2-16} - \frac{1}{x+4}$$

$$= \frac{1}{(x-4)(x+4)} - \frac{x-4}{(x-4)(x+4)}$$

$$= \frac{5-x}{x^2-16}$$

$$b) \frac{1}{3x^2-3} + \frac{1}{x+1}$$

$$= \frac{1}{3(x-1)(x+1)} + \frac{3x-3}{3(x-1)(x+1)}$$

$$= \frac{3x-2}{3x^2-3}$$

$$c) \frac{2}{x+2} - \frac{5}{x^2-x-6}$$

$$= \frac{2x-6}{(x+2)(x-3)} - \frac{5}{(x+2)(x-3)}$$

$$= \frac{2x-11}{x^2-x-6}$$

$$d) \frac{x+1}{x^2-4} + \frac{3}{x^2+3x+2}$$

$$= \frac{x^2+2x+1+3x-6}{(x-2)(x+2)(x+1)}$$

$$= \frac{x^2+5x-5}{(x-2)(x+2)(x+1)}$$

$$e) \frac{4x+12}{x^2-9} - \frac{3}{x+3}$$

$$= \frac{4x+12}{(x-3)(x+3)} - \frac{3x-9}{(x-3)(x+3)}$$

$$= \frac{x+21}{x^2-9}$$

$$f) \frac{1}{x^2+x-12} - \frac{1}{x^2+3x-4}$$

$$= \frac{1}{(x+4)(x-3)} - \frac{1}{(x+4)(x-1)}$$

$$= \frac{(x-1)-(x-3)}{(x+4)(x-3)(x-1)}$$

$$= \frac{2}{(x+4)(x-3)(x-1)}$$

4. a) $\frac{500}{x}$

b) $\frac{500}{x+3}$

c) $\frac{500}{x} + \frac{500}{x+3}$
 $= \frac{500x + 1500 + 500x}{x(x+3)}$
 $= \frac{1000x + 1500}{x(x+3)}$
 $= \frac{100(10x + 15)}{x^2 + 3x}$

Ex 7C

1. a) $\frac{7}{x} \times \frac{2}{x}$
 $= \frac{14}{x^2}$

b) $\frac{4^2}{x} \times \frac{x^1}{2y}$
 $= \frac{2}{y}$

c) $\frac{5^1}{x} \times \frac{3^1}{10} \times \frac{x^1 y}{3^1}$
 $= \frac{y}{2}$

d) $\frac{2xy}{z} \times \frac{5}{4x^2}$
 $= \frac{5y}{2xz}$

e) $\frac{3^1}{2x^2} \times \frac{x^4}{9^1}$
 $= \frac{1}{6x}$

f) $\frac{(x+5)(x+2)}{x+3} \times \frac{2(x+3)}{x+2}$
 $= 2(x+5)$

2. a) $\frac{5}{x} \times \frac{2y}{1}$
 $= \frac{10y}{x}$

b) $\frac{x \times 5x}{3}$
 $= \frac{5x^2}{3}$

c) $\frac{4x^2}{5} \times \frac{3x^2}{1}$
 $= \frac{12x^4}{5}$

d) $\frac{(x+5) \times (x-5)}{3}$
 $= \frac{x^2 - 25}{3}$

e) $\frac{5}{4x^2 - 4} \times \frac{(x+1)}{1}$
 $= \frac{5}{4(x+1)(x-1)} \times \frac{(x+1)}{1}$
 $= \frac{5}{4x-4}$

f) $\frac{(x+3)}{1} \times \frac{x^2+9}{x^2-9}$
 $= \frac{x+3}{1} \times \frac{x^2+9}{(x-3)(x+3)}$
 $= \frac{x^2+9}{x-3}$

g) $\frac{3x}{x^2+3x-28} \times \frac{x^2-49}{x^2}$
 $= \frac{3x}{(x+7)(x-4)} \times \frac{(x-7)(x+7)}{x^2}$
 $= \frac{3(x-7)}{x(x-4)}$

h) $\frac{x+3}{x^2+3x+2} \times \frac{x^2-4}{x^2+4x+3}$
 $= \frac{x+3}{(x+1)(x+2)} \times \frac{(x-2)(x+2)}{(x+1)(x+3)}$
 $= \frac{x-2}{(x+1)^2}$

i) $\frac{x^2+x-12}{x^2-x-6} \times \frac{x^2-2x-8}{x^2-16}$
 $= \frac{(x+4)(x-3)}{(x-3)(x+2)} \times \frac{(x-4)(x+2)}{(x-4)(x+4)}$
 $= 1$

