

Unit 1 Expressions and Functions1.3. Applying algebraic and trigonometric skills to functions

$$\begin{aligned} 1. \quad a) \quad i) \quad & g(f(2)) \\ & = g(2^2 + 2) \\ & = g(6) \\ & = 18 - 1 \\ & = 17 \end{aligned}$$

$$\begin{aligned} ii) \quad & g(f(-1)) \\ & = g((-1)^2 + (-1)) \\ & = g(0) \\ & = 0 - 1 \\ & = -1 \end{aligned}$$

$$\begin{aligned} b) \quad i) \quad & f(g(x)) \\ & = f(3x - 1) \\ & = (3x - 1)^2 + (3x - 1) \\ & = 9x^2 - 6x + 1 + 3x - 1 \\ & = 9x^2 - 3x \end{aligned}$$

$$\begin{aligned} ii) \quad & g(f(x)) \\ & = g(x^2 + x) \\ & = 3(x^2 + x) - 1 \\ & = 3x^2 + 3x - 1 \end{aligned}$$

$$\begin{aligned} iii) \quad & f(f(k)) \\ & = f(k^2 + k) \\ & = (k^2 + k)^2 + (k^2 + k) \\ & = k^4 + 2k^3 + k^2 + k^2 + k \\ & = k^4 + 2k^3 + 2k^2 + k \end{aligned}$$

$$c) \quad g(x) = 3x - 1$$

$$y = 3x - 1$$

$$x = \frac{y + 1}{3}$$

$$3y = x + 1$$

$$y = \frac{1}{3}(x + 1) \Rightarrow g^{-1}(x) = \frac{1}{3}(x + 1)$$

$$\begin{aligned}
 2. \quad a) \quad f(h(x)) &= f\left(\frac{1}{x^2} + 1\right) \\
 &= \frac{2}{3\left(\frac{1}{x^2} + 1\right) - 3} \\
 &= \frac{2}{\frac{3}{x^2} + 3 - 3} \\
 &= \frac{2}{\frac{3}{x^2}} \\
 &= \frac{2x^2}{3}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(x) &= \frac{2}{3x-3} \\
 y &= \frac{2}{3x-3} \\
 x &= \frac{2}{3y-3}
 \end{aligned}$$

$$\begin{aligned}
 x(3y-3) &= 2 \\
 3y-3 &= \frac{2}{x} \\
 3y &= \frac{2}{x} + 3 \\
 y &= \frac{2}{3x} + 1
 \end{aligned}$$

$$f^{-1}(x) = \frac{2}{3x} + 1$$

$$\text{Domain} = \left\{ x : x \in \mathbb{R}, x \neq 0 \right\}$$

$$3. \quad a) \quad \text{Domain} = \left\{ x : x \in \mathbb{R}, x \neq -\frac{1}{2} \right\}$$

$$b) \quad f(x) = \frac{1}{2x+1}$$

$$y = \frac{1}{2x+1}$$

$$x = \frac{1}{2y+1}$$

$$x(2y+1) = 1$$

$$2y+1 = \frac{1}{x}$$

$$2y = \frac{1}{x} - 1$$

$$y = \frac{1}{2x} - \frac{1}{2}$$

$$y = \frac{1-x}{2x}$$

$$f^{-1}(x) = \frac{1-x}{2x}$$

$$c) \quad \frac{1-x}{2x} = \frac{1}{2x+1}$$

$$1-x = \frac{2x}{2x+1}$$

$$(2x+1)(1-x) = 2x$$

$$2x - 2x^2 + 1 - x - 2x = 0$$

$$1 - x - 2x^2 = 0$$

$$(1-2x)(1+x) = 0$$

$$x = \frac{1}{2}, \quad x = -1$$

$$\begin{array}{r} 1 - 2 - 2 \\ 1 + 1 \quad \underline{+1} \\ -1 \end{array}$$

$$\begin{aligned}
 4a) \quad g(x) &= f(h(x)) \\
 &= f\left(\frac{1}{x-1}\right) \\
 &= 2\left(\frac{1}{x-1}\right) + 1 \\
 &= \frac{2}{x-1} + \frac{x-1}{x-1} \\
 &= \frac{x+1}{x-1}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad g(x) &= \frac{x+1}{x-1} \\
 y &= \frac{x+1}{x-1} \\
 x &= \frac{y+1}{y-1}
 \end{aligned}$$

