AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Graphing Functions

When sketching functions we should remember to

- Use neatly drawn and labelled axes.
- Label the function with its name.
- Focus on key points e.g. intercepts, turning points giving their coordinates whenever possible.

Examples

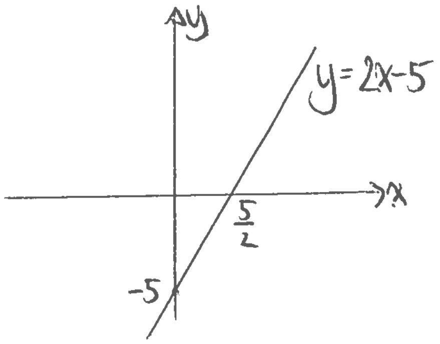
Make a neat sketch of the following functions.

(a) $f(x) = 2x - 5$

$y = 2x - 5$

Straight line gradient 2
y intercept $(0,5)$

Cuts x-axis $y = 0$
$2x - 5 = 0$
$x = \frac{5}{2}$ $\left(\frac{5}{2}, 0\right)$



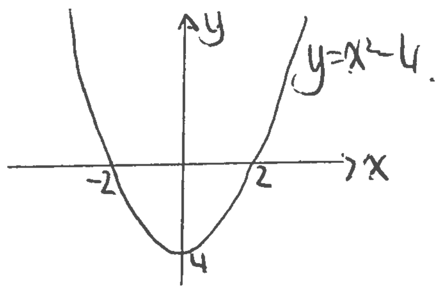
(b) $f(x) = x^2 - 4$

$y = x^2 - 4$

parabola $+x^2$ U

Cuts x-axis $x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$.

Cuts y-axis $x = 0$ $y = -4$



Using function notation, find an additional point on each graph.

(a) $f(x) = 2x - 5$

eg $x = 2$
$f(2) = 2 \times 2 - 5$
$= 4 - 5$
$= -1$ $(2, -1)$

(b) $f(x) = x^2 - 4$

eg $x = 1$
$f(x) = 1^2 - 4$
$= 1 - 4$
$= -3$ $(1, -3)$

AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Transformations of Functions

For the graph of a function y = f (x) the following are the results of transformations

y = f (x) + a moves y = f (x) vertically — up for a > 0, Down for a < 0

add a to each y co-ord

y = f (x + a) moves y = f (x) horizontally — to the left for a > 0, To the right for a < 0

In brackets with the x changes x co-ords 'opposite of what you might think'

subtract a from each x co-ord

y = - f (x) reflects y = f (x) in the x – axis

y co-ords change sign.

y = f (- x) reflects y = f (x) in the y – axis

x co-ords change sign.

y = k f (x) stretch or compress y = f (x) vertically — stretch for k > 1, Compress for k<1

multiply each y co-ord by k.

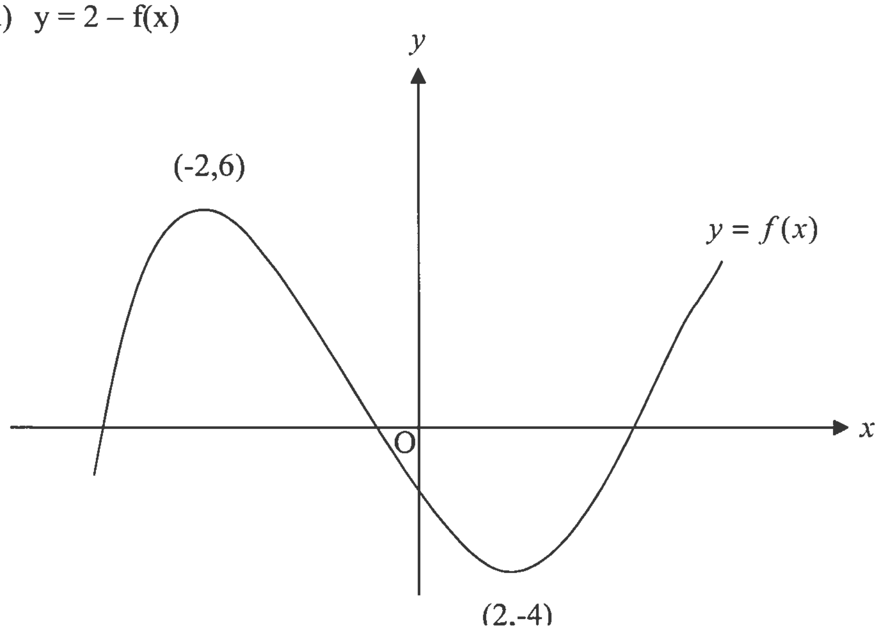
y = f (kx) stretch or compress y=f(x) horizontally — compress for k > 1, Stretch for k < 1

divide each x co-ord by k.

Example

Given the graph y = f(x) as shown, sketch and annotate on separate diagrams

a) y = f(-x)
b) y = f(x) + 4
c) y = f(x-2)
d) y = 2 – f(x)



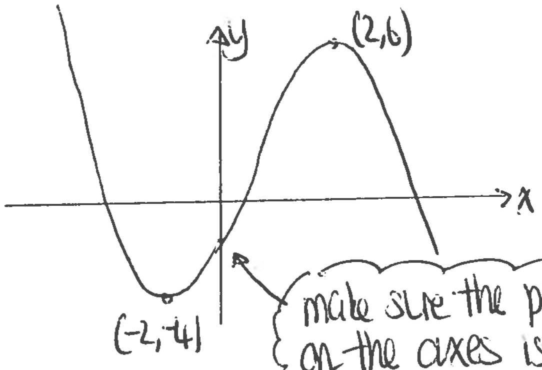
(a) $y = f(-x)$

$\uparrow$ change sign of x co-ords

Reflection in y-axis

$(-2,6) \rightarrow (2,6)$

$(2,-4) \rightarrow (-2,-4)$



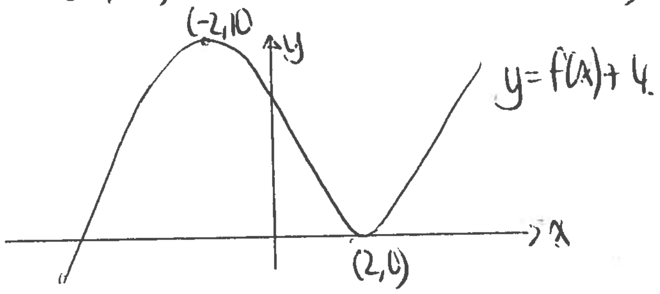
* Mark on new co-ords of any given points

make sure the position relative to O on the axes is correct.

(b) $y = f(x) + 4$

$\nwarrow$ move up 4 $\rightarrow$ add 4 to y co-ords.

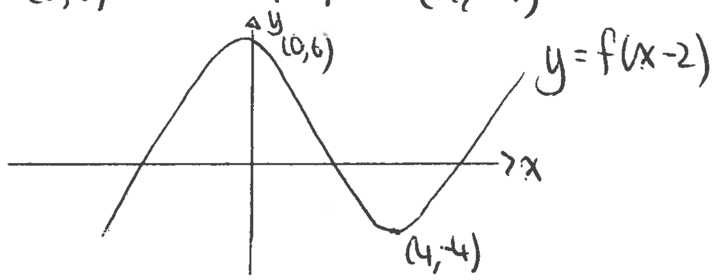
$(-2,6) \rightarrow (-2,10)$ $(2,-4) \rightarrow (2,0)$



(c) $y = f(x-2)$

$\nwarrow$ shift graph 2 places to right $\rightarrow$ add 2 to x-coords.