Ex 7D

$$1. a) 3x^4 \div \frac{x^2}{5}$$

$$= \frac{3x^4}{1} \times \frac{5}{x^2}$$

$$= 15x^2$$

$$b) x^3y \div \frac{3x^2}{y}$$

$$= \frac{x^3y}{1} \times \frac{y}{3x^2}$$

$$= \frac{xy^2}{3}$$

$$c) \frac{4x^3}{7} \div 2x^2$$

$$= \frac{4x^3}{7} \times \frac{1}{2x^2}$$

$$= \frac{2x}{7}$$

$$d) \frac{2xy^2}{3} \div 6x^2y$$

$$= \frac{2xy^2}{3} \times \frac{1}{6x^2y}$$

$$= \frac{y}{9x}$$

$$e) \frac{2x^2}{3} \div \frac{5x^3}{6}$$

$$= \frac{2x^2}{3} \times \frac{6}{5x^3}$$

$$= \frac{4}{5x}$$

$$f) -\frac{2x^2y^3}{5} \div \frac{4x^2y^3}{8}$$

$$= -\frac{2x^2y^3}{5} \times \frac{8}{4x^2y^3}$$

$$= -\frac{y}{2x}$$

$$2. a) \frac{3}{5x} \div \frac{6}{x^2} \times \frac{5}{4x}$$

$$= \frac{3}{5x} \times \frac{x^2}{6} \times \frac{5}{4x}$$

$$= \frac{1}{8}$$

$$b) \frac{yz^2}{4x} \times \frac{x^2}{y} \div \frac{(xz)^2}{2}$$

$$= \frac{yz^2}{4x} \times \frac{x^2}{y} \times \frac{2}{x^2z^2}$$

$$= \frac{1}{2x}$$

$$c) \frac{3x-3}{x+2} \div \frac{x^2-1}{x^2+4x+4}$$

$$= \frac{3(x-1)}{x+2} \times \frac{(x+2)(x+2)}{(x-1)(x+1)}$$

$$= \frac{3(x+2)}{x+1}$$

$$d) \frac{x^3+7x^2+12x}{y^2-9} \div \frac{x^2+4x}{y-3}$$

$$= \frac{x(x^2+7x+12)}{(y-3)(y+3)} \times \frac{y-3}{x(x+4)}$$

$$= \frac{x(x+3)(x+4)}{(y-3)(y+3)} \times \frac{y-3}{x(x+4)}$$

$$= \frac{x+3}{y+3}$$

$$e) -\frac{b}{x^2+bx} \div \frac{b}{x+b}$$

$$= -\frac{b}{x(x+b)} \times \frac{x+b}{b}$$

$$= -\frac{1}{x}$$

$$f) \frac{3x+3}{x^2+4x+4} \div \frac{3x^2+6x+3}{x^2-4}$$

$$= \frac{3(x+1)}{(x+2)(x+2)} \times \frac{(x-2)(x+2)}{3(x^2+2x+1)}$$

$$= \frac{3(x+1)}{(x+2)(x+2)} \times \frac{(x-2)(x+2)}{3(x+1)(x+1)}$$

$$= \frac{x-2}{(x+2)(x+1)}$$

$$\begin{aligned} 3. \quad a) \quad & 1 \div \frac{x}{\frac{y}{x}} \\ &= \frac{1}{1} \times \frac{y}{x} \\ &= \frac{y}{x} \end{aligned}$$

$$\begin{aligned} b) \quad & \frac{1}{\frac{x}{x+1}} \\ &= \frac{1}{1} \times \frac{x+1}{x} \\ &= \frac{x+1}{x} \end{aligned}$$

$$\begin{aligned} c) \quad & \frac{1}{\frac{1}{x-2}} \\ &= \frac{1}{1} \times \frac{x-2}{1} \\ &= x-2 \end{aligned}$$

$$\begin{aligned} d) \quad & \frac{a/b}{c/d} \\ &= \frac{a}{b} \times \frac{d}{c} \\ &= \frac{ad}{bc} \end{aligned}$$

Chapter 8 Determining the Gradient of a Straight Line Given two Points

Ex 8A

1. a) A(2,1) B(3,4)

$$\begin{aligned} m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 1}{3 - 2} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

b) C(-1,6) D(0,4)

$$\begin{aligned} m_{CD} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 6}{0 - (-1)} \\ &= \frac{-2}{1} \\ &= -2 \end{aligned}$$

c) E(-1,-3) F(1,5)

$$\begin{aligned} m_{EF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - (-3)}{1 - (-1)} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

d) G(-7,2) H(-4,-1)

$$\begin{aligned} m_{GH} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 2}{-4 - (-7)} \\ &= -\frac{3}{3} \\ &= -1 \end{aligned}$$

e) J(-2,5) K(1,7)

$$\begin{aligned} m_{JK} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7 - 5}{1 - (-2)} \\ &= \frac{2}{3} \end{aligned}$$

f) M(-7,3) N(-3,-2)

$$\begin{aligned} m_{MN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 3}{-3 - (-7)} \\ &= \frac{-5}{4} \end{aligned}$$

g) P(-7,-8) Q(-3,-2)

$$\begin{aligned} m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-8)}{-3 - (-7)} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \end{aligned}$$

h) R(-11,4) S(-2,-8)

$$\begin{aligned} m_{RS} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-8 - 4}{-2 - (-11)} \\ &= \frac{-12}{9} \\ &= -\frac{4}{3} \end{aligned}$$

i) T(9,-10) U(-3,5)

$$\begin{aligned} m_{TU} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - (-10)}{-3 - (9)} \\ &= \frac{15}{6} \\ &= \frac{5}{2} \end{aligned}$$

2. a) $A(-3, 5)$ $B(7, 5)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 5}{7 - (-3)}$$

$$= \frac{0}{10}$$

$$= 0$$

b) x -axis

c) both have $y = 5$

3. a) $C(5, 4)$ $D(5, -7)$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-7 - 4}{5 - 5}$$

$$= \frac{-11}{0}$$

$$= \text{undefined}$$

b) y -axis

c) both have $x = 5$

4. a) $T(-\frac{1}{2}, -\frac{3}{2})$ $U(-\frac{9}{4}, \frac{15}{4})$ b) $V(\frac{1}{3}, \frac{1}{5})$ $W(-\frac{16}{9}, \frac{41}{15})$

$$m_{TU} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{15}{4} - (-\frac{3}{2})}{-\frac{9}{4} - (-\frac{1}{2})}$$

$$= \frac{\frac{21}{4}}{-\frac{7}{4}}$$

$$= \frac{3 \cdot 7}{4} \times \left(-\frac{4}{7}\right)$$

$$= -3$$

$$m_{VW} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\frac{41}{15} - \frac{1}{5}}{-\frac{16}{9} - \frac{1}{3}}$$

$$= \frac{\frac{38}{15}}{-\frac{19}{9}}$$

$$= \frac{38^2}{15 \cdot 19} \times \left(-\frac{9}{19}\right)^3$$

$$= -\frac{3}{5}$$

5. $A(5, 1)$ $B(8, y)$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{y - 1}{8 - 5}$$

$$2 = \frac{y - 1}{3}$$

$$6 = y - 1$$

$$y = 7$$

6. $C(-1, -7)$ $D(x, -5)$

$$m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{2}{5} = \frac{-5 - (-7)}{x - (-1)}$$

$$-\frac{2}{5} = \frac{2}{x+1}$$

$$x+1 = -5$$

$$x = -6$$

7. $E(3/2, 5/3)$ $F(1/2, y)$

$$m_{EF} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-\frac{3}{4} = \frac{y - 5/3}{1/2 - 3/2}$$

$$-\frac{3}{4} = \frac{y - 5/3}{-1}$$

$$y - 5/3 = 3$$

$$y = 14/3$$

8. $G(a, a^2)$ $H(-2, 4)$

$$m_{GH} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - a^2}{-2 - a}$$

$$= \frac{(2-a)(2+a)}{-(2+a)}$$

$$= -(2-a)$$

$$= a-2$$

9. a) At E.

b) A to B and E to F

$$c) m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1800 - 0}{2 - 1}$$

$$= \frac{1800}{1}$$

$$= 1800 \text{ km/min}^2$$

$$30 \text{ km/hr} = 1800 \text{ km/min}$$

$$A(1, 0) \quad B(2, 1800)$$

$$d) m_{EF} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{900 - 2700}{6 - 5}$$

$$= -\frac{1800}{1}$$

$$= -1800 \text{ km/min}^2$$

$$E(5, 2700) \quad F(6, 900)$$

TRUE as acc = 1800
dec = -1800

e) $m=0 \Rightarrow$ constant speed

0. a) (09:00, 0) (13:00, 4)

$$\begin{aligned} m &= \frac{4-0}{13-9} \\ &= \frac{4}{4} \\ &= 1^\circ\text{C per hr.} \end{aligned}$$

b) (07:00, -6.5) (08:00, -2)

$$\begin{aligned} m &= \frac{-2 - (-6.5)}{8-7} \\ &= \frac{4.5}{1} \\ &= 4.5^\circ\text{C per hr} \end{aligned}$$