$$x(y-1) = y+1$$

$$xy - x = y + 1$$

$$xy - y = x + 1$$

$$y(x-1) = x+1$$

$$y = \frac{x+1}{x-1}$$

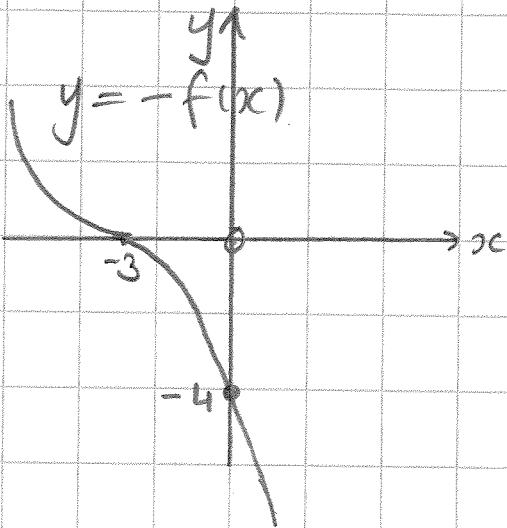
$$g^{-1}(x) = \frac{x+1}{x-1} = g(x)$$

$$5. a) y = 4 \sin x^\circ$$

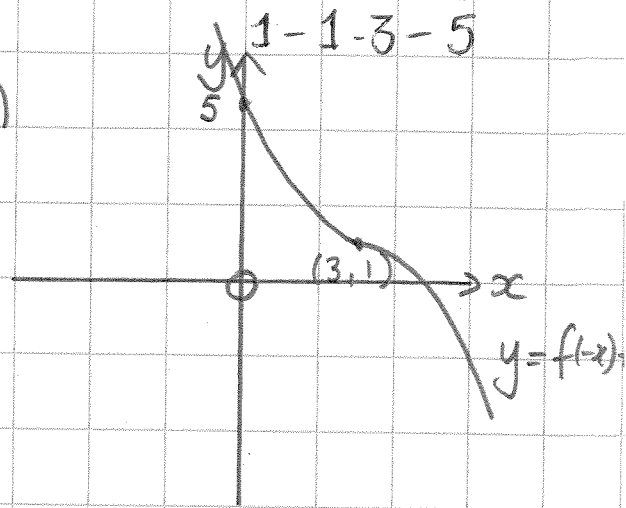
$$b) y = 20 \cos 2x^\circ$$

$$c) y = 4 \sin 4x^\circ + 2$$

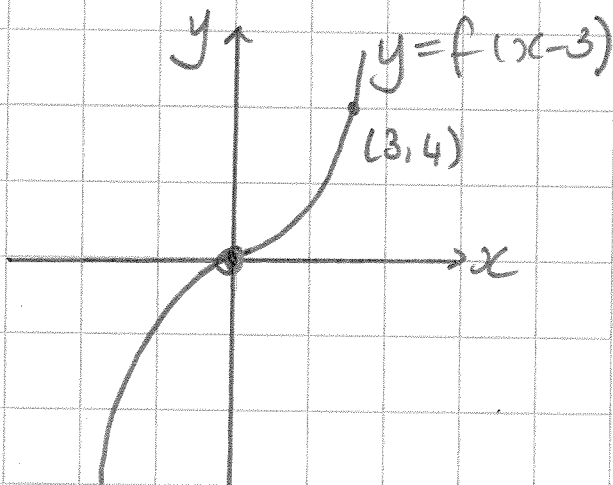
6. a)



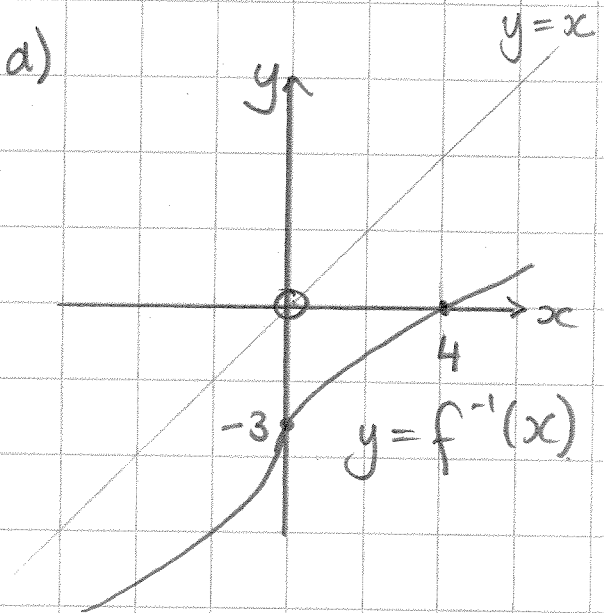
b)



c)



d)