$(-2,6) \rightarrow (0,6)$ $(2,-4) \rightarrow (4,-4)$



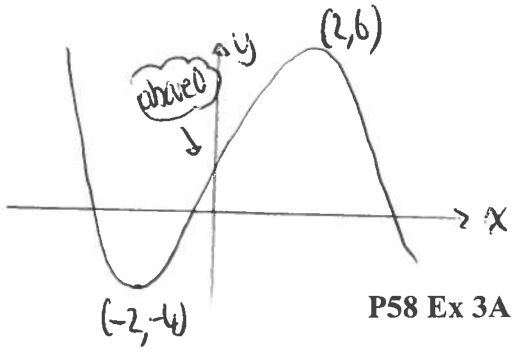
(d) $y = 2 - f(x)$

$= -f(x) + 2$

$\nwarrow$ reflection in x-axis then add 2 to y co-ords.

$(-2,6) \rightarrow (-2,-6) \rightarrow (-2,-4)$

$(2,-4) \rightarrow (2,4) \rightarrow (2,6)$



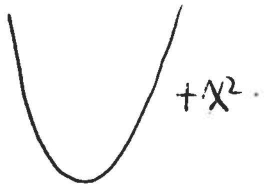
P58 Ex 3A

AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Revision of Quadratics

Of form $f(x) = ax^2 + bx + c, \quad a \neq 0$

Graph is a parabola



if $a > 0$ minimum

if $a < 0$ maximum

Sketching Quadratics

Example

Sketch $y = 5 - 4x - x^2$ $\quad -x^2 \quad \wedge$ shape.

cuts x-axis $\quad$ put $y = 0$

$$5 - 4x - x^2 = 0$$
$$x^2 + 4x - 5 = 0$$
$$(x+5)(x-1) = 0$$
$$x = -5 \quad x = 1$$

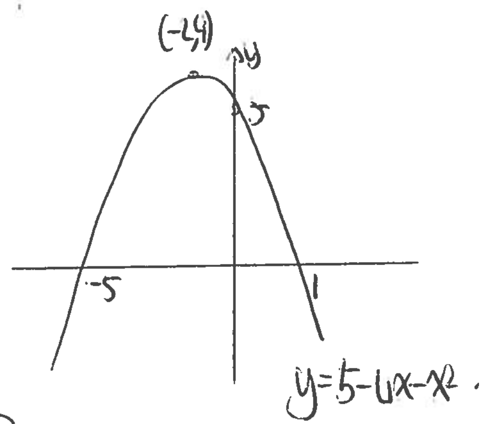
cuts y-axis $\quad$ put $x = 0 \quad y = 5$

max TP $\quad x = \frac{-5+1}{2} \leftarrow$ half way between zeros

$$= -2$$

and $y = 5 - 4(-2) - (-2)^2$

$$= 5 + 8 - 4$$
$$= 9 \qquad (-2, 9)$$



Completing the square

Consider	**$y = x^2$**		
$y = x^2 + 1$	∪	min TP	$(0,1)$
$y = 2x^2$	∪	min TP	$(0,0)$
$y = (x-1)^2$	∪	min TP	$(1,0)$
$y = (x-1)^2 + 2$	∪	min TP	$(1,2)$
$y = (x+3)^2 + 4$	∪	min TP	$(-3,4)$

> In fact for $y = a(x+p)^2 + q$ the turning point is $(-p,q)$ and the axis of symmetry is $x = -p$

It is therefore very helpful to write quadratics of the form $y = ax^2 + bx + c$ in the form $y = a(x+p)^2 + q$

This is called "**Completing the Square**"

Examples

1) Write $x^2 + 8x + 3$ in the form $(x+p)^2 + q$

$$x^2+8x+3$$
$$=(x+4)^2-4^2+3$$
$$=(x+4)^2-13$$

2) Write $y = 4 + x - x^2$ in the form $y = a(x+p)^2 + q$

$$y = 4+x-x^2$$
$$=-x^2+x+4$$
$$=-(x^2-x)+4$$
$$=-\left(\left(x-\tfrac{1}{2}\right)^2-\left(\tfrac{1}{2}\right)^2\right)+4$$
$$=-\left(x-\tfrac{1}{2}\right)^2+\tfrac{1}{4}+4$$
$$=-\left(x-\tfrac{1}{2}\right)^2+\tfrac{17}{4}$$

3) Write $2x^2 - 12x + 23$ in the form $a(x+p)^2 + q$ and hence sketch $y = 2x^2 - 12x + 23$

$y = 2x^2 - 12x + 23 = 2(x^2 - 6x) + 23$

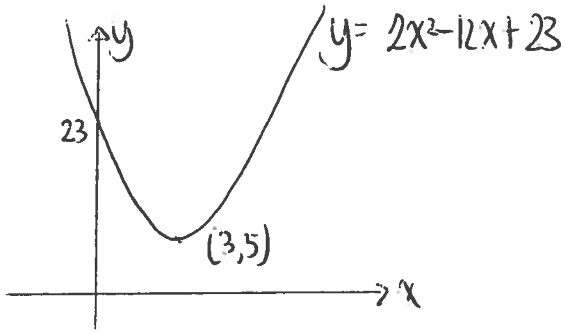
$= 2((x-3)^2 - (-3)^2) + 23$

$= 2(x-3)^2 - 18 + 23$

$y = 2(x-3)^2 + 5$

parabola U min TP (3, 5)

cuts y-axis $x=0$ $y=23$.



4) (a) Write $f(x) = 9 + 4x + 2x^2$ in the form $p + q(x+r)^2$

(b) Hence state the maximum value of $\frac{14}{9+4x+2x^2}$

(a) $f(x) = 9 + 4x + 2x^2$

$= 2x^2 + 4x + 9$

$= 2(x^2 + 2x) + 9$

$= 2((x+1)^2 - 1^2) + 9$

$= 2(x+1)^2 + 7$

U shape minimum value 7.

so $\frac{14}{9+4x+2x^2}$ has maximum value $\frac{14}{7} = 2$.

P60 Ex 3B, p65 Ex 3C

Solving Quadratic Inequations

We solve quadratic inequations by looking at the graph of an appropriate quadratric.

- Solve **equation** to find roots
- Sketch graph and look for solutions
- Mark solutions as region(s) on the x-axis

Examples

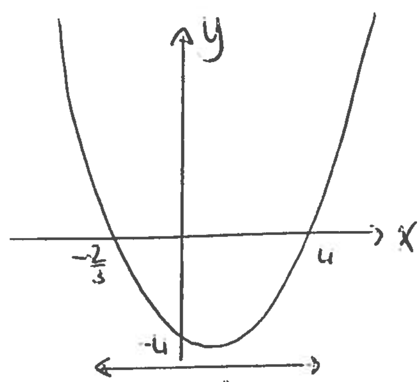
1) Solve $5x^2 - 18x - 8 < 0$

Consider $y = 5x^2 - 18x - 8$ $+x^2$ parabola $\cup$

Cuts x-axis $y = 0$ $\Rightarrow$ $5x^2 - 18x - 8 = 0$

$(5x+2)(x-4) = 0$

$x = -\frac{2}{5}$ or $x = 4$



Cuts y-axis $x = 0$ $y = -8$

Note don't need turning point.

We want $5x^2 - 18x - 8 < 0$

$y < 0$ ie graph below x-axis

so $-\frac{2}{3} < x < 4$ $\leftarrow$ (don't include end values as < 0)

2) Solve $12 - 5x - 2x^2 \leq 0$

Consider $y = 12 - 5x - 2x^2$ $(-x^2)$ parabola $\cap$.