c) Between 18:00 and 00:00

Ex 8B

1. a) P(1, 4) Q(6, 6) b) R(-5, -3) S(0, -1)

$$\begin{aligned} m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 4}{6 - 1} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} m_{RS} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - (-3)}{0 - (-5)} \\ &= \frac{2}{5} \end{aligned}$$

c) $m_{PQ} = m_{RS}$ so PQ and RS are parallel

2. T(-3/2, -5) U(3/2, 3)

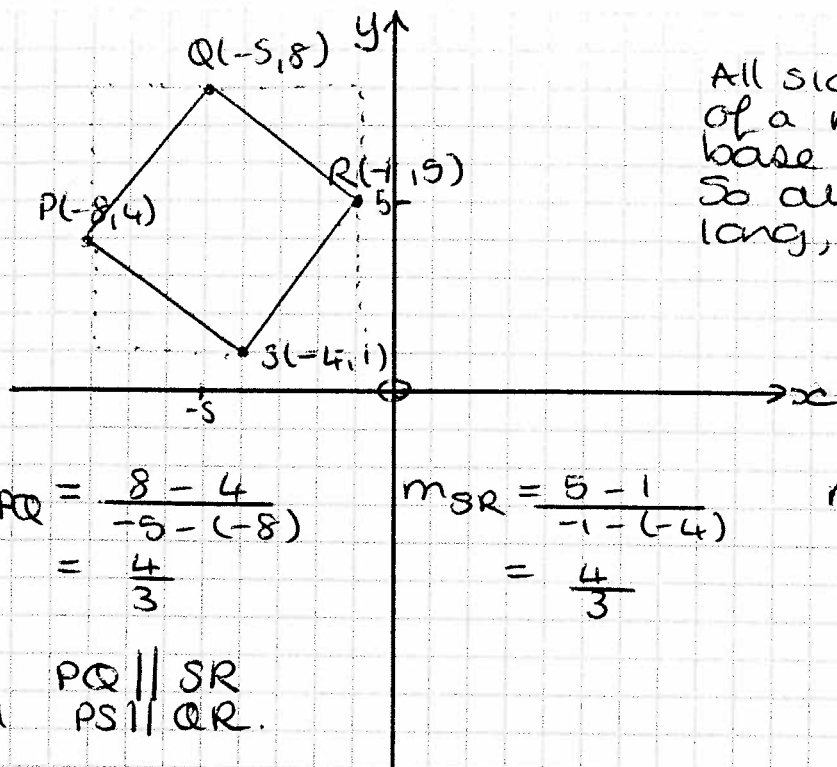
V(7/2, 2/5) W(15/2, 42/5)

$$\begin{aligned} m_{TU} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (-5)}{3/2 - (-3/2)} \\ &= \frac{8}{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} m_{VW} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{42/5 - 2/5}{15/2 - 7/2} \\ &= \frac{8}{4} \\ &= 2 \end{aligned}$$

$m_{TU} = m_{VW}$ so TU and VW are parallel

3.



All sides are the hypotenuse of a right angled triangle, base = 3 units, height = 4 units. So all sides are 5 units long, by Pythagoras.

$$m_{PQ} = \frac{8-4}{-5-(-8)} = \frac{4}{3}$$

$$m_{SR} = \frac{5-1}{-1-(-4)} = \frac{4}{3}$$

$$m_{PS} = \frac{1-4}{-4-(-8)} = \frac{-3}{4}$$

$$m_{QR} = \frac{5-8}{-1-(-5)} = \frac{-3}{4}$$

So $PQ \parallel SR$
and $PS \parallel QR$.

Since all 4 sides equal in length and $PQ \parallel SR$ and $PS \parallel QR$ then PQRS is a rhombus.

Ex 8C

1 a) $\tan 30^\circ = 0.6$ (to 1 dp)

b) $\tan 45^\circ = 1$

c) $\tan 120^\circ = -1.7$ (to 1 dp)

d) $\tan 135^\circ = -1$

e) $\tan 150^\circ = -0.6$ (to 1 dp)

f) $\tan 180^\circ = 0$

2. a) $\tan \theta = 2$
 $\theta = 63.4^\circ$ (3sf)

b) $\tan \theta = \frac{1}{2}$
 $\theta = 26.6^\circ$ (3sf)

c) $\tan \theta = -3$
 $\theta = -71.6$
 \Rightarrow angle = 108.4°
 $= 108^\circ$ (3sf)

d) $\tan \theta = -\frac{3}{5}$
 $\theta = -31.0$
 \Rightarrow angle = 149° (to 3sf)

e) $\tan \theta = \frac{7}{2}$
 $\theta = 74.1^\circ$ (3sf)

f) $\tan \theta = -\frac{13}{4}$
 $\theta = -72.9^\circ$
 \Rightarrow angle = 107° (3sf)

3. $m = \frac{5-2}{5-(-4)}$
 $= \frac{3}{9}$
 $= \frac{1}{3}$

$\tan \theta = \frac{1}{3}$
 $\theta = 18.4^\circ$ (to 1 dp)

$$4. \quad m = \frac{-3 - 1}{3 - (-5)}$$

$$= \frac{-4}{8}$$

$$= -\frac{1}{2}$$

$$\tan \theta = -\frac{1}{2}$$

$$\theta = -26.565^\circ$$

$$\Rightarrow \text{angle} = 153.435^\circ \text{ (3dp)}$$

$$5. a) \quad m_{OA} = \frac{7}{3}$$

$$\tan \theta = \frac{7}{3}$$

$$\theta = 66.8^\circ$$

$$m_{OB} = \frac{2}{5}$$

$$\tan \theta = \frac{2}{5}$$

$$\theta = 21.8^\circ$$

$$\angle AOB = 66.8 - 21.8$$

$$= 45^\circ$$

$$b) \quad m_{AB} = \frac{2 - 7}{5 - 3}$$

$$= -\frac{5}{2}$$

$$6. \quad m_{JK} = \frac{9 - (-9)}{4 - (-2)}$$

$$= \frac{18}{6}$$

$$= 3$$

$$\tan \theta = 3$$

$$\theta = 71.6^\circ$$

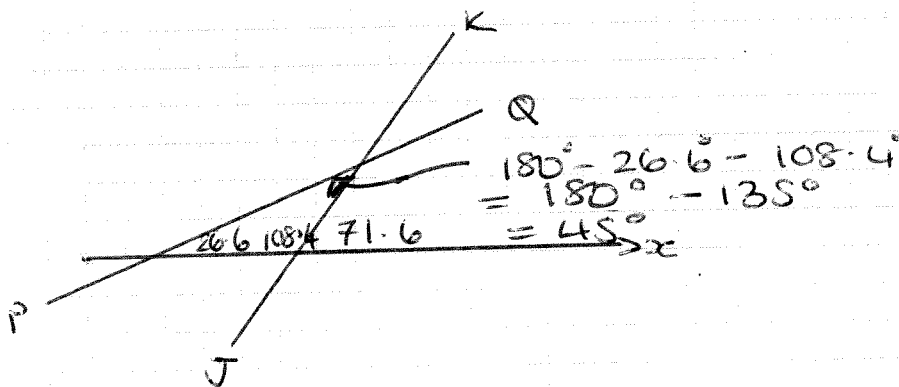
$$m_{PQ} = \frac{5 - (-3)}{9 - (-7)}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

