

Unit 1 Expressions and Functions1.2. Applying trigonometric skills to manipulating expressions

$$\begin{aligned} 1 \text{ a) } \sqrt{3} \cos x^\circ + \sin x^\circ &= k \sin(x+\alpha)^\circ \\ &= k(\sin x^\circ \cos \alpha^\circ + \cos x^\circ \sin \alpha^\circ) \\ &= (k \cos \alpha^\circ) \sin x^\circ + (k \sin \alpha^\circ) \cos x^\circ \end{aligned}$$

$$k \cos \alpha^\circ = 1$$

$$k \sin \alpha^\circ = \sqrt{3}$$

$$\begin{aligned} k &= \sqrt{1+3} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \tan \alpha^\circ &= \frac{\sqrt{3}}{1} \\ \alpha^\circ &= 60^\circ \end{aligned}$$

$$\sqrt{3} \cos x^\circ + \sin x^\circ = 2 \sin(x+60)^\circ$$

b) minimum value of a fraction when denominator maximised.

$$\begin{aligned} &\sqrt{3} \cos x^\circ + \sin x^\circ + 4 \\ &= 2 \sin(x+60)^\circ + 4 \end{aligned}$$

$$\begin{aligned} \text{so max value} &= 2+4 \\ &= 6 \end{aligned}$$

$$\text{when } \sin(x+60)^\circ = 1$$

$$(x+60)^\circ = 90^\circ$$

$$x^\circ = 30^\circ$$

$$2. \quad h = 3 \sin 15t^\circ + 4 \cos 15t^\circ + 25$$

$$\begin{aligned} a) \quad 3 \sin 15t^\circ + 4 \cos 15t^\circ &= k \cos(15t - \alpha)^\circ \\ &= k (\cos 15t^\circ \cos \alpha^\circ + \sin 15t^\circ \sin \alpha^\circ) \\ &= (k \cos \alpha^\circ) \cos 15t^\circ + (k \sin \alpha^\circ) \sin 15t^\circ \end{aligned}$$

$$k \cos \alpha^\circ = 4$$

$$k \sin \alpha^\circ = 3$$

$$\begin{aligned} k &= \sqrt{16+9} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \tan \alpha^\circ &= \frac{3}{4} \\ \alpha^\circ &= 36.9^\circ \end{aligned}$$