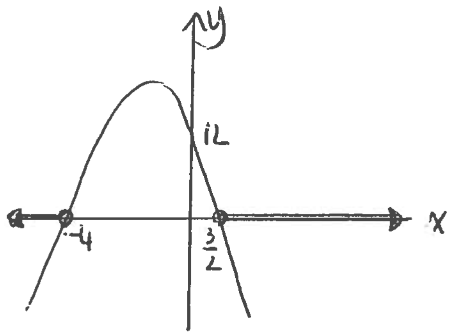
Cuts x-axis $y = 0$

$12 - 5x - 2x^2 = 0$

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$x = \frac{3}{2}$ or $x = -4$



Cuts y-axis $x = 0 \Rightarrow y = 12$.

We want $12 - 5x - 2x^2 \leq 0$

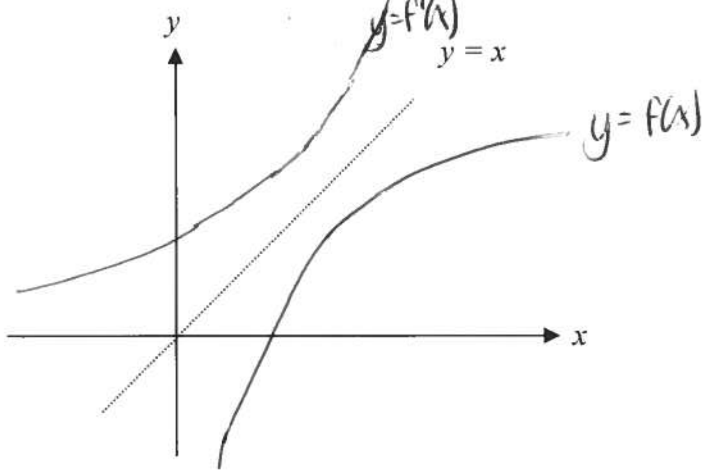
graph below x-axis

ie $x \leq -4$ or $x \geq \frac{3}{2}$.

Exercise :- p168 Ex 7K Q3

Graphs of Inverse Functions

To find the graph $y = f^{-1}(x)$ we reflect the graph $y = f(x)$ in the line $y = x$

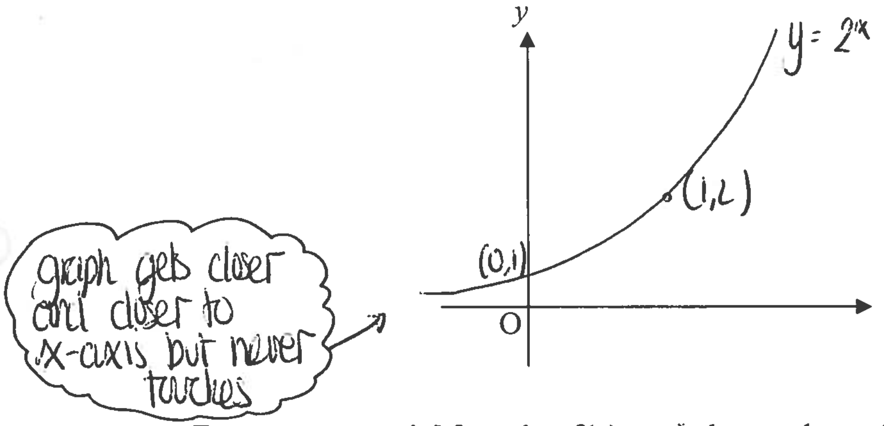


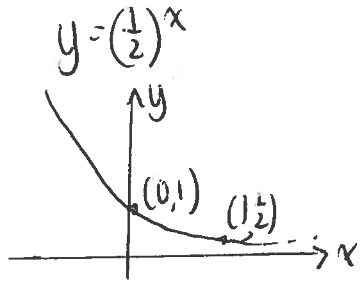
Exponential Functions

f(x) = a^x , $x \in \mathbb{R}$ is called an <u>exponential function</u> to the base **a**, $a \in \mathbb{R}$, $a > 0$, $a \neq 1$

e.g. $f(x) = 2^x$ is an exponential function to the base 2

Note
a > 1 shape.
0<a<1 shape.





For an **exponential function f(x) = a^x** the graph $y = f(x)$ **passes through (0,1) and (1,a)**

$f(0) = a^0 = 1$ for all $a \in \mathbb{R}$
$f(1) = a^1 = a$ for all $a \in \mathbb{R}$

LEARN

It never touches the x-axis since for $x \rightarrow -\infty$, $a^x \rightarrow 0$ but never equals 0

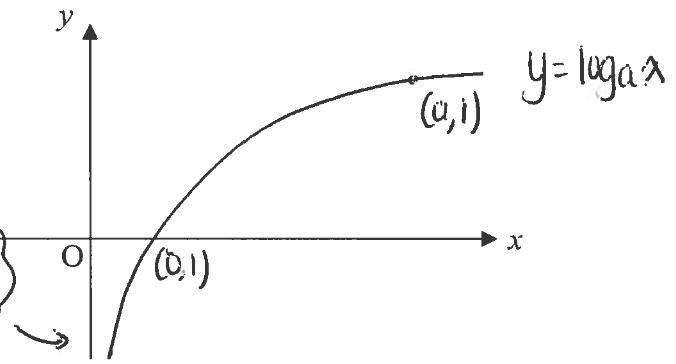
Logarithmic Functions

The inverse function of $f(x) = a^x$ is called the logarithmic function to base **a** written $\mathbf{\log_a x}$

$$\textbf{If } f(x) = a^x \textbf{ then } f^{-1}(x) = \log_a x$$
$$\textbf{If } f(x) = \log_a x \textbf{ then } f^{-1}(x) = a^x$$

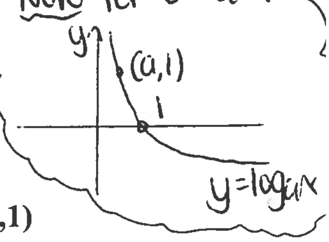
It follows, by considering exponential graphs that for $f(x) = \log_a x$ the graph $y = f(x)$ will look like this. (Exponential reflected in line $y = x$)

$a > 1$



graph gets closer and closer to y-axis but doesn't touch

Note for $0 < a < 1$



For a **logarithmic function** $\mathbf{f(x) = \log_a x}$ **the graph goes through (1,0) and (a,1)**

$\log_a 1 = 0$ for all $a \in \mathbb{R}$
$\log_a a = 1$ for all $a \in \mathbb{R}$

* * LEARN

The graph never touches the y-axis.