$$\tan \theta = \frac{1}{2}$$

$$\theta = 26.6^\circ$$



Ex 8D

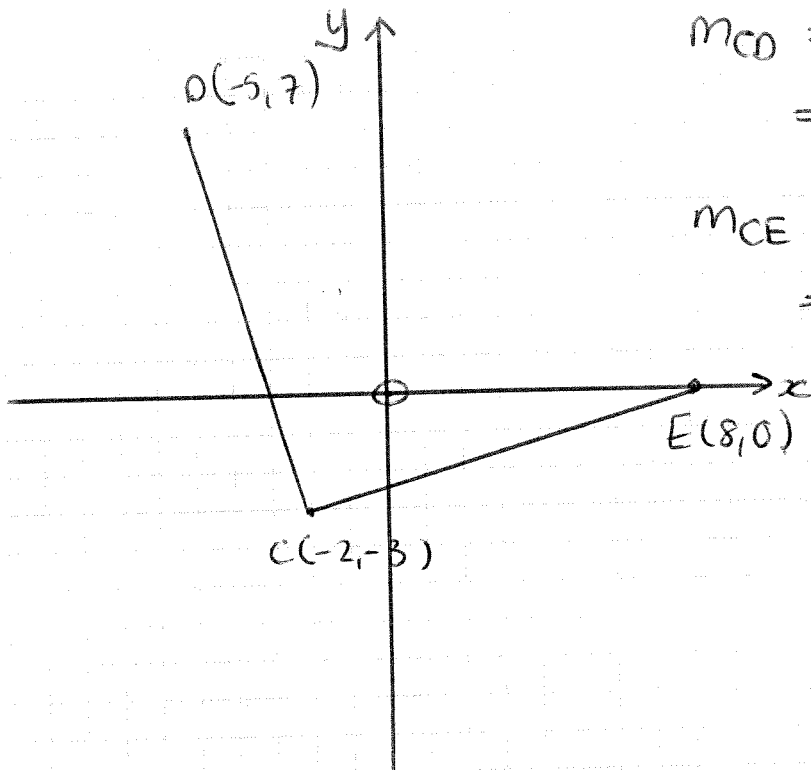
1. a) $-4/3$ b) $2/5$ c) -3 d) $-1/5$
 e) $5/4$ f) 1 g) 2 h) -1

$$2. \quad m_{AB} = \frac{11 - (-4)}{-3 - (-5)}$$

$$= \frac{15}{2}$$

$$m_{\text{perp}} = -\frac{2}{15}$$

3.



$$m_{CD} = \frac{7 - (-3)}{-5 - (-2)}$$

$$= \frac{10}{-3}$$

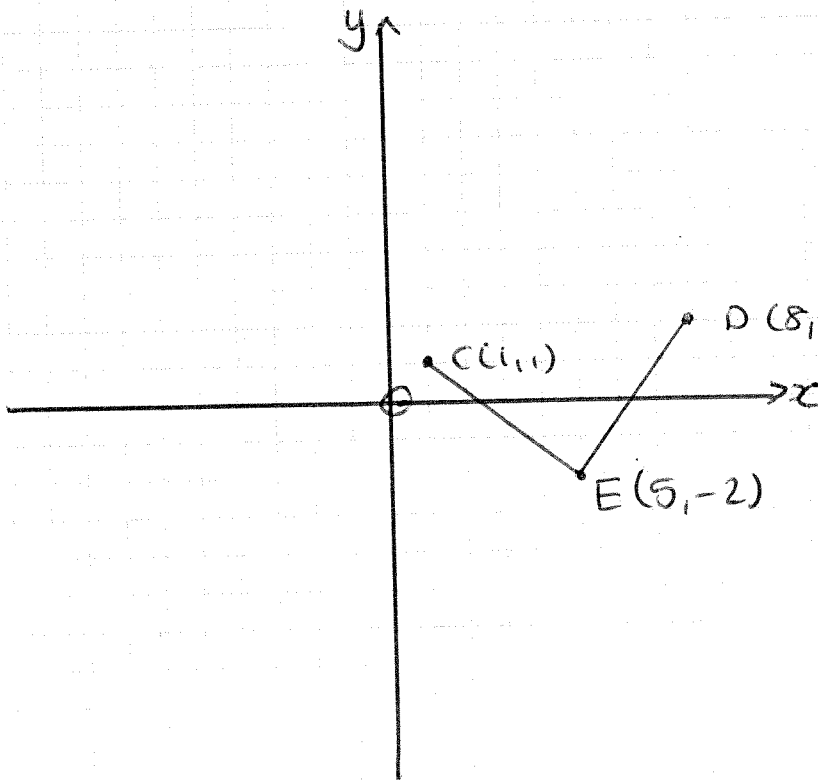
$$m_{CE} = \frac{0 - (-3)}{8 - (-2)}$$

$$= \frac{3}{10}$$

$$m_{CD} \times m_{CE} = -1$$

So $CD \perp CE$

4.