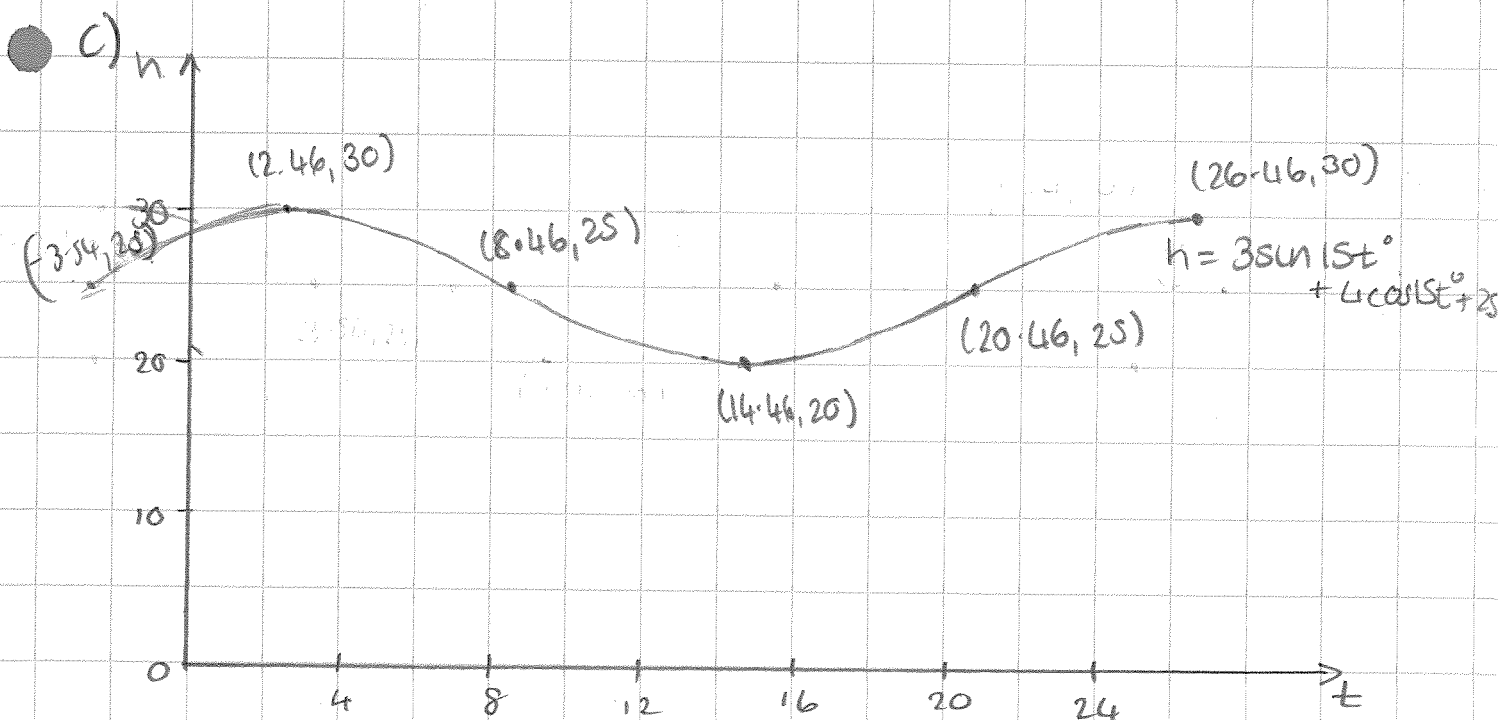
$$\begin{aligned} &3 \sin 15t^\circ + 4 \cos 15t^\circ + 25 \\ &= 5 \cos(15t - 36.9)^\circ + 25 \end{aligned}$$

$$\begin{aligned} b) \quad \text{minimum} &= -5 + 25 \\ &= 20 \text{ m} \end{aligned}$$

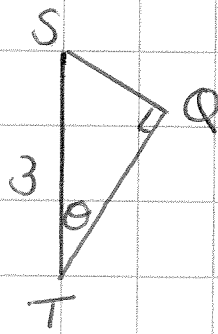
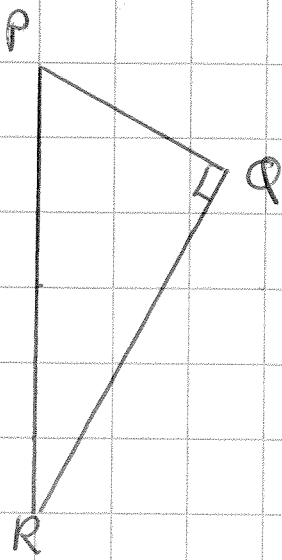
$$\text{when } (15t - 36.9)^\circ = 180^\circ$$

$$15t^\circ = 216.9^\circ$$

$$t^\circ = 14.46 \quad \text{ie. around } 14:30.$$



3. a)