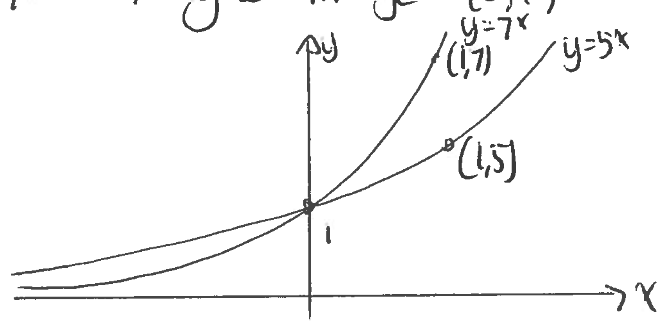
Examples

1) Sketch and annotate the graphs of $y = 5^x$ and $y = 7^x$ on the same diagram.

a^x graph — shape

$y = 5^x \rightarrow$ goes through $(0,1)$ and $(1,5)$

$y = 7^x \rightarrow$ goes through $(0,1)$ and $(1,7)$

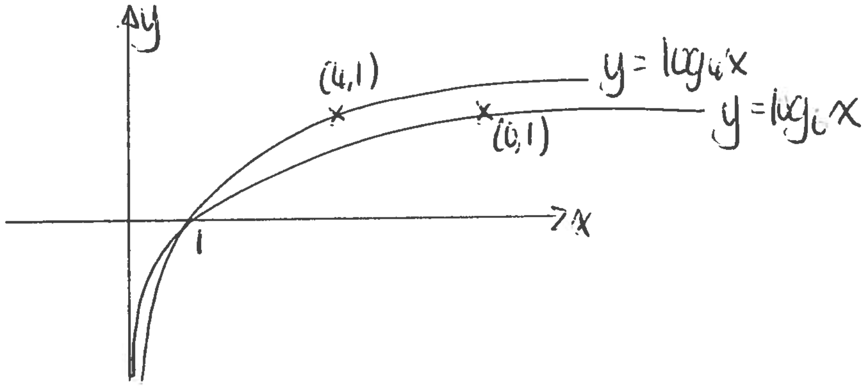


2) Sketch and annotate the graphs of $y = \log_4 x$ and $\log_6 x$ on the same diagram

log graph — shape

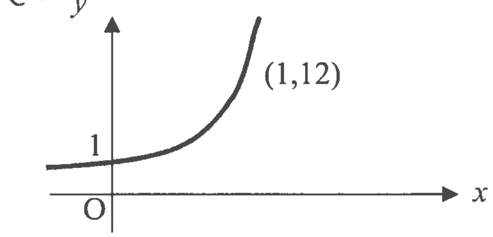
$y = \log_4 x$ goes through $(1,0)$ and $(4,1)$

$y = \log_6 x$ goes through $(1,0)$ and $(6,1)$



3) Identify the graphs below

(a)



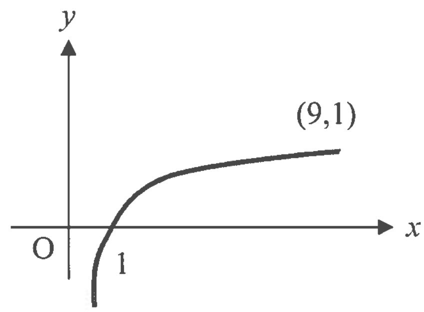
shape → exponential

Through point $(1, 12)$

$a = 12$

$y = 12^x$

(b)



shape → log

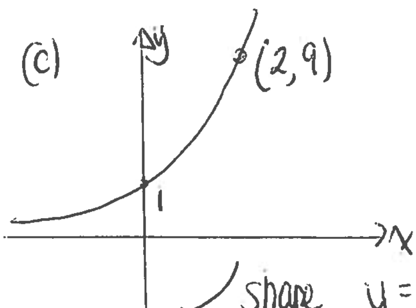
$y = \log_a x$

point $(9, 1)$

$a = 9$

$y = \log_9 x$

(c)



shape $y = a^x$

point $(2,9)$ $\quad 9 = a^2$ $\quad a = 3$ $\quad y = 3^x$.

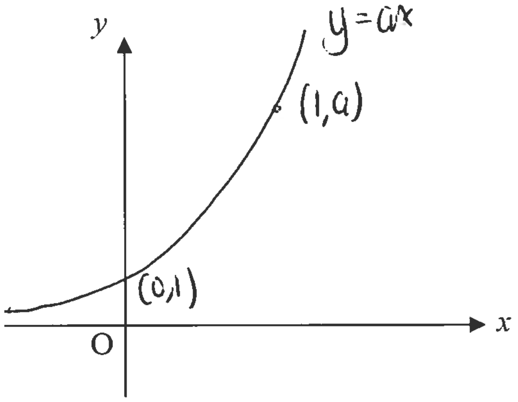
p68 Ex3D 6 ① ②

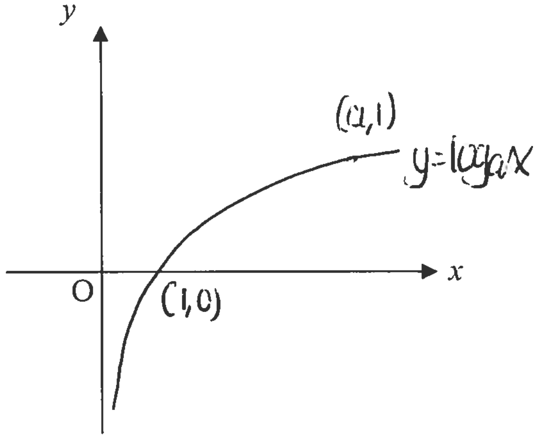
p73 Ex3E ① ②

Exponential and Logarithmic Transformations

As with others
Remember

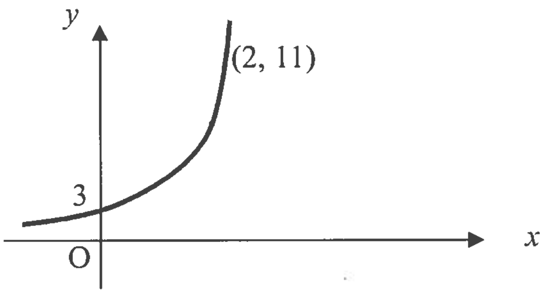
$y = a^x$	**through (0,1) and (1,a)**	and
$y = \log_a x$	**through (1,0) and (a,1)**	





Examples

1) Find the value of **a** and **b** in graph of $y = \mathbf{a}^x + \mathbf{b}$ shown



$y = a^x + b$

$(0,1) \rightarrow (0,3)$

+2 so $b = 2$.

or Substitute (0, 3)
x y

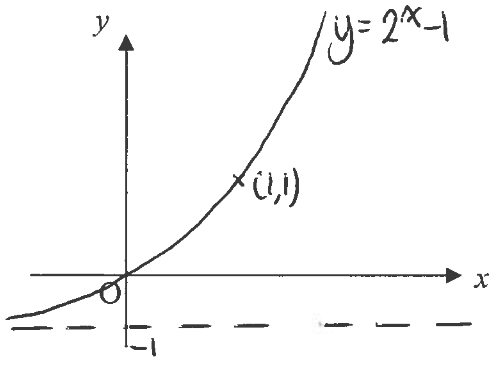
$3 = a^0 + b$
$3 = 1 + b$
$b = 2$.

Also point (2, 11)
substitute with $b = 2$.

$11 = a^2 + b$
$a^2 = 9$
$a = 3$

So $y = 3^x + 2$.