$$m_{EC} = \frac{-2 - 1}{5 - 1}$$

$$= \frac{-3}{4}$$

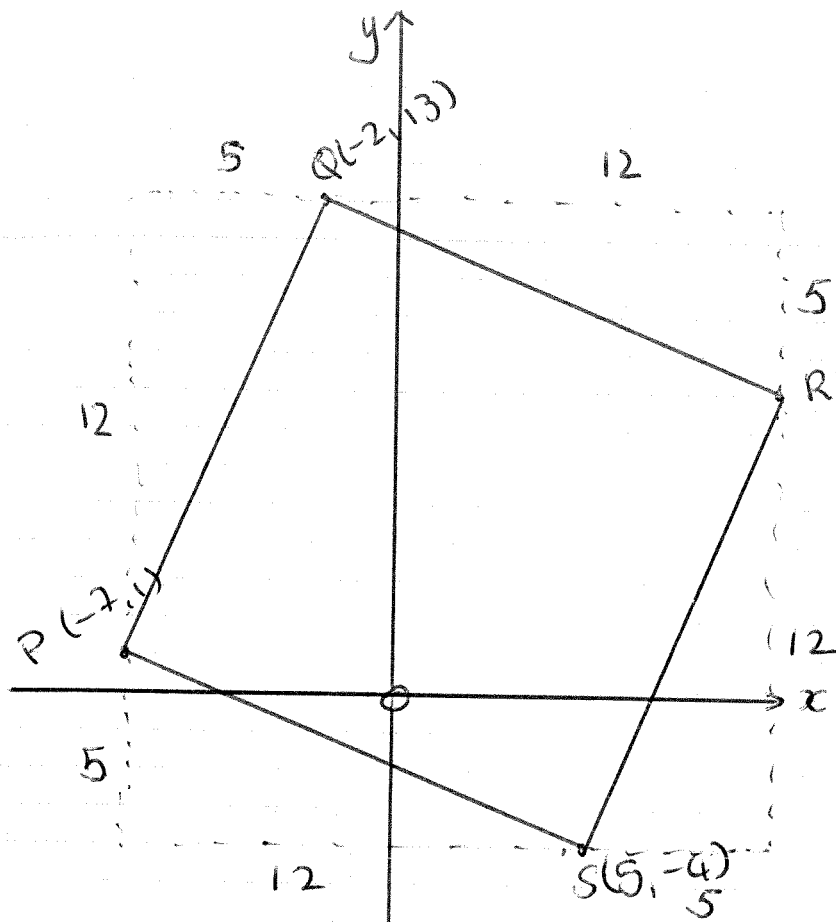
$$m_{ED} = \frac{2 - (-2)}{8 - 5}$$

$$= \frac{4}{3}$$

$$m_{EC} \times m_{ED} = -1$$

So $\triangle CDE$ right-angled at E

5.



✓ lengths
Parallel
Perpendicular.

$$m_{PQ} = \frac{13 - 4}{-2 - (-7)} = \frac{9}{5}$$

$$m_{QR} = \frac{8 - 13}{10 - (-2)} = \frac{-5}{12}$$

$$m_{RS} = \frac{-4 - 8}{5 - 10} = \frac{-12}{-5} = \frac{12}{5}$$

$$m_{SP} = \frac{4 - (-4)}{-7 - 5} = \frac{8}{-12} = -\frac{2}{3}$$

• each side is the hypotenuse on a 12 unit by 5 unit right-angled triangle
So all sides are 13 units long.

• $m_{PQ} = m_{RS}$ so $PQ \parallel RS$

$m_{QR} = m_{SP}$ so $QR \parallel SP$

so 2 pairs of parallel sides.

• $m_{PQ} \times m_{RS} = -1$ $\angle P = 90^\circ$

$m_{PQ} \times m_{QR} = -1$ $\angle Q = 90^\circ$

$m_{QR} \times m_{RS} = -1$ $\angle R = 90^\circ$

$m_{RS} \times m_{SP} = -1$ $\angle S = 90^\circ$

So all angles are 90°

So PQRS is a square

Chapter 9 Calculating the Length of an Arc or the Area of a Sector of a Circle

Ex 9A

$$1. a) \text{ Arc} = \frac{70}{360} \times \pi \times 24 \\ = 14.7 \text{ cm (to 1 dp)}$$

$$\text{Sector} = \frac{70}{360} \times \pi \times 12^2 \\ = 88.0 \text{ cm}^2 \text{ (to 1 dp)}$$

$$b) \text{ Arc} = \frac{35}{360} \times \pi \times 12 \\ = 3.7 \text{ cm (to 1 dp)}$$

$$\text{Sector} = \frac{35}{360} \times \pi \times 6^2 \\ = 11.0 \text{ cm}^2 \text{ (to 1 dp)}$$

$$c) \text{ Arc} = \frac{15}{360} \times \pi \times 28 \\ = 3.7 \text{ cm}$$

$$\text{Sector} = \frac{15}{360} \times \pi \times 14^2 \\ = 25.7 \text{ cm}^2 \text{ (to 1 dp)}$$

$$d) \text{ Arc} = \frac{130}{360} \times \pi \times 3.6 \\ = 4.1 \text{ m (to 1 dp)}$$

$$\text{Sector} = \frac{130}{360} \times \pi \times 1.8^2 \\ = 3.7 \text{ m}^2 \text{ (to 1 dp)}$$

$$e) \text{ Arc} = \frac{175}{360} \times \pi \times 16.8 \\ = 25.7 \text{ m (to 1 dp)}$$

$$\text{Sector} = \frac{175}{360} \times \pi \times 8.4^2 \\ = 107.8 \text{ m}^2 \text{ (to 1 dp)}$$

$$f) \text{ Arc} = \frac{115}{360} \times \pi \times 30 \\ = 30.1 \text{ cm (to 1 dp)}$$

$$\text{Sector} = \frac{115}{360} \times \pi \times 15^2 \\ = 225.8 \text{ cm}^2 \text{ (to 1 dp)}$$

$$2. a) \text{ Arc} = \frac{135}{360} \times \pi \times 24 \\ = 28.3 \text{ cm (to 1 dp)}$$

$$\text{Perimeter} = 28.3 + 12 + 12 \\ = 52.3 \text{ cm (to 1 dp)}$$

$$b) \text{ Sector} = \frac{135}{360} \times \pi \times 12^2 \\ = 169.6 \text{ cm}^2 \\ \text{(to 1 dp)}$$

$$3. a) \text{ Arc} = \frac{310}{360} \times \pi \times 50 \\ = 135.3 \text{ mm (to 1 dp)}$$

$$\text{Perimeter} = 135.3 + 25 + 25 \\ = 185.3 \text{ mm (to 1 dp)}$$

$$b) \text{ Sector} = \frac{310}{360} \times \pi \times 25^2 \\ = 1690.8 \text{ mm}^2 \\ \text{(to 1 dp)}$$

