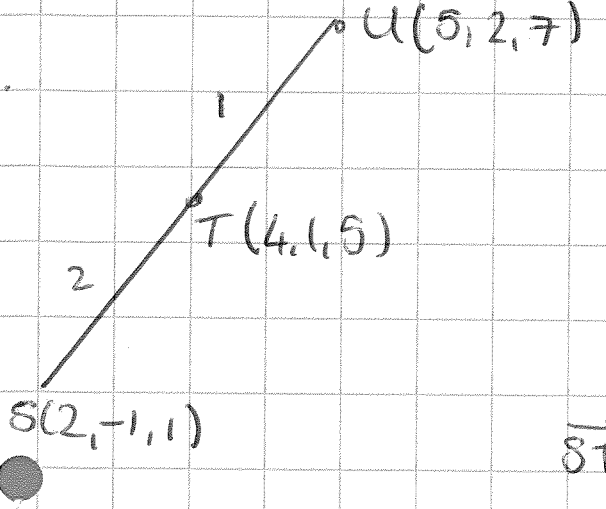
Higher Maths
Unit 1 Expressions and Functions
Mixed Questions

1.



$\vec{ST} = \underset{\sim}{t} - \underset{\sim}{s}$

$= \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$

$\vec{TU} = \underset{\sim}{u} - \underset{\sim}{t}$

$= \begin{pmatrix} 5 \\ 2 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$\vec{ST} = 2\vec{TU}$

so T divides $\vec{SU}$ in the ratio 2:1

2. $\log_a 9 = 2/3$

$y = a^x \Leftrightarrow x = \log_a y$

$a^{2/3} = 9$

(square root each side)

$\sqrt[3]{a^2} = 9$

$\sqrt[3]{a} = 3$

(cube each side)

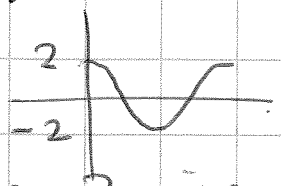
$a = 27$

3. $9 - 4\sin(x - \pi/5)$

min value of $4\sin(x - \pi/5)$ is -4

so max value of $9 - 4\sin(x - \pi/5)$ is $9 + 4 = \underline{13}$

4. $y = 2\cos(px) + q$



- $y = 2\cos x$ so moved up 3 ie $q = 3$

  $y = 2\cos(px) + 3$

- period of graph is $\pi/2$, so 4 waves in 2π. so $p = 4$

ie. $y = 2\cos 4x + 3$

5. $x^2 - 16 \neq 0$. So $x^2 - 16 = 0$

$(x-4)(x+4) = 0$

$x = 4, \; x = -4$

So $x \neq 4, \; x \neq -4$

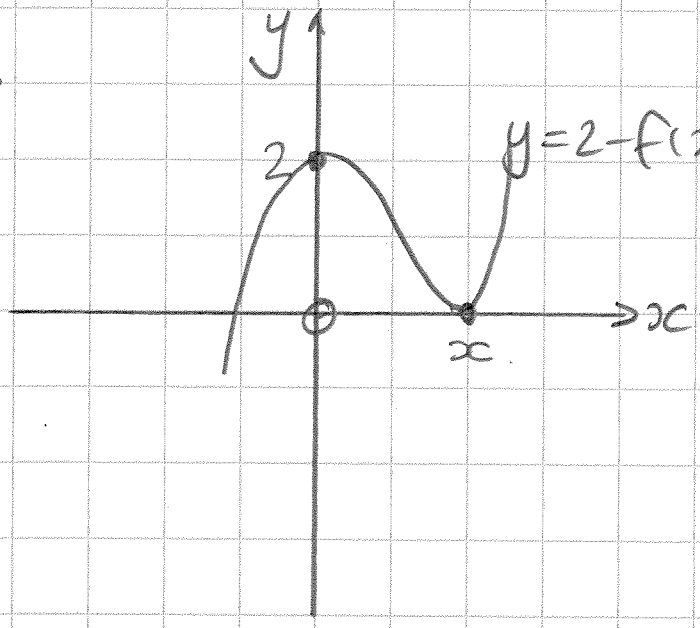
Domain $= \{x : x \in \mathbb{R}, x \neq \pm 4\}$

6. $|\underset{\sim}{t}| = \sqrt{16+0+9}$

$= 5$

$\underset{\sim}{u} = \frac{1}{5}\underset{\sim}{t}$ ie $k = \frac{1}{5}$

7.