2) Sketch $y = 2^x - 1$



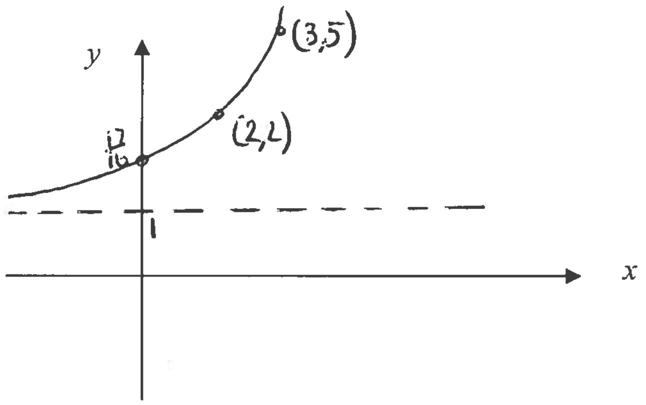
$y = 2^x$ goes through $(0,1)$ $(1,2)$

$y = 2^x - 1$ ← subtract 1 from y co-ords

$(0,1) \to (0,0)$

$(1,2) \to (1,1)$

3) Sketch $y = 4^{(x-2)} + 1$



$y = 4^x$ goes through $(0,1)$ $(1,4)$

$y = 4^{(x-2)} + 1$ ← add 1 to y co-ords (shift up)

↑ add 2 to x co-ords (shift right)

$(0,1) \to (2,2)$

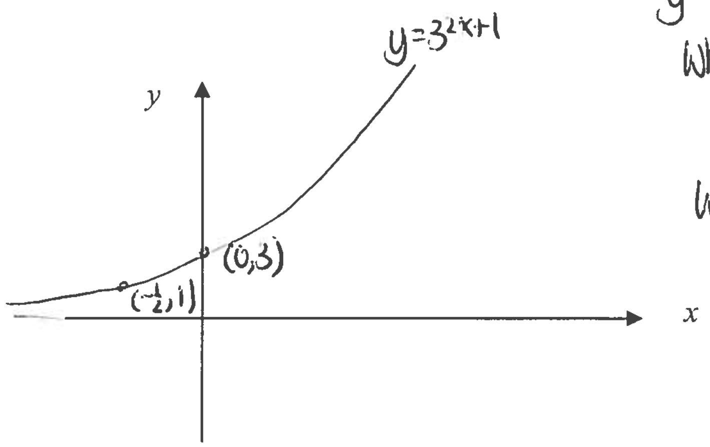
$(1,4) \to (3,5)$

cuts y-axis when $x = 0$

$y = 4^{-2} + 1$

$= \frac{17}{16}$.

4) Sketch $y = 3^{2x+1}$



$y = 3^x$ goes through $(0,1)$ $(1,3)$

When $2x + 1 = 0$ $y = 1$

ie $x = -\frac{1}{2}$ point $(-\frac{1}{2}, 1)$

When $2x + 1 = 1$ $y = 3$

$x = 0$ $(0, 3)$

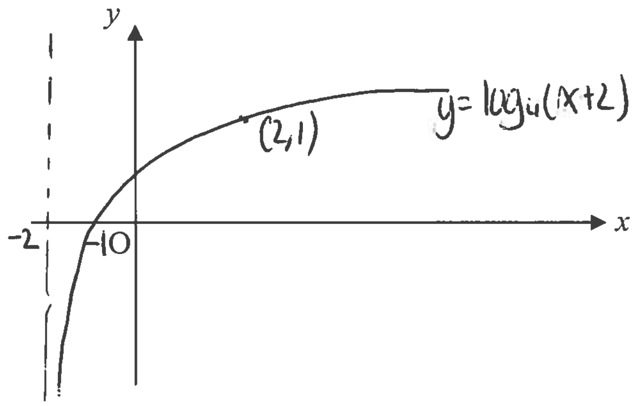
or $y = 3^{2x+1}$

$= 3^{2(x+\frac{1}{2})}$

half x then subtract $\frac{1}{2}$ from x co-ords

P69 Ex 3D Question 3

5) Sketch $y = \log_4(x + 2)$



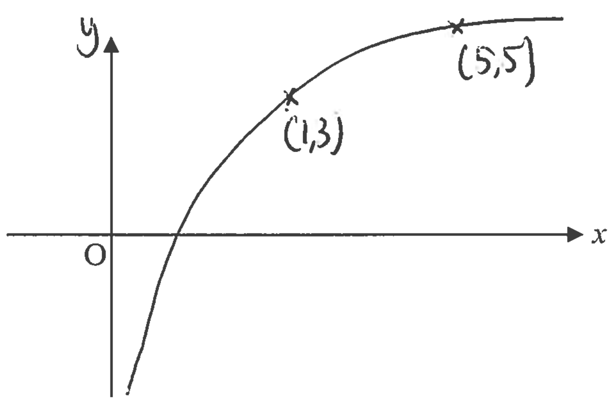
$y = \log_4 x$ goes through $(1,0)$ $(4,1)$

$y = \log_4 (x+2)$ subtract 2 from x co-ords

$(1,0) \to (-1,0)$ (shift $\xleftarrow{2}$)

$(4,1) \to (2,1)$

6) Sketch $y = \log_5 x^2 + 3$



$y = \log_5 x^2 + 3$

$= 2\log_5 x + 3$

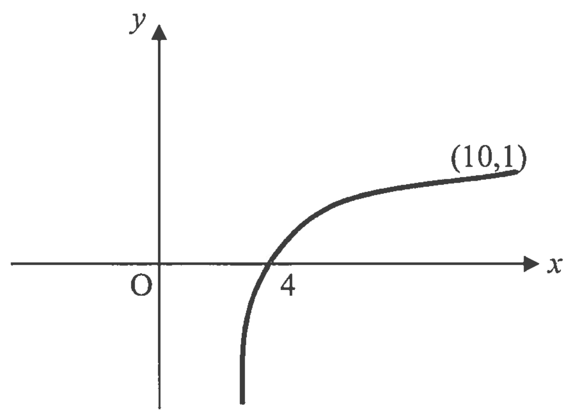
like $y = \log_5 x$

$(1,0) \to (1,0) \to (1,3)$

$(5,1) \to (5,2) \to (5,5)$

y co-ords ×2 y co-ords +3 (up 3)

7) What is the equation of the graph shown.



shape ⇒ log graph.

$(1,0) \to (4,0)$

+3 to x coords

so $y = \log_a(x-3)$

Substitute $(10,1)$

$1 = \log_a(10-3)$

$1 = \log_a 7$

$a = 7$

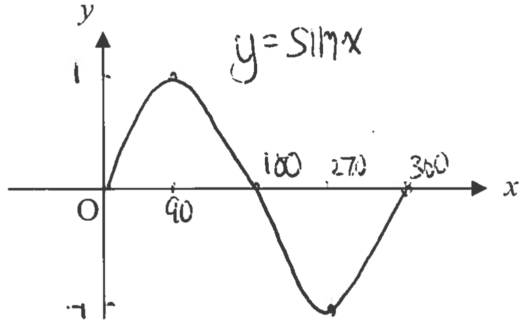
Equation $y = \log_7(x-3)$

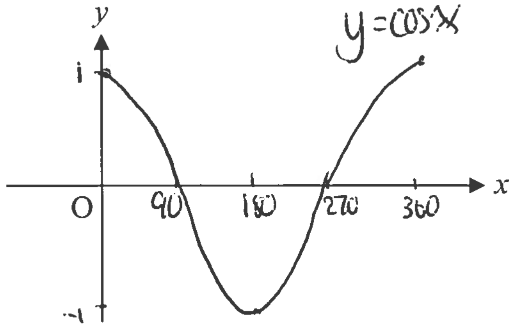
p73 Ex 3D Questions 3 to 5

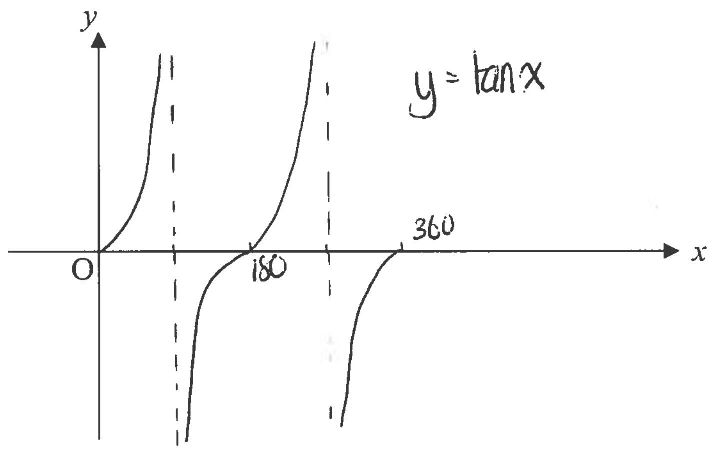
AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Trigonometric Functions and Graphs