$$4. \quad a) i) \text{ Outer Arc} = \frac{90}{360} \times \pi \times 18$$

$$= 14.1 \text{ cm (to 1 dp)}$$

$$\text{Inner Arc} = \frac{90}{360} \times \pi \times 12$$

$$= 9.4 \text{ cm (to 1 dp)}$$

$$\text{Perimeter} = 14.1 + 9.4 + 3 + 3$$

$$= 29.5 \text{ cm (to 1 dp)}$$

$$ii) \text{ Large Sector} = \frac{90}{360} \times \pi \times 9^2$$

$$= 254.5 \text{ cm}^2$$

$$\text{Small Sector} = \frac{90}{360} \times \pi \times 6^2$$

$$= 28.3 \text{ cm}^2$$

$$\text{Area} = 254.5 - 28.3$$

$$= 226.2 \text{ cm}^2 \text{ (to 1 dp)}$$

$$b) i) \text{ Outer Arc} = \frac{40}{360} \times \pi \times 38$$

$$= 13.3 \text{ cm (to 1 dp)}$$

$$\text{Inner Arc} = \frac{40}{360} \times \pi \times 24$$

$$= 8.4 \text{ cm (to 1 dp)}$$

$$\text{Perimeter} = 13.3 + 8.4 + 7 + 7$$

$$= 35.7 \text{ cm (to 1 dp)}$$

$$ii) \text{ Large Sector} = \frac{40}{360} \times \pi \times 19^2$$

$$= 126.0 \text{ cm}^2 \text{ (to 1 dp)}$$

$$\text{Small Sector} = \frac{40}{360} \times \pi \times 12^2$$

$$= 50.3 \text{ cm}^2 \text{ (to 1 dp)}$$

$$\text{Area} = 126.0 - 50.3$$

$$= 75.7 \text{ cm}^2 \text{ (to 1 dp)}$$

$$5. \quad \text{Sector} = \frac{90}{360} \times \pi \times 15^2$$

$$= 176.7145868$$

$$\text{triangle} = \frac{1}{2} \times 15 \times 15$$

$$= 112.5$$

$$\text{Segment} = 176.7145868 - 112.5$$

$$= 64.2 \text{ cm}^2 \text{ (to 3 sf)}$$

6. 10 sectors per circle

$$a) \text{ Arc} = \frac{1}{10} \times \pi \times 20$$

$$= 6.3 \text{ cm (to 1 dp)}$$

$$b) \text{ Radius} = 40 \times 6.3$$

$$= 252 \text{ cm (nearest cm)}$$

$$7. \quad \text{Arc} = \frac{25}{60} \times \pi \times 30$$

$$= 39.3 \text{ cm (to 1 dp)}$$

Ex 9B

1 a) i) Arc = $\frac{60}{360} \times \pi \times 24$
= 4π cm

ii) Sector = $\frac{60}{360} \times \pi \times 12^2$
= 24π cm²

c) i) Arc = $\frac{120}{360} \times \pi \times 30$
= 10π cm

ii) Sector = $\frac{120}{360} \times \pi \times 15^2$
= 75π cm²

2. a) i) Arc = $\frac{12}{360} \times \pi \times 36$
= $\frac{6}{5}\pi$ cm

ii) Sector = $\frac{12}{360} \times \pi \times 18^2$
= $\frac{54}{5}\pi$ cm²

c) i) Arc = $\frac{135}{360} \times \pi \times 24$
= 9π m

ii) Sector = $\frac{135}{360} \times \pi \times 12^2$
= 54π m²

3. a) Sector = $\frac{30}{360} \times \pi \times r^2$
 $3\pi = \frac{1}{12} \pi r^2$
 $r^2 = 36$
 $r = 6$ cm

c) $\frac{\pi}{16} = \frac{90}{360} \times \pi \times r^2$
 $\frac{\pi}{16} = \frac{1}{4} \pi r^2$
 $\frac{1}{4} = r^2$
 $r = \frac{1}{2}$ cm

b) i) Arc = $\frac{30}{360} \times \pi \times 36$
= 3π cm

ii) Sector = $\frac{30}{360} \times \pi \times 18^2$
= 9π cm²

b) i) Arc = $\frac{150}{360} \times \pi \times 48$
= 20π cm

ii) Sector = $\frac{150}{360} \times \pi \times 24^2$
= 240 cm²

b) $20\pi = \frac{72}{360} \times \pi \times r^2$
 $20\pi = \frac{1}{5} \pi r^2$
 $100 = r^2$
 $r = 10$ cm.

$$4. \quad 3 \times \text{Blade} = 3 \times \frac{60}{360} \times \pi \times 6^2$$

$$= 18\pi \text{ cm}^2$$

$$\text{centre bits} = 3 \times \frac{60}{360} \times \pi \times 2^2$$

$$= 2\pi \text{ cm}^2$$

$$\text{Total} = 18\pi + 2\pi$$

$$= 20\pi \text{ cm}^2$$

Ex 9c

$$1. \quad a) \quad 10 = \frac{\theta}{360} \times \pi \times 24$$

$$3600 = \theta \times 24\pi$$

$$48^\circ = \theta$$

(nearest $^\circ$)

$$b) \quad 28 = \frac{\theta}{360} \times \pi \times 28$$

$$10080 = 28\pi \times \theta$$

$$\theta = 115^\circ \text{ (nearest } ^\circ \text{)}$$

$$c) \quad 26.2 = \frac{\theta}{360} \times \pi \times 16.8$$

$$9432 = 16.8\pi \times \theta$$

$$\theta = 179^\circ \text{ (nearest } ^\circ \text{)}$$

$$d) \quad 54 = \frac{\theta}{360} \times \pi \times 36$$

$$19440 = 36\pi \times \theta$$

$$\theta = 172^\circ \text{ (nearest } ^\circ \text{)}$$

$$e) \quad 1.8 = \frac{\theta}{360} \times \pi \times 1.8^2$$

$$648 = 3.24\pi \times \theta$$

$$\theta = 64^\circ \text{ (nearest } ^\circ \text{)}$$

$$f) \quad 72 = \frac{\theta}{360} \times \pi \times 18^2$$

$$25920 = 324\pi \times \theta$$

$$\theta = 25^\circ \text{ (nearest } ^\circ \text{)}$$

$$2. \quad a) \quad 4\pi = \frac{\theta}{360} \times \pi \times 20$$

$$1440 = 20\theta$$

$$\theta = 72^\circ$$

$$b) \quad \text{Sector} = \frac{72}{360} \times \pi \times 10^2$$

$$= 20\pi \text{ cm}^2$$

$$3. \quad a) \quad \frac{5}{60} \times 360 = 30^\circ$$

$$b) \quad 2\pi = \frac{30}{360} \times \pi \times d$$

$$720 = 30d$$

$$d = 24 \text{ cm}$$

$$\text{So } r = 12 \text{ cm.}$$

$$4. \quad 50 = \frac{\theta}{360} \times \pi \times 12^2$$

$$18000 = 144\pi \times \theta$$

$$\theta = 39.8^\circ \text{ (to 1 dp)}$$

Chapter 10 Calculating the Volume of a Standard Sol

Ex 10A

1. a) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 7^3$
 $= 1436.8 \text{ cm}^3 \text{ (to 1 dp)}$

b) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 12^3$
 $= 7238.2 \text{ cm}^3 \text{ (to 1 dp)}$

c) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 4.5^3$
 $= 381.7 \text{ cm}^3 \text{ (to 1 dp)}$

d) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 0.1^3$
 $= 0.004 \text{ cm}^3 \text{ (1 sf)}$

e) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 0.125^3$
 $= 0.008 \text{ cm}^3 \text{ (to 1 sf)}$

f) $V = \frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times \pi \times 1.25^3$
 $= 8.2 \text{ m}^3 \text{ (to 1 dp)}$

2. $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$
 $= \frac{2}{3} \times \pi \times 12^3$
 $= 3619.11$
 $= 3620 \text{ cm}^3 \text{ (3 sf)}$

3. $V = \frac{4}{3}\pi r^3$
 $80 = \frac{4}{3} \times \pi \times r^3$
 $60 = \pi r^3$
 $\frac{60}{\pi} = r^3$
 $r = 2.7 \text{ cm (to 1 dp)}$

4. a) Large = x × small
 $\frac{4}{3} \times \pi \times 20^3 = x \times \frac{4}{3} \times \pi \times 4^3$
 $8000 = 64x$
 $x = 125 \text{ balls}$

b) Large = x × small
 $\frac{4}{3} \times \pi \times 5^3 = x \times \frac{4}{3} \times \pi \times (\frac{1}{2})^3$
 $125 = 0.125x$
 $x = 1000 \text{ balls}$

c) Large = $x \times$ small
 $\frac{4}{3} \times \pi \times 1^3 = x \times \frac{4}{3} \times \pi \times \left(\frac{1}{5}\right)^3$
 $1 = 0.008x$
 $x = 125$ balls.