	$f(x)$	$-f(x)$	$2-f(x)$
	$(0,0)$	$(0,0)$	$(0,2)$
	$(x,2)$	$(x,-2)$	$(x,0)$

8. $2\log_3 x - \log_3(x+1)$

$= \log_3 x^2 - \log_3(x+1)$

$= \log_3\left(\frac{x^2}{x+1}\right)$

9. a) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ $\quad \overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

$= \begin{pmatrix}3\\0\\3\end{pmatrix} - \begin{pmatrix}1\\-1\\2\end{pmatrix}$ $\quad = \begin{pmatrix}-2\\3\\4\end{pmatrix} - \begin{pmatrix}1\\-1\\2\end{pmatrix}$

$= \begin{pmatrix}2\\1\\1\end{pmatrix}$ $\quad = \begin{pmatrix}-3\\4\\2\end{pmatrix}$

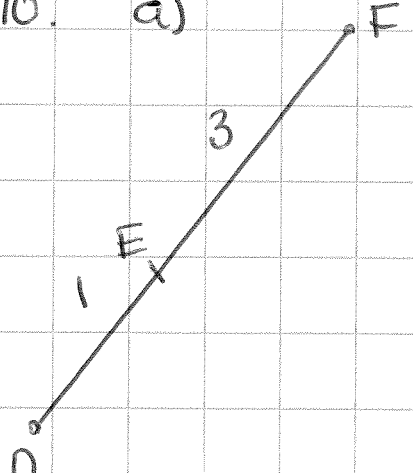
b) $\cos B\hat{A}C = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|}$

$= \dfrac{-6+4+2}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|}$

$= 0$

$B\hat{A}C = 90°$

10. a)



$\overrightarrow{EF} = 3\overrightarrow{DE}$

$\underset{\sim}{f} - \underset{\sim}{e} = 3(\underset{\sim}{e} - \underset{\sim}{d})$

$\underset{\sim}{f} - \underset{\sim}{e} = 3\underset{\sim}{e} - 3\underset{\sim}{d}$

$4\underset{\sim}{e} = 3\underset{\sim}{d} + \underset{\sim}{f}$

$= \begin{pmatrix}21\\-6\\3\end{pmatrix} + \begin{pmatrix}-1\\2\\5\end{pmatrix}$

$= \begin{pmatrix}20\\-4\\8\end{pmatrix}$

$\underset{\sim}{e} = \begin{pmatrix}5\\-1\\2\end{pmatrix}$ $\quad E(5, -1, 2)$

b) $\vec{EG} = \underset{\sim}{g} - \underset{\sim}{e}$ $\quad\quad$ $\vec{DF} = \underset{\sim}{f} - \underset{\sim}{d}$

$= \begin{pmatrix} 6 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix}$ $\quad\quad$ $= \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$ $\quad\quad$ $= \begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$

$\vec{EG} \cdot \vec{DF} = -8 - 4 + 12$

$= 0$

So $\vec{EG}$ and $\vec{DF}$ are perpendicular.

11. a) i) $f(g(x)) = f(x^2 - 2k)$

$= 2(x^2 - 2k) + k$

$= 2x^2 - 4k + k$

$= 2x^2 - 3k$

ii) $g(f(x)) = g(2x + k)$

$= (2x + k)^2 - 2k$

$= 4x^2 + 4kx + k^2 - 2k$

12. $y = ae^{bx}$

$\ln y = \ln a + bx \ln e$

$\ln y = bx + \ln a$

x	4.3	6.1
$\ln y$	-3.611	-5.521

$$b = \frac{-5.521 - (-3.611)}{6.1 - 4.3}$$

$= -1.061$

$\ln y = -1.061x + \ln a$

$(4.3, -3.611)$

$-3.611 = -1.061 \times 4.3 + \ln a$

$\ln a = 0.9513$

$a = 2.589$

$y = 2.589\, e^{-1.061x}$