7. a)

$$y = 6 \sin 2x^\circ$$

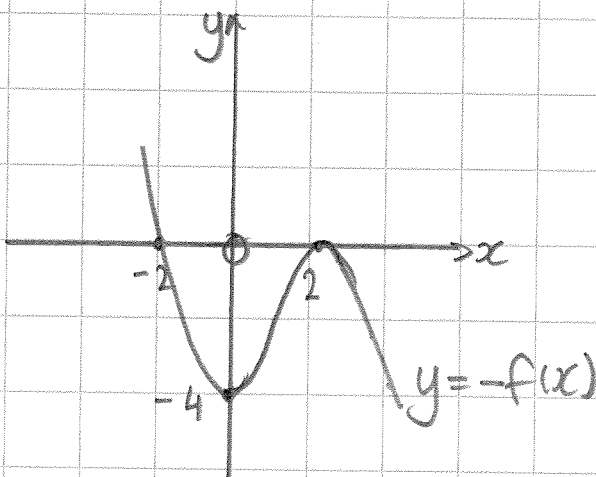
b)

$$y = -10 \cos 8x^\circ$$

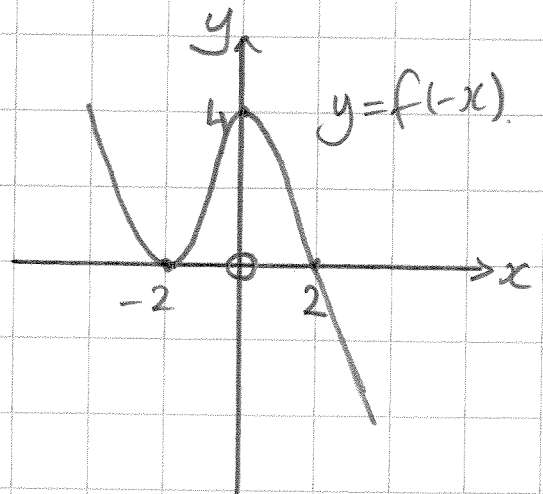
c)

$$y = 3 \cos 4x^\circ + 1$$

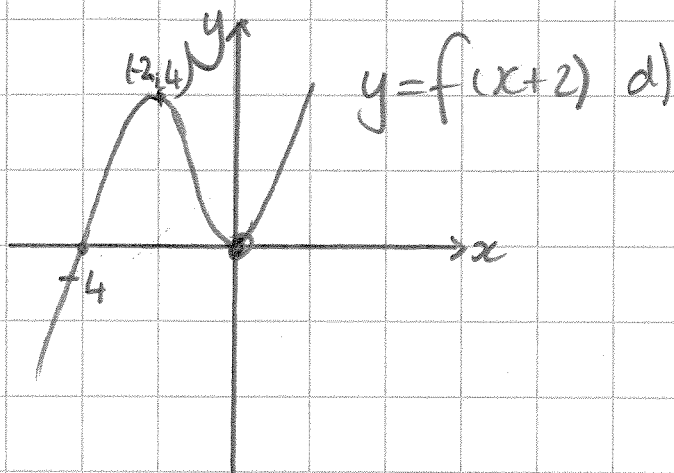
8. a)



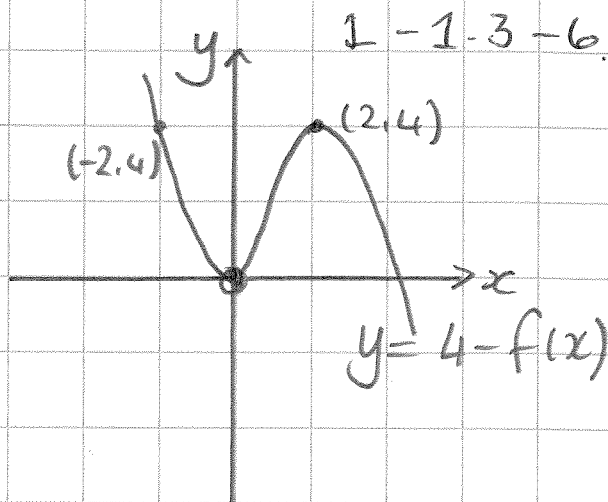
b)



c)



d)