5. a) $3 \times 1 \times 2$
 $= 6$ spheres

b) $V_{\text{sphere}} = \frac{4}{3} \times \pi \times 3^3$
 $= 36\pi \text{ cm}^3$

6 spheres = $216\pi \text{ cm}^3$

Box = $18 \times 6 \times 12$
 $= 1296 \text{ cm}^3$

Water = $1296 - 216\pi$
 $= 617.4 \text{ cm}^3$ (to 1 dp)

6. a) $V_{\text{sphere}} = \frac{4}{3} \times \pi \times 4^3$
 $= \frac{256\pi}{3}$

$V_{\text{cyl.}} = \pi r^2 h$

$\frac{256\pi}{3} = \pi \times 4^2 \times h$

$h = 1.7 \text{ cm}$ (to 1 dp)

b) $V_{\text{sphere}} = \frac{4}{3} \times \pi \times 3^3$
 $= 36\pi$

$V_{\text{cyl.}} = \pi r^2 h$

$36\pi = \pi \times 4^2 \times h$

$h = 2.25 \text{ cm}$

7. a) $V_{\text{cyl.}} = \pi r^2 h$
 $= \pi \times 4 \times 8$
 $= 32\pi$

$V_{\text{sp}} = \frac{4}{3} \pi r^3$

$32\pi = \frac{4}{3} \pi r^3$

$24 = r^3$

$r = 2.9 \text{ cm}$ (to 1 dp)

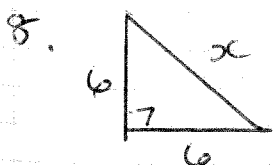
b) $V_{\text{cyl.}} = \pi \times 20^2 \times 5$
 $= 2000\pi$

$V_{\text{sp}} = \frac{4}{3} \pi r^3$

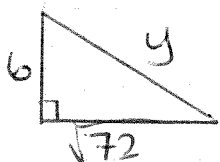
$2000\pi = \frac{4}{3} \pi r^3$

$1500 = r^3$

$r = 11.4 \text{ cm}$ (to 1 dp)



$x^2 = 36 + 36$
 $= 72$ 1 face diagonal
 $x = \sqrt{72} \text{ cm}$



$y^2 = 72 + 36$
 $= 108$ space diagonal
 $y = \sqrt{108}$ = diameter
 $y = 6\sqrt{3} \text{ cm}$

\Rightarrow radius = $3\sqrt{3} \text{ cm}$

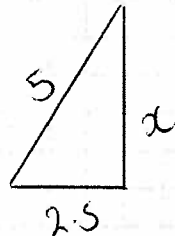
$V = \frac{4}{3} \times \pi \times (3\sqrt{3})^3$
 $= 587.7 \text{ cm}^3$ (4sf)

Exercise 10B

$$\begin{aligned} 1. a) \quad V &= \frac{1}{3} Ah \\ &= \frac{1}{3} \times 36 \times 7 \\ &= 84 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} b) \quad V &= \frac{1}{3} Ah \\ &= \frac{1}{3} \times 15 \times 7 \\ &= 35 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} c) \quad V &= \frac{1}{3} Ah \\ &= \frac{1}{3} \times 10.8 \times 11 \\ &= 39.7 \text{ cm}^3 \end{aligned}$$



$$\begin{aligned} x^2 &= 5^2 - 2.5^2 \\ &= 18.75 \\ x &= 4.33 \text{ cm (to 2 dp)} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 5 \times 4.33 \\ &= 10.8 \end{aligned}$$

$$\begin{aligned} 2. \quad V &= \frac{1}{3} Ah \\ 30 &= \frac{1}{3} \times 8 \times h \\ h &= 11.25 \text{ cm} \end{aligned}$$

$$\begin{aligned} 3. \quad V_{\text{cube}} &= 12^3 \\ &= 1728 \text{ cm}^3 \\ V_{\text{pyr}} &= 1728 \div 6 \\ &= 288 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} V &= \frac{1}{3} Ah \\ 288 &= \frac{1}{3} \times 144 \times h \\ h &= 6 \text{ cm} \end{aligned}$$

$$\begin{aligned} 4. \quad V_{\text{cube}} &= (2x)^3 \\ &= 8x^3 \\ V_{\text{pyr}} &= \frac{1}{2} Ah \\ &= \frac{1}{2} \times (2x)^2 \times x \\ &= 2x^3 \end{aligned}$$

$$\begin{aligned} \text{Total Volume} &= 8x^3 + 2x^3 \\ &= 10x^3 \end{aligned}$$

$$\begin{aligned} a) \quad V &= 20 \times 3^3 \\ &= 540 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} b) \quad V &= 20 \times 0.5^3 \\ &= 2.5 \text{ m}^3 \end{aligned}$$

Exercise 10C

$$\begin{aligned} 1. a) \quad V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 3^2 \times 7 \\ &= 66.0 \text{ cm}^3 \text{ (to 1 dp)} \end{aligned}$$

$$\begin{aligned} b) \quad V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 2^2 \times 5 \\ &= 20.9 \text{ cm}^3 \text{ (to 1 dp)} \end{aligned}$$

$$\begin{aligned} c) \quad V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 5^2 \times \frac{1}{\pi} \\ &= 8.3 \text{ cm}^3 \text{ (to 1 dp)} \end{aligned}$$

$$\begin{aligned} d) \quad V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 0.5^2 \times 0.25 \\ &= 0.1 \text{ m}^3 \text{ (to 1 dp)} \end{aligned}$$

