CfE Higher Maths

Unit 1 Expressions and Functions

1.4 Applying Geometric Skills to Vectors

① (a) (i) $\overrightarrow{AC} = \underline{c} - \underline{a}$

$= \begin{pmatrix}5\\-4\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}3\\-7\end{pmatrix}}}$

(ii) $\overrightarrow{CB} = \underline{b} - \underline{c}$

$= \begin{pmatrix}-4\\1\end{pmatrix} - \begin{pmatrix}5\\-4\end{pmatrix}$

$= \underline{\underline{\begin{pmatrix}-9\\5\end{pmatrix}}}$

(b) $|\overrightarrow{AB}| = \sqrt{(-6)^2 + (-2)^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40}$

$= \underline{\underline{2\sqrt{10}}}$

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$= \begin{pmatrix}-4\\1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$

$= \begin{pmatrix}-6\\-2\end{pmatrix}$

② (i) $\underline{a} + 2\underline{b} = \begin{pmatrix}2\\-1\\4\end{pmatrix} + 2\begin{pmatrix}0\\1\\-2\end{pmatrix}$

$= \begin{pmatrix}2\\-1\\4\end{pmatrix} + \begin{pmatrix}0\\2\\-4\end{pmatrix}$

$= \begin{pmatrix}2\\1\\0\end{pmatrix}$

(ii) $3\underline{c} - \underline{a} = 3\begin{pmatrix}6\\1\\-4\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix}$

$= \begin{pmatrix}18\\3\\-12\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix}$

$= \begin{pmatrix}16\\4\\-16\end{pmatrix}$

(b) $2\underline{a} - \underline{b} = 2\begin{pmatrix}2\\-1\\4\end{pmatrix} - \begin{pmatrix}0\\1\\-2\end{pmatrix}$

$= \begin{pmatrix}4\\-2\\8\end{pmatrix} - \begin{pmatrix}0\\1\\-2\end{pmatrix}$

$= \begin{pmatrix}4\\-3\\10\end{pmatrix}$

$|2\underline{a} - \underline{b}| = \sqrt{4^2 + (-3)^2 + 10^2}$

$= \sqrt{16 + 9 + 100}$

$= \sqrt{125}$

$= \underline{\underline{5\sqrt{5}}}$

(3) $\underline{F_1} + \underline{F_2} = 2\underline{i} + \underline{j} - 3\underline{k} + \underline{i} + 4\underline{k}$

$= 3\underline{i} + \underline{j} + \underline{k}$

$|\underline{F_1} + \underline{F_2}| = \sqrt{3^2 + 1^2 + 1^2}$

$= \sqrt{9+1+1}$

$= \underline{\underline{\sqrt{11}}}$

(4) $2\overrightarrow{PQ} = \overrightarrow{RS}$

$2(\underline{q} - \underline{p}) = \underline{s} - \underline{r}$

$\underline{s} = 2\underline{q} - 2\underline{p} + \underline{r}$

$= 2\begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \\ -5 \end{pmatrix}$

$\underline{s} = \begin{pmatrix} -4 \\ 0 \\ 12 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \\ -5 \end{pmatrix}$

$\underline{s} = \begin{pmatrix} -10 \\ -5 \\ 3 \end{pmatrix}$

$\underline{\underline{S(10, 5, 3)}}$

(5) $\overrightarrow{EF} = \underline{f} - \underline{e}$

$= \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 6 \\ -3 \end{pmatrix}$

$\overrightarrow{FG} = \underline{g} - \underline{f}$

$= \begin{pmatrix} -1 \\ 16 \\ -9 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 12 \\ -6 \end{pmatrix}$

$= 2\begin{pmatrix} -1 \\ 6 \\ -3 \end{pmatrix}$

so $2\overrightarrow{EF} = \overrightarrow{FG}$

EF is parallel to FG and since F is a common point E, F and G must be collinear.

Ratio EF : FG is $\underline{\underline{1:2}}$

(6) $\underline{v_1} = 3\underline{i} + 2\underline{j} - \underline{k}$ $\quad$ $\underline{v_2} = 2\underline{i} - \underline{j} + 4\underline{k}$

$\underline{v_1} \cdot \underline{v_2} = 6 - 2 - 4$

$= 0$ so $\underline{v_1}$ and $\underline{v_2}$ are perpendicular.