Reminder







Recall transformations $y = a\sin bx + c$

a – amplitude
Period = 360/**b** (**b** waves in 360)
Moved **c** vertically

moves up or down

Also now for $y = a\sin(x+d) + c$

height from centre line

moves to left or right

as above for **a** and **c**
moved **d** to left for **d>0**
moved **d** to right for **d<0**

Example

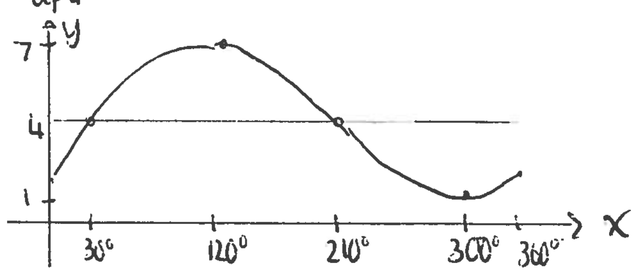
30° to right

1) Sketch $y = 3\sin(x-30)° + 4$, $0 \le x \le 360$

3 from centre line

up 4

max $3+4 = 7$ min $-3+4 = 1$



2) Given $f(x) = 2\sin(3x - 60)° + 5$ where $0 \le x \le 120$, sketch the graph and find the greatest and least values of $f(x)$ and the values of x for which they occur.

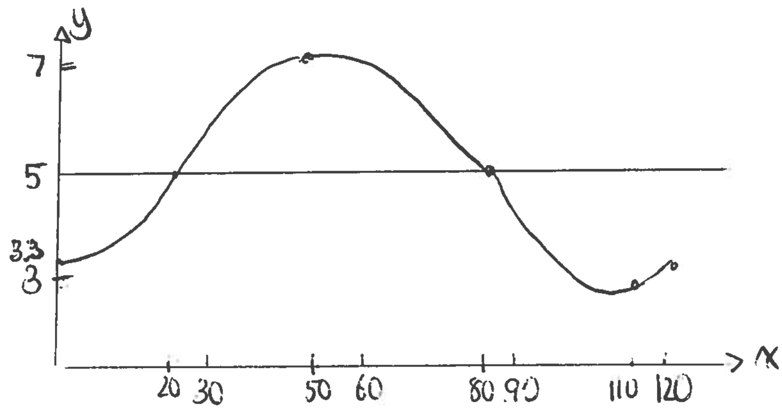
$f(x) = 2\sin(3x - 60) + 5$ ← up 5

2 from centre — period $360 \div 3 = 120°$ — $60°$ to right

max $f(x) = 2+5 = 7$ when angle $= 90°$
$3x - 60 = 90$
$3x = 150$
$x = 50°$

min $f(x) = -2+5 = 3$ when angle $= 270$
$3x - 60 = 270$
$x = 110°$

when $x = 0$
$f(x) = 2\sin(-60) + 5$
$= 3.3$



3) Given $f(x) = 4 - 3\cos\left(x + \frac{\pi}{3}\right)$ where $0 \le x \le 2\pi$ find the greatest value of $f(x)$ and the value of x at which it occurs.

$f(x) = -3\cos\left(x + \frac{\pi}{3}\right) + 4$

max $3+4 = 7$ when angle $= \pi$
$x + \frac{\pi}{3} = \pi$
$x = \frac{2\pi}{3}$

min $-3+4 = 1$ when angle $= 0$ or 2π
$x + \frac{\pi}{3} = 0$ or 2π
$x = \frac{5\pi}{3}$.

select.

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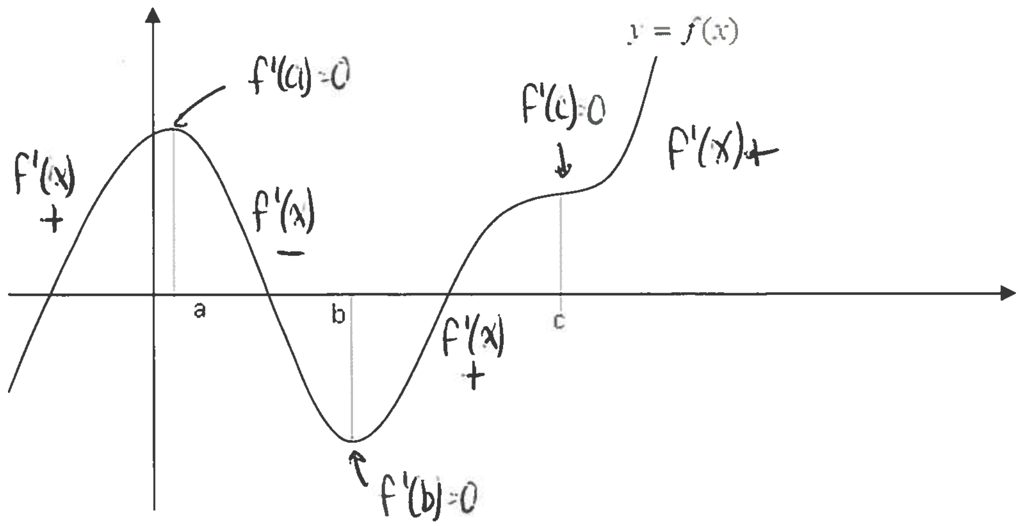
AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

* later

Graph of the Derived Function

Suppose for a function f (x) the graph looks like this



with stationary points at (a, f(a)) , (b, f(b)) , (c, f(c))

We know then that $f'(a) = 0$
And $f'(b) = 0$
And $f'(c) = 0$

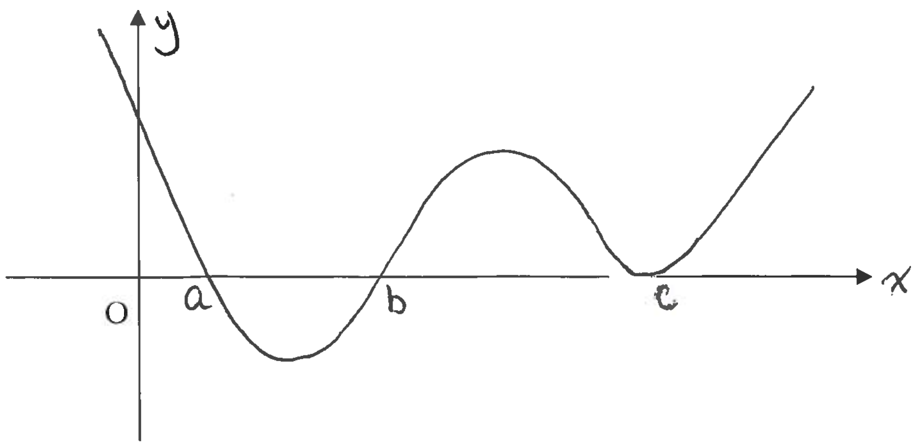
* Stationary points on y=f(x) become zeros on y=f'(x)

i.e. the graph of f '(x) crosses the x-axis at x = a, x = b, x = c

What about before, between and after these points ?