$$2. \quad V = \frac{1}{3} \pi r^2 h$$

$$64 = \frac{1}{3} \times \pi \times 3^2 \times h$$

$$192 = 9\pi \times h$$

$$h = 6.8 \text{ cm (to 1 dp)}$$

$$3. \quad V = \frac{1}{3} \pi r^2 h$$

$$124 = \frac{1}{3} \times \pi \times r^2 \times 21$$

$$372 = 21\pi \times r^2$$

$$r^2 = 5.63 \dots$$

$$r = 2.4 \text{ cm (to 1 dp)}$$

$$4. \quad \text{Large Cone} = \frac{1}{3} \times \pi \times 4^2 \times 12$$

$$= 64\pi$$

$$\text{Small Cone} = \frac{1}{3} \times \pi \times 3^2 \times 3$$

$$= 9\pi$$

$$\text{Frustrum} = 64\pi - 9\pi$$

$$= 172.8 \text{ cm}^3 \text{ (to 1 dp)}$$

$$5. \quad V_{\text{cyl.}} = \pi r^2 h$$

$$= \pi \times 1.5^2 \times 18$$

$$= 40.5\pi$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$= 13.5\pi$$

$$\text{Metal removed} = 40.5\pi - 13.5\pi$$

$$= 84.8 \text{ cm}^3 \text{ (to 1 dp)}$$

Exercise 10D

$$V = \frac{1}{3} \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$= \frac{1}{3} \times \pi \times 4^2 \times 12 + \frac{2}{3} \pi \times 4^3$$

$$= 335.1032164$$

$$= 335 \text{ cm}^3 \text{ (to 3 sf)}$$

$$2. \quad \text{Silver} = \pi r^2 h$$

$$= \pi \times 11^2 \times 28$$

$$= 10643.71591$$

$$\text{White} = \pi r^2 h$$

$$= \pi \times 14^2 \times 28$$

$$= 17241.06048$$

$$\text{Gold} = \text{White} - \text{Silver}$$

$$= 17241.06048 - 10643.71591$$

$$= 6597.344573$$

$$= 6597.3 \text{ mm}^3 \text{ (to 1 dp)}$$

$$\begin{aligned}
 3. \quad V &= lbh - \frac{1}{3}\pi r^2 h \\
 &= 7 \times 5 \times 5 - \frac{1}{3} \times \pi \times 2^2 \times 6 \\
 &= 175 - 25.13274123 \\
 &= 149.8672588 \\
 &= 149.9 \text{ cm}^3 \text{ (to 1 dp)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad V &= \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3 \\
 &= \pi \times 3^2 \times 5 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 \\
 &= 122.5221135 \\
 &= 122.5 \text{ m}^3 \text{ (to 1 dp)}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \text{Pill} &= \pi r^2 h \\
 &= \pi \times 8^2 \times 3.375 \\
 &= 216\pi
 \end{aligned}$$

$$\begin{aligned}
 \text{Capsule} &= \pi r^2 h + \frac{4}{3} \pi r^3 \\
 216\pi &= 9\pi h + 36\pi \\
 180\pi &= 9\pi h \\
 h &= 20 \text{ mm}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad a) \quad V &= lbh + \frac{1}{2}bhL \\
 &= 8 \times 7 \times 10 + \frac{1}{2} \times 7 \times 4 \times 8 \\
 &= 560 + 112 \\
 &= 672 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 b) \quad V &= 16 \times 14 \times 20 + \frac{1}{2} \times 14 \times 8 \times 16 \\
 &= 4480 + 896 \\
 &= 5376 \text{ cm}^3
 \end{aligned}$$

Chapter 11 Rounding to a Given Number of Significant Figures

Ex 11A

1. a) 5

b) 4

c) 2

d) 5

e) 2

f) 3

g) 2

h) 1

2. a) 2

b) 1

c) 3

d) 4

e) 4

f) 6

g) 6

h) 5

3. a) 2

b) 4

c) 4

d) 2

e) 3

f) 5

g) 7

h) 8

4. a) 3

b) 4

c) 1

d) 2

5. a) 2

b) 1

c) 2

d) 1

e) 2

f) 3

Ex 11B

1. a) 3421
 $= 3000$ (to 1 sf)

b) 87
 $= 90$ (to 1 sf)

c) 9523
 $= 10\,000$ (to 1 sf)

d) $14\,980$
 $= 10\,000$ (to 1 sf)

e) 0.0385
 $= 0.04$ (to 1 sf)

f) 0.0049
 $= 0.005$ (to 1 sf)

g) 0.349
 $= 0.3$ (to 1 sf)

h) 0.955
 $= 1.0$ (to 1 sf)

i) 232.45
 $= 200$ (to 1 sf)

j) 8.09
 $= 8$ (to 1 sf)

k) 200.5
 $= 200$ (to 1 sf)

l) 9.885
 $= 10$ (to 1 sf)

m) 0.71×10^8
 $= 6 \times 10^7$ (to 1 sf)

n) 3.018×10^5
 $= 3 \times 10^5$ (to 1 sf)

o) 8.0×10^{-5}
 $= 8 \times 10^{-5}$ (to 1 sf)

$$p) 3.50 \times 10^{-7} \\ = 4 \times 10^{-7} \text{ (to 3sf)}$$

$$2. a) 6975 \\ = 7000 \text{ (to 2sf)}$$

$$b) 30055 \\ = 30060 \text{ (to 4sf)}$$

$$c) 449 \\ = 400 \text{ (to 1sf)}$$

$$d) 35230 \\ = 35200 \text{ (to 3sf)}$$

$$e) 0.845 \\ = 0.85 \text{ (to 2sf)}$$

$$f) 0.0379 \\ = 0.038 \text{ (to 2sf)}$$

$$g) 0.30509 \\ = 0.305 \text{ (to 3sf)}$$

$$h) 0.0030055 \\ = 0.003006 \text{ (to 4sf)}$$

$$i) 24.542 \\ = 24.5 \text{ (to 3sf)}$$

$$j) 700.034 \\ = 700 \text{ (to 2sf)}$$

$$k) 840.078 \\ = 840.1 \text{ (to 4sf)}$$

$$l) 450.0045 \\ = 450.00 \text{ (to 5sf)}$$

$$1. a) 2.3 \times 2.3 \times 1.2 \\ = 5.29 \times 1.2 \\ = 6.3 \times 1.2 \\ = 6.36 \\ = 6.4 \text{ (to 2sf)}$$

$$3.6^2 \times \pi \div 2 \\ = 12.96 \times \pi \div 2 \\ = 13 \times \pi \div 2 \\ = 40.8407045 \div 2 \\ = 20.5 \\ = 21 \text{ (to 2sf)}$$

$$\sqrt{(2.5)^2 + (1.5)^2} \\ = \sqrt{6.25 + 2.25} \\ = \sqrt{8.5} \\ = 2.915475947 \\ = 2.9 \text{ (to 2sf)}$$

$$b) 2.3 \times 2.3 \times 1.2 \\ = 6.348 \\ = 6.3 \text{ (to 2sf)}$$

$$3.6^2 \times \pi \div 2 \\ = 20.3575204 \\ = 20 \text{ (to 2sf)}$$

$$\sqrt{2.5^2 + 1.5^2} \\ = \sqrt{8.5} \\ = 2.915475947 \\ = 2.9 \text{ (to 2sf)}$$

c) Different

Different

Same