(7) $\underline{u} = \begin{pmatrix} \sqrt{2} \\ -1 \\ 1 \end{pmatrix}$ $\quad$ $|\underline{u}| = \sqrt{(\sqrt{2})^2 + (-1)^2 + 1^2}$

$= \sqrt{2+1+1}$

$= \sqrt{4}$

$= 2.$

$\underline{v} = \frac{1}{2} \begin{pmatrix} \sqrt{2} \\ -1 \\ 1 \end{pmatrix}$

(8) $\underline{a} \cdot (\underline{a} - \underline{b}) = \underline{a} \cdot \underline{a} - \underline{a} \cdot \underline{b}$

$= |\underline{a}|^2 - |\underline{a}||\underline{b}| \cos\theta$

$= 2^2 - 2 \times 2\sqrt{2} \times \cos 45$

$= 4 - 2 \times 2\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 4 - 4$

$= 0$

Since the scalar product is 0 $\underline{a}$ and $\underline{a} - \underline{b}$ are perpendicular. The angle between them is 90°.

(9) (a) $\underline{a} \cdot \underline{b} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$

$= 4 - 12 + 4$

$= -4 < 0$ so the angle between $\underline{a}$ and $\underline{b}$ is obtuse.

(b) $\underline{u} = 2\underline{a}$ $\quad$ $\underline{v} = \underline{a} + \underline{b}$

$= \begin{pmatrix} 4 \\ 6 \\ -4 \end{pmatrix}$ $\quad$ $= \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$

$\underline{u} \cdot \underline{v} = \begin{pmatrix} 4 \\ 6 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$

$= 16 - 6 + 16$

$= 26$

$|\underline{u}| = \sqrt{16+36+16}$

$= \sqrt{68}$

$|\underline{v}| = \sqrt{16+1+16}$

$= \sqrt{33}$

$\cos\theta = \dfrac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|}$

$\cos\theta = \dfrac{26}{\sqrt{68}\sqrt{33}}$

$\underline{\underline{\theta = 56.7^\circ}}$

(c) $\underline{c} \cdot \underline{a} = 0$

$\begin{pmatrix} 2 \\ k \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = 0$

$4 + 3k - 10 = 0$

$3k = 6$

$\underline{k = 2}$

10) $\vec{BA} = \underline{a} - \underline{b}$

$= \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b}$

$= \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = \begin{pmatrix}-2\\3\\-4\end{pmatrix} \cdot \begin{pmatrix}-3\\3\\-2\end{pmatrix}$$
$$= 6+9+8$$
$$= 23$$

$$|\overrightarrow{BA}| = \sqrt{4+9+16}$$
$$= \sqrt{29}$$

$$|\overrightarrow{BC}| = \sqrt{9+9+4}$$
$$= \sqrt{22}$$

$$\cos A\hat{B}C = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{|\overrightarrow{BA}|\,|\overrightarrow{BC}|}$$
$$= \frac{23}{\sqrt{29}\,\sqrt{22}}$$
$$A\hat{B}C = 24.4^\circ$$

(11) (a) $\times 3$
$$4 : 1 : 2$$
$$12 \quad 3 \quad 6$$

$p = 3 \;,\; q = 6$

(b) $A(12, 0, 0)$ $\quad C(0, 3, 0)$

$$\overrightarrow{CF} = \underline{f} - \underline{c}$$
$$= \begin{pmatrix}12\\3\\6\end{pmatrix} - \begin{pmatrix}0\\3\\0\end{pmatrix}$$
$$= \begin{pmatrix}12\\0\\6\end{pmatrix}$$

$$\overrightarrow{CA} = \underline{a} - \underline{c}$$
$$= \begin{pmatrix}12\\0\\0\end{pmatrix} - \begin{pmatrix}0\\3\\0\end{pmatrix}$$
$$= \begin{pmatrix}12\\-3\\0\end{pmatrix}$$

$$\overrightarrow{CF} \cdot \overrightarrow{CA} = \begin{pmatrix}12\\0\\6\end{pmatrix} \cdot \begin{pmatrix}12\\-3\\0\end{pmatrix}$$
$$= 144$$

$$|\overrightarrow{CF}| = \sqrt{144+0+36}$$
$$= \sqrt{180}$$
$$|\overrightarrow{CA}| = \sqrt{144+9+0}$$
$$= \sqrt{153}$$

$$\cos A\hat{C}F = \frac{\overrightarrow{CF} \cdot \overrightarrow{CA}}{|\overrightarrow{CF}|\,|\overrightarrow{CA}|}$$
$$= \frac{144}{\sqrt{180}\,\sqrt{153}}$$

$\cos A\hat{C}F = 0.8677\ldots$

$\underline{\underline{A\hat{C}F = 29.8°}}$ (1dp)