$$\cos \theta = \frac{TQ}{3}$$

$$TQ = 3 \cos \theta$$

$$\text{So } RQ = 2 + 3 \cos \theta$$

$$\sin \theta = \frac{SQ}{3}$$

$$SQ = 3 \sin \theta$$

$$\text{So } PQ = 1 + 3 \sin \theta$$

$$b) \quad PR^2 = PQ^2 + RQ^2$$

$$= (1 + 3 \sin \theta)^2 + (2 + 3 \cos \theta)^2$$

$$= 1 + 6 \sin \theta + 9 \sin^2 \theta + 4 + 12 \cos \theta + 9 \cos^2 \theta$$

$$= 12 \cos \theta + 6 \sin \theta + 5 + 9(\sin^2 \theta + \cos^2 \theta)$$

$$= 12 \cos \theta + 6 \sin \theta + 14$$

$$a = 12 \quad b = 6 \quad c = 14$$

$$c) \quad 12 \cos \theta + 6 \sin \theta = k \cos(x - \alpha)$$

$$= k(\cos x \cos \alpha + \sin x \sin \alpha)$$

$$= (k \cos \alpha) \cos x + (k \sin \alpha) \sin x$$

$$k \cos \alpha = 12$$

$$k \sin \alpha = 6$$

$$k = \sqrt{144 + 36}$$

$$= \sqrt{180}$$

$$= 6\sqrt{5}$$

$$\tan \alpha = \frac{6}{12}$$

$$\alpha = 0.46$$

$$12 \cos \theta + 6 \sin \theta + 14 = 6\sqrt{5} \cos(x - 0.46) + 14$$

$$\text{max of } PR^2 = 6\sqrt{5} + 14$$

$$\text{max of } PR = 5.2 \text{ (to 1 dp)}$$

$$\begin{aligned}
 4. \text{ a)} \quad & 4\sin(x+30)^\circ - \cos x^\circ \\
 &= 4(\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ) - \cos x^\circ \\
 &= 4\sin x^\circ \times \frac{\sqrt{3}}{2} + 4\cos x^\circ \times \frac{1}{2} - \cos x^\circ \\
 &= 2\sqrt{3}\sin x^\circ + \cos x^\circ
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & 2\sqrt{3}\sin x^\circ + \cos x^\circ = k\cos(x-\alpha)^\circ \\
 &= (k\cos\alpha^\circ)\cos x^\circ + (k\sin\alpha^\circ)\sin x^\circ
 \end{aligned}$$

$$k\cos\alpha^\circ = 1$$

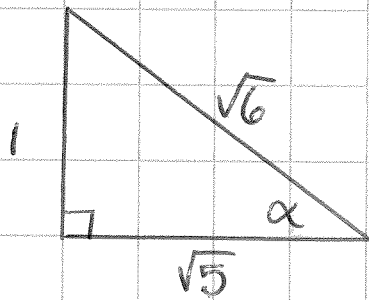
$$k\sin\alpha^\circ = 2\sqrt{3}$$

$$\begin{aligned}
 k &= \sqrt{1+12} & \tan\alpha^\circ &= \frac{2\sqrt{3}}{1} \\
 &= \sqrt{13}
 \end{aligned}$$

$$\alpha^\circ = 73.9^\circ$$

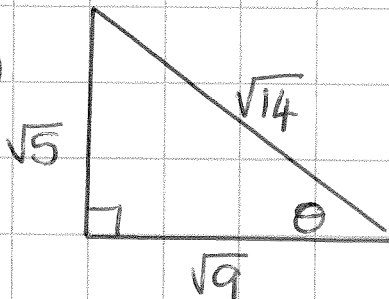
$$2\sqrt{3}\sin x^\circ + \cos x^\circ = \sqrt{13}\cos(x-73.9^\circ)^\circ$$

5.



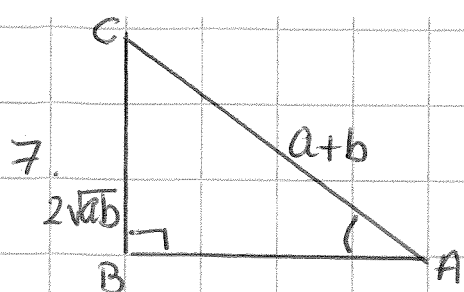
$$\begin{aligned}
 \sin 2\alpha &= 2\sin\alpha\cos\alpha \\
 &= 2 \times \frac{1}{\sqrt{6}} \times \frac{\sqrt{5}}{\sqrt{6}} \\
 &= \frac{2\sqrt{5}}{6} \\
 &= \frac{\sqrt{5}}{3}
 \end{aligned}$$