From the graph of f(x) we see that $f'(x) > 0$ for $x < a$ (since increasing)
And $f'(x) < 0$ for $a<x<b$ (since decreasing)
And $f'(x) > 0$ for $b<x<c$ (since increasing)
And $f'(x) > 0$ for $x > c$ (since increasing)

So graph of f '(x) looks like this

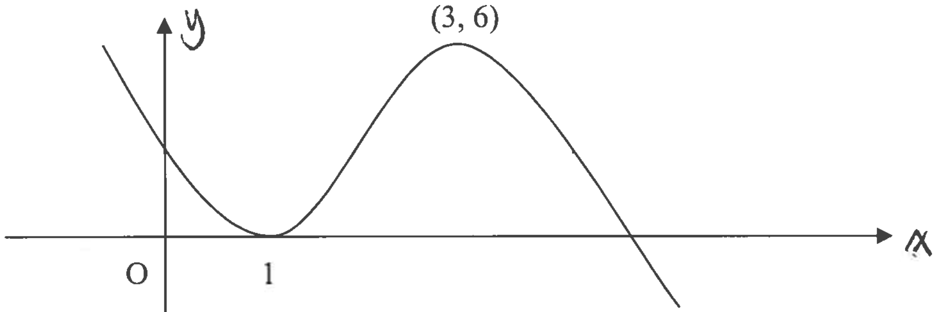


AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

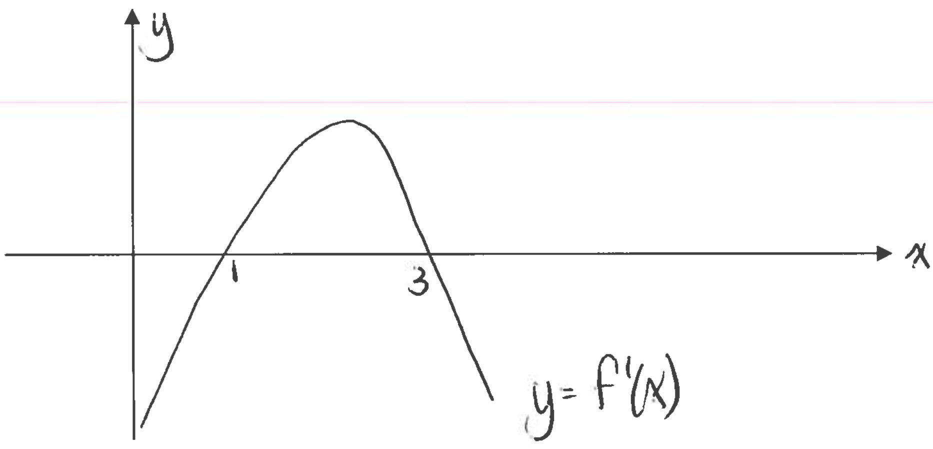
Note
If f(x) is a quadratic (parabola) then f ′(x) will be linear (straight line)
If f(x) is a cubic then f ′(x) will be a quadratic

Example

For the cubic function shown sketch the graph of the derived function.



$f'(x)$	$-$	0	$+$	0	$-$
	below x-axis	when $x=1$	above x-axis	when $x=3$	below x-axis



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Functions and Graphs

Set Notation

$\in$ means ' belongs to'
$\notin$ means 'does not belong to'
{ }or ϕ means the 'empty set' (no elements in it)
$\subset$ means subset

e.g. $2 \in \{2,4,6,8\}$
$3 \notin \{2,4,6,8\}$
The set of even prime numbers bigger than 2 is { }
$\{4,8\} \subset \{2,4,6,8\}$

Some Standard Sets

Set of Natural Numbers $\mathbb{N} = \{1,2,3,4,.......\}$

Set of Whole Numbers $\mathbb{W} = \{0,1,2,3,....\}$

Set of Integers $\mathbb{Z} = \{..., -2,-1,0,1,2,3,....\}$

Set of Rational Numbers $\mathbb{Q}$ = {all numbers which can be written as a fraction}

Set of Real Numbers $\mathbb{R}$ = {all numbers rational and irrational}

Set Builder Notation

We can define sets efficiently using set builder notation
e.g. $\{2,3,4,5,6,....,20\}$ can be defined by $\{x : 2 \le x \le 20, x \in \mathbb{W}\}$

or $\{x \in W : 2 \leq x \leq 20\}$

Examples

1) List the set $A = \{x : x < 10, x \in \mathbb{N}\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

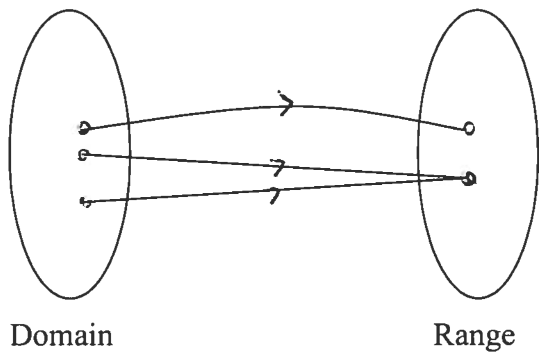
2) Write in set builder notation the set $B = \{-3,-2,-1,0,1,2\}$

$B = \{x \in Z : -3 \leq x \leq 2\}$

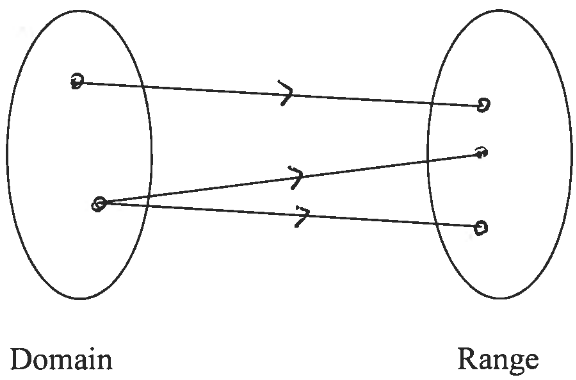
Functions

A function (or mapping) from set P to set Q is a rule that links each element in set P with one and only one element in set Q.

The set of elements in P (the start) is called the domain
The set of elements they go to in Q (the finish) is called the range



Represents a function f



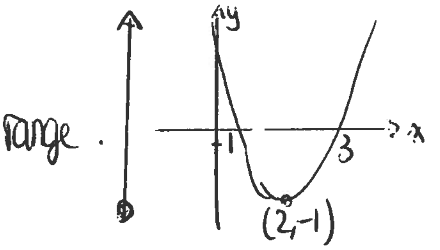
Does not represent a function f since one element in the domain goes to two different elements in the range.

Example

Illustrate and state the range of the following functions (use set builder notation)

(a) $f(x) = x^2 - 4x + 3$

Graph $y = (x-3)(x-1)$

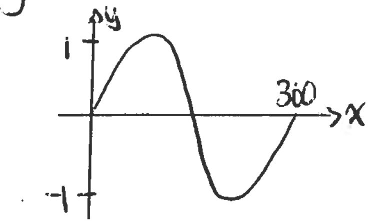


min TP
$x = 2$
$y = 4 - 8 + 3$
$= -1$

range $\{y \in R : y \geq -1\}$.

(b) $f(x) = \sin x°, \quad 0 \leq x \leq 360$

$y = \sin x$



range $\{y \in R : -1 \leq y \leq 1\}$.

Remember : For any function, each element in the Domain must link to ONE element in the Range.
Therefore, we cannot allow our Domain to contain values for which there would be no defined image in the Range.

Notably, our function must not be asked to (a) divide by zero
(b) square root a negative

because there would be no defined output.

Example

Write down any restrictions on the domain of the following functions and hence state the largest suitable domains.

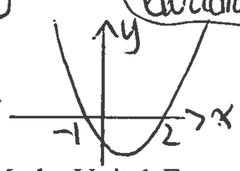
(a) $f(x) = \frac{1}{x^2-3x+2}$ (b) $f(x) = \sqrt{(x+1)(x-2)}$

(a) can't have zero on bottom of fraction
so $x^2 - 3x + 2 \neq 0$
$(x-2)(x-1) \neq 0$
$x \neq 2$ and $x \neq 1$

largest suitable domain
$\{x \in R : x \neq 2, 1\}$.