⑫ (a) $3\overrightarrow{EF} = \overrightarrow{GH}$

$3(\underline{f} - \underline{e}) = \underline{h} - \underline{g}$

$\underline{h} = 3\underline{f} - 3\underline{e} + \underline{g}$

$= \begin{pmatrix} +3 \\ 9 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \\ 2 \end{pmatrix}$

$\underline{h} = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$ $\underline{\underline{H(4, 1, -1)}}$

(b) $\overrightarrow{EH} = \underline{h} - \underline{e}$

$= \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$ $|\overrightarrow{EH}| = \sqrt{25 + 1 + 4}$

$= \underline{\underline{\sqrt{30}}}$

⑬ $\underline{F_1} \cdot \underline{F_2} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix}$

$= 10 - 8 - 2$

$= 0$ so $\underline{F_1}$ and $\underline{F_2}$ are perpendicular.

⑭ $\underline{V} = 2\underline{i} + \underline{j} - 2\underline{k}$

$|\underline{V}| = \sqrt{2^2 + 1^2 + (-2)^2}$

$= \sqrt{4 + 1 + 4}$

$= 3$

unit vector $= \frac{1}{3}(2\underline{i} + \underline{j} - 2\underline{k})$

$= \frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} - \frac{2}{3}\underline{k}$

(15) $\frac{AC}{CB} = \frac{3}{2}$

$2AC = 3CB$

If A, B, C are collinear.

$\Rightarrow 2\overrightarrow{AC} = 3\overrightarrow{CB}$

$2(\underline{c}-\underline{a}) = 3(\underline{b}-\underline{c})$

$2\underline{c} - 2\underline{a} = 3\underline{b} - 3\underline{c}$

$5\underline{c} = 2\underline{a} + 3\underline{b}$

$\underline{c} = \frac{1}{5}(2\underline{a} + 3\underline{b})$

$= \frac{1}{5}\left(\begin{pmatrix}-4\\-2\\8\end{pmatrix} + \begin{pmatrix}9\\12\\-3\end{pmatrix}\right)$

$= \frac{1}{5}\begin{pmatrix}5\\10\\5\end{pmatrix}$

$\underline{c} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ $\quad$ $\underline{\underline{C = (1,2,1)}}$

(16) $\overrightarrow{PQ} = \underline{q} - \underline{p}$

$= \begin{pmatrix}-2\\-6\\4\end{pmatrix} - \begin{pmatrix}-8\\3\\1\end{pmatrix}$

$= \begin{pmatrix}6\\-9\\3\end{pmatrix}$

$= 3\begin{pmatrix}2\\-3\\1\end{pmatrix}$

$\overrightarrow{QR} = \underline{r} - \underline{q}$

$= \begin{pmatrix}2\\-12\\6\end{pmatrix} - \begin{pmatrix}-2\\-6\\4\end{pmatrix}$

$= \begin{pmatrix}4\\-6\\2\end{pmatrix}$

$= 2\begin{pmatrix}2\\-3\\1\end{pmatrix}$

$2\overrightarrow{PQ} = 3\overrightarrow{QR}$

so PQ and QR are parallel and since Q is a common point, P, Q and R are collinear.

$PQ : QR = 3 : 2$

17 (a) $R(0, 4, -6)$ $S(6, 10, -15)$

(b) $\vec{PR} = \underline{r} - \underline{p}$

$= \begin{pmatrix}0\\4\\-6\end{pmatrix} - \begin{pmatrix}-8\\-4\\6\end{pmatrix}$

$= \begin{pmatrix}8\\8\\-12\end{pmatrix}$

$= 4\begin{pmatrix}2\\2\\-3\end{pmatrix}$

$\vec{RS} = \underline{s} - \underline{r}$

$= \begin{pmatrix}6\\10\\-15\end{pmatrix} - \begin{pmatrix}0\\4\\-6\end{pmatrix}$

$= \begin{pmatrix}6\\6\\-9\end{pmatrix}$

$= 3\begin{pmatrix}2\\2\\-3\end{pmatrix}$

so $3\vec{PR} = 4\vec{RS}$

and PR and RS are parallel and since R is a common point P, R and S are collinear.