$$9. \quad a) \quad f(g(-3)) = f\left(\frac{-3+9}{4}\right)$$

$$= f\left(\frac{3}{2}\right)$$

$$= 2 \times \frac{3}{2} - 3$$

$$= 0.$$

$$b) \quad g(f(x)) = g(2x-3)$$

$$= \frac{(2x-3)+9}{4}$$

$$= \frac{2x+6}{4}$$

$$= \frac{x+3}{2}$$

$$c) \quad f(x) = 2x-3$$

$$y = 2x-3$$

$$x = \frac{y+3}{2}$$

$$x+3 = y$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$

$$= g(f(x)).$$

$$10. a) \text{ Domain} = \{x : x \in \mathbb{R}, x > 0\}$$

$$b) \quad g(x) = \frac{2}{\sqrt{x}}$$

$$y = \frac{2}{\sqrt{x}}$$

$$x = \frac{2}{\sqrt{y}}$$

$$\sqrt{y} = \frac{2}{x}$$

$$y = \frac{4}{x^2}$$

$$g^{-1}(x) = \frac{4}{x^2}$$

$$\begin{aligned} c) \quad f(x) &= g^{-1}(g^{-1}(x)) \\ &= g^{-1}\left(\frac{4}{x^2}\right) \\ &= \frac{4}{\left(\frac{4}{x^2}\right)^2} \\ &= 4 \div \frac{16}{x^4} \\ &= 4 \times \frac{x^4}{16} \\ &= \frac{x^4}{4} \end{aligned}$$

$$\begin{aligned}
 11. \quad a) \quad h(f(2)) &= h(2^2+3) \\
 &= h(7) \\
 &= 7+21 \\
 &= 28
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(h(x)) &= f(7+3x) \\
 &= (7+3x)^2+3 \\
 &= 49+42x+9x^2+3 \\
 &= 9x^2+42x+52
 \end{aligned}$$

$$c) \quad x^2+3=7+3x$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x=4, \quad x=-1$$

$$\begin{array}{r}
 1-4 \quad -4 \\
 1+1 \quad +1 \\
 \hline
 -3
 \end{array}$$

$$12. \quad f(x) = 3x(x-1) + (3a+3)$$

$$f(k) = f(a+1)$$

$$= 3(a+1)(a+1-1) + (3a+3)$$

$$= 3(a+1)(a) + (3a+3)$$

$$= 3a^2 + 3a + 3a + 3$$

$$= 3a^2 + 6a + 3$$

$$= 3(a^2 + 2a + 1)$$

$$= 3(a+1)^2$$

$$= 3k^2$$

13. a) $f(2) = h(2) = 7$

$$f(2) = 4p - 1 = 7$$

$$4p = 8$$

$$p = 2$$

$$f(x) = 2x^2 - 1$$

$$h(2) = \frac{10 + q}{2} = 7$$

$$10 + q = 14$$

$$q = 4$$

$$h(x) = \frac{5x + 4}{2}$$

b) $h(f(x)) = h(2x^2 - 1)$
 $= \frac{5(2x^2 - 1) + 4}{2}$

$$= \frac{10x^2 - 5 + 4}{2}$$

$$= 5x^2 - \frac{1}{2}$$

c) $2[h(f(x))] - 4 = k[f(x)]$

$$2(5x^2 - \frac{1}{2}) - 4 = k(2x^2 - 1)$$

$$10x^2 - 1 - 4 = 2kx^2 - k$$

$$10x^2 - 2kx^2 + k - 1 - 4 = 0$$

$$(10 - 2k)x^2 + (k - 5) = 0$$

$$10 - 2k = 0 \quad \text{and} \quad k - 5 = 0$$

$$\text{a. } k = 5$$

14. $y = p \log_3(x + k)$

$$y = \log_3 x$$

$$(1, 0) \xrightarrow{(x+3)} (-2, 0)$$

$$(3, 1)$$

$$y = p \log_3(x + 3)$$

(6, 4) would be (9, 4) on $p \log_3 x$

$$y = 2 \log_3(x + 3)$$

$$\Rightarrow x = 9$$

$$\log_3 9 = 2$$

$$p = 2$$

$$\begin{aligned} 15. \quad a) \quad f(g(x)) &= f(x-1) \\ &= 2(x-1)^2 + 5 \\ &= 2(x^2 - 2x + 1) + 5 \\ &= 2x^2 - 4x + 2 + 5 \\ &= 2x^2 - 4x + 7 \end{aligned}$$