(b) can't square root a negative
so $(x+1)(x-2) \geq 0$
$x \leq -1$ or $x \geq 2$

Quadratic inequation → SKETCH graph



largest suitable domain
$\{x \in R : x \leq -1$ or $x \geq 2\}$.

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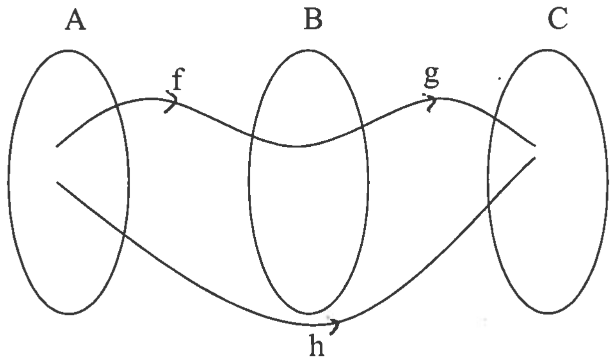
AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Composite Functions

Supposing we do two functions **f** and **g** one after the other on a domain A.



h(x) is the same as doing f(x) followed by g(x)
We write.
$h(x) = g(f(x))$
f(x) first since in brackets

Then we say that **h** is a <u>composite function</u> and **h(x) = g(f(x))**
(**g** of **f** of x)

In general $\mathbf{f(g(x)) \neq g(f(x))}$

ORDER IS IMPORTANT
f(g(x)) means we do g(x) <u>first</u>

<u>Examples</u>

1) $f(x) = x + 3$ and $g(x) = x^2$

a) Find an expression for $h(x) = g(f(x))$ and evaluate h(4)
b) Find an expression for $k(x) = f(g(x))$ and evaluate k(-2)

(a)
$$h(x) = g(f(x))$$
$$= g(x+3)$$
$$= (x+3)^2$$

* start with inside of bracket.

$$h(4) = (4+3)^2$$
$$= 7^2$$
$$= 49$$

(b)
$$k(x) = f(g(x))$$
$$= f(x^2)$$
$$= x^2+3$$
$$k(-2) = (-2)^2+3$$
$$= 7$$

2) The functions f and g defined on suitable domains are given by

$$f(x) = \frac{1}{x^2 - 4} \text{ and } g(x) = 2x + 1$$

a) Find an expression for h(x) = g(f(x)). Give your answer as a single fraction.
b) State a suitable domain for h.

$$h(x) = g(f(x))$$
$$= g\left(\frac{1}{x^2-4}\right)$$
$$= 2\left(\frac{1}{x^2-4}\right) + 1$$
$$= \frac{2}{x^2-4} + 1$$
$$= \frac{2}{x^2-4} + \frac{x^2-4}{x^2-4}$$
$$h(x) = \frac{x^2-2}{x^2-4}$$

(b) Can't have zero on bottom of fraction

$$x^2-4 \neq 0$$
$$x^2 \neq 4$$
$$x \neq \pm 2$$

remember two answers when square rooting

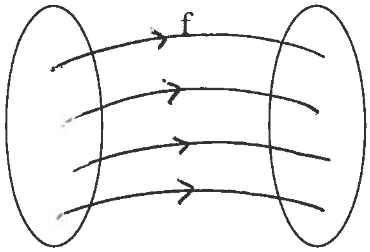
domain $\{x \in R : x \neq \pm 2\}$.

P87 Ex 4B

<u>One-to-One Correspondence</u>

A one-to-one correspondence occurs when a function is such that every value x in the domain relates to a different value in the range.

exactly one arrow <u>from</u> each point



Domain Range

exactly one arrow <u>to</u> each point

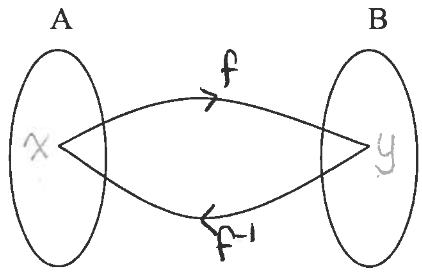
i.e. If f(x) is a **one-to-one correspondence** then $\boxed{x_1 = x_2 \Leftrightarrow f(x_1) = f(x_2)}$

e.g. $f(x) = x^2$ is <u>not</u> a one-to-one correspondence since $f(-2) = 4 = f(2)$

AS 1.3 Applying Algebraic and Trigonometric Skills to Functions

Inverse of a Function

When a function is a one-to-one correspondence from set A to set B then an inverse function exists from B to A that 'undoes' function f.



If $y = f(x)$ then $x = f^{-1}(y)$

We call this the **inverse** of f and write it as f^{-1}

In this case $\mathbf{f^{-1}(f(x)) = x = f(f^{-1}(x))}$

$$\boxed{\mathbf{f(g(x)) = x \Leftrightarrow} \textbf{ f and g are inverse functions of each other}}$$

Examples   Determine the inverse functions in questions 1-4 below, where they exist

1) $f(x) = x + 3$

If $y = f(x)$ then $x = f^{-1}(y)$

$y = x + 3$

$x = y - 3$ } (change subject to x)

$f^{-1}(y) = y - 3$

$f^{-1}(x) = x - 3$.

2) $g(x) = \frac{x}{2}$

If $y = g(y)$ then $x = g^{-1}(y)$

$y = \frac{x}{2}$

$x = 2y$

$g^{-1}(y) = 2y$

$g^{-1}(x) = 2x$

3) $f(x) = 2(x + 3)$

If $y = f(x)$ then $x = f^{-1}(y)$

$y = 2(x+3)$

$\frac{y}{2} = x + 3$

$x = \frac{y}{2} - 3$

$f^{-1}(y) = \frac{y}{2} - 3$

$f^{-1}(x) = \frac{x}{2} - 3$.

4) $g(x) = x^3 - 4$

If $y = g(x)$ then $x = g^{-1}(y)$

$y = x^3 - 4$

$y + 4 = x^3$

$x = \sqrt[3]{y+4}$

$g^{-1}(y) = \sqrt[3]{y+4}$

$g^{-1}(x) = \sqrt[3]{x+4}$.

5) A function is defined as $f(x) = \frac{2}{x} + 3$. Determine a formula for $f^{-1}(x)$, the inverse function, and hence evaluate $f^{-1}(1)$. State a suitable domain for $f^{-1}(x)$

if $y = f(x)$ then $x = f^{-1}(y)$

$y = \frac{2}{x} + 3$

$y - 3 = \frac{2}{x}$

$x(y-3) = 2$

$x = \frac{2}{y-3}$

$f^{-1}(y) = \frac{2}{y-3}$

$f^{-1}(x) = \frac{2}{x-3}$

$f^{-1}(1) = \frac{2}{1-3}$

$= -1$

domain $\{x \in R : x \neq 3\}$

6) The functions f and g are defined on suitable domains by $f(x) = \log_3 x$ and $g(x) = 9x^4$.
Find an expression for the function $h(x) = f(g(x))$ expressing your answer in the form $h(x) = A + B\log_3 x$.

$h(x) = f(g(x))$

$= f(9x^4)$

$= \log_3 9x^4$

$= \log_3 9 + \log_3 x^4$

$= \log_3 3^2 + 4\log_3 x$

$= 2\log_3 3 + 4\log_3 x$

$= 2 + 4\log_3 x$.

P89 Ex 4C, p91 Ex 4D