(c) $\vec{RP} = \underline{p} - \underline{r}$

$= \begin{pmatrix}-8\\-4\\6\end{pmatrix} - \begin{pmatrix}0\\4\\-6\end{pmatrix}$

$= \begin{pmatrix}-8\\-8\\12\end{pmatrix}$

$\vec{RQ} = \underline{q} - \underline{r}$

$\begin{pmatrix}6\\-10\\-15\end{pmatrix} - \begin{pmatrix}0\\4\\-6\end{pmatrix}$

$= \begin{pmatrix}6\\-14\\-9\end{pmatrix}$

(d) $\vec{RP} \cdot \vec{RQ} = \begin{pmatrix}-8\\-8\\12\end{pmatrix} \cdot \begin{pmatrix}6\\-14\\-9\end{pmatrix}$

$= -48 + 112 - 108$

$= -44$

$|\vec{RP}| = \sqrt{64+64+144}$

$= \sqrt{272}$

$|\vec{RQ}| = \sqrt{36+196+81}$

$= \sqrt{313}$

$\cos P\hat{R}Q = \dfrac{\vec{RP} \cdot \vec{RQ}}{|\vec{RP}|\,|\vec{RQ}|}$

$= \dfrac{-44}{\sqrt{272}\sqrt{313}}$

$\underline{\underline{P\hat{R}Q = 98.7°}}$

18 (a)

$$\underline{u}.\underline{v} = 2\underline{s}.(\underline{t}-\underline{s})$$
$$= 2\underline{s}.\underline{t} - 2\underline{s}.\underline{s}$$
$$= 2|\underline{s}||\underline{t}|\cos 60° - 2|\underline{s}|^2$$
$$= 2 \times 1 \times 2 \times \frac{1}{2} - 2 \times 1^2$$
$$= 2-2$$
$$= 0$$

(b) The angle between $\underline{u}$ and $\underline{v}$ is 90°.

19 (a)

$$\overrightarrow{ZY} = \underline{y} - \underline{z} = \begin{pmatrix}12\\20\\0\end{pmatrix} - \begin{pmatrix}8\\22\\-1\end{pmatrix} = \begin{pmatrix}4\\-2\\1\end{pmatrix}$$

$$\overrightarrow{YX} = \underline{x} - \underline{y} = \begin{pmatrix}20\\16\\2\end{pmatrix} - \begin{pmatrix}12\\20\\0\end{pmatrix} = \begin{pmatrix}8\\-4\\2\end{pmatrix} = 2\begin{pmatrix}4\\-2\\1\end{pmatrix}$$

so $2\overrightarrow{ZY} = \overrightarrow{YX}$

so ZY and YX are parallel and since there is a common point Y, X, Y and Z are collinear.

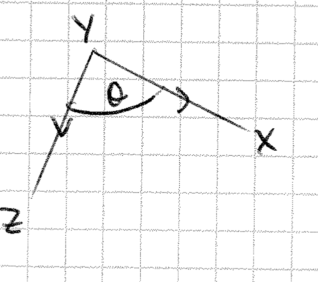
(b) ratio ZY : YX is 1 : 2.

so if ZY = 42 km, YX = 84 km.

and ZX = 42 + 84

= 126 km.

(c)



$$\overrightarrow{YZ} = \underline{z} - \underline{y} = \begin{pmatrix}8\\22\\-1\end{pmatrix} - \begin{pmatrix}50\\15\\-8\end{pmatrix} = \begin{pmatrix}-42\\7\\7\end{pmatrix}$$

$$\overrightarrow{YX} = \underline{x} - \underline{y} = \begin{pmatrix}20\\16\\2\end{pmatrix} - \begin{pmatrix}50\\15\\-8\end{pmatrix} = \begin{pmatrix}-30\\1\\10\end{pmatrix}$$

$$\vec{YZ} \cdot \vec{YX} = \begin{pmatrix} -42 \\ 7 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} -30 \\ 1 \\ 10 \end{pmatrix}$$

$$= 1337$$

$$|\vec{YZ}| = \sqrt{1764+49+49}$$

$$= \sqrt{1862}$$

$$|\vec{YX}| = \sqrt{900+1+100}$$

$$= \sqrt{1001}$$

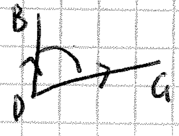
$$\cos X\hat{Y}Z = \frac{\vec{YZ} \cdot \vec{YX}}{|\vec{YZ}|\,|\vec{YX}|}$$

$$= \frac{1337}{\sqrt{1862}\,\sqrt{1001}}$$

$$\underline{\underline{X\hat{Y}Z = 11.7^\circ}}$$

(20) (a) B $(8, -2, -3)$ D $(0, 2, -3)$ G $(8, 2, 3)$

(b) $B\hat{D}G$



$$\vec{DB} = \underline{b} - \underline{d}$$

$$= \begin{pmatrix} 8 \\ -2 \\ -3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix}$$

$$\vec{DG} = \underline{g} - \underline{d}$$

$$= \begin{pmatrix} 8 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 0 \\ 6 \end{pmatrix}$$

$$\vec{DB} \cdot \vec{DG} = \begin{pmatrix} 8 \\ -4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 0 \\ 6 \end{pmatrix}$$

$$= 64$$

$$|\vec{DB}| = \sqrt{64+16+0}$$

$$= \sqrt{80}$$

$$|\vec{DG}| = \sqrt{64+0+36}$$

$$= 10$$

$$\cos B\hat{D}G = \frac{64}{\sqrt{80} \times 10}$$

$$\underline{\underline{B\hat{D}G = 44.3^\circ}}$$

21 (a) $\vec{QP} = \vec{RS}$

$$\underline{p} - \underline{q} = \underline{s} - \underline{r}$$

$$\underline{s} = \underline{p} - \underline{q} + \underline{r}$$

$$= \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 5 \\ -7 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$S(1,2,3)$

(b) $\vec{SQ} = \underline{q} - \underline{s}$

$$= \begin{pmatrix} 5 \\ -7 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -9 \\ -1 \end{pmatrix}$$

$\vec{PR} = \underline{r} - \underline{p}$

$$= \begin{pmatrix} -1 \\ -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ -3 \\ -5 \end{pmatrix}$$

$$\vec{SQ} \cdot \vec{PR} = \begin{pmatrix} 4 \\ -9 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -3 \\ -5 \end{pmatrix}$$

$$= -32 + 27 + 5$$

$$= 0$$

so SQ and PR are perpendicular.

22 (a) $|v_2| = 2|v_1|$

$$\sqrt{8^2 + (\sqrt{24})^2 + (a\sqrt{3})^2} = 2(\sqrt{4^2 + 1^2 + (\sqrt{8})^2})$$

$$64 + 24 + 3a^2 = 4(16 + 1 + 8)$$

$$88 + 3a^2 = 100$$

$$3a^2 = 12$$

$$a^2 = 4$$

$$a = 2 \quad (\text{since } a > 0$$

(b) $\underline{v_1} \cdot \underline{v_2} = \begin{pmatrix} 4 \\ 1 \\ \sqrt{8} \end{pmatrix} \cdot \begin{pmatrix} 8 \\ \sqrt{24} \\ 2\sqrt{3} \end{pmatrix} = 32 + \sqrt{24} + 2\sqrt{24}$

$= 32 + 3\sqrt{24}$

> 0 so angle between $\underline{V}_1$ and $\underline{V}_2$ is acut

(23) $\underline{p} \cdot \underline{q} = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix} \cdot \begin{pmatrix} k \\ -3 \\ -2 \end{pmatrix}$

$= k - 3k - 4$

$= -2k - 4.$

If $\underline{p}$ and $\underline{q}$ are perpendicular $\underline{p} \cdot \underline{q} = 0$

$-2k - 4 = 0$

$k = -2$

PTO.

(24)

$$\vec{KM} = 3\vec{KL}$$

$$\underline{m} - \underline{k} = 3(\underline{l} - \underline{k})$$

$$\underline{m} = 3\underline{l} - 2\underline{k}$$

$$= 3\begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} - 2\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix} - \begin{pmatrix} 0 \\ -2 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ -4 \\ 7 \end{pmatrix}$$

$$M = (9, -4, 7)$$

(25)

$$\underline{p} \cdot (\underline{p} - \underline{q}) = 18$$

$$\underline{p} \cdot \underline{p} - \underline{p} \cdot \underline{q} = 18$$

$$|\underline{p}|^2 - \underline{p} \cdot \underline{q} = 18$$

$$\underline{p} \cdot \underline{q} = |\underline{p}|^2 - 18$$

$$= 9 - 18$$

$$= -9$$

(26)

$$\vec{KL} = \underline{l} - \underline{k}$$

$$= \begin{pmatrix} 5 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$$

$$\vec{KM} = \underline{m} - \underline{k}$$

$$= \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$$

$$\vec{KL} \cdot \vec{KM} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ -4 \end{pmatrix}$$

$$= 8 - 2 - 16$$

$$= -10$$

$$|\vec{KL}| = \sqrt{4+4+16}$$

$$= \sqrt{24}$$

$$|\vec{KM}| = \sqrt{16+1+16}$$

$$= \sqrt{33}$$

$$\cos L\hat{K}M = \frac{\vec{KL}\cdot\vec{KM}}{|\vec{KL}||\vec{KM}|}$$

$$= \frac{-10}{\sqrt{24}\sqrt{33}}$$

$$\cos L\hat{K}M = -0.3553\ldots$$

$$\underline{\underline{L\hat{K}M = 110.8^\circ}}$$