6. a)



$$\sin\theta = \frac{\sqrt{5}}{\sqrt{14}} \quad \cos\theta = \frac{\sqrt{9}}{\sqrt{14}}$$

$$\begin{aligned}
 \text{b)} \quad \sin 2\theta &= 2\sin\theta\cos\theta \\
 &= 2 \times \frac{\sqrt{5}}{\sqrt{14}} \times \frac{\sqrt{9}}{\sqrt{14}} \\
 &= \frac{2\sqrt{45}}{14} = \frac{3\sqrt{5}}{7}
 \end{aligned}$$

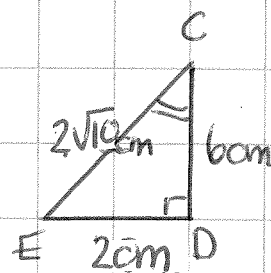
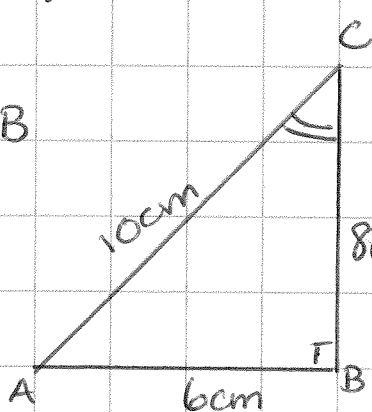


$$\begin{aligned} AB^2 &= (a+b)^2 - (2\sqrt{ab})^2 \\ &= a^2 + 2ab + b^2 - 4ab \\ &= a^2 - 2ab + b^2 \end{aligned}$$

$$AB = a - b$$

$$\begin{aligned} P &= (a+b) + (a-b) + 2\sqrt{ab} \\ &= 2a + 2\sqrt{ab} \\ &= 2(a + \sqrt{ab}) \end{aligned}$$

8. $ACE = ACB - ECB$



using pyth.

$$\cos ACE = \cos (ACB - ECB)$$

$$= \cos ACB \cos ECB + \sin ACB \sin ECB$$

$$= \frac{8}{10} \times \frac{6}{2\sqrt{10}} + \frac{6}{10} \times \frac{2}{2\sqrt{10}}$$

$$= \frac{48 + 12}{20\sqrt{10}}$$

$$= \frac{60}{20\sqrt{10}}$$

$$= \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{3\sqrt{10}}{10}$$

$$\begin{aligned}
 9. \text{ a) } 12 \cos x^\circ + 5 \sin x^\circ &= k \sin(x - \alpha)^\circ \\
 &= k (\sin x^\circ \cos \alpha^\circ - \cos x^\circ \sin \alpha^\circ) \\
 &= (k \cos \alpha^\circ) \sin x^\circ - (k \sin \alpha^\circ) \cos x^\circ
 \end{aligned}$$

$$k \cos \alpha^\circ = 5$$

$$k \sin \alpha^\circ = -12$$

$$\begin{aligned}
 k &= \sqrt{144 + 25} \\
 &= 13
 \end{aligned}$$

$$\tan \alpha^\circ = -\frac{12}{5} \quad \begin{array}{c} \swarrow A \\ \text{---} \\ \searrow S \end{array}$$

$$\alpha^\circ = 292.6^\circ$$

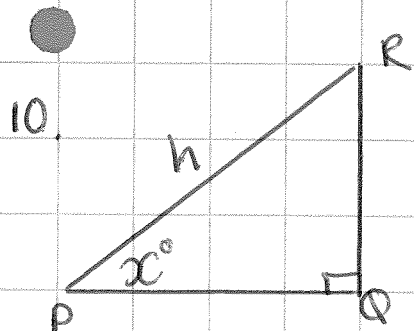
$$12 \cos x^\circ + 5 \sin x^\circ = 13 \sin(x - 292.6)^\circ$$

● b) minimum of a fraction occurs when the denominator is maximised

$$\begin{aligned}
 \text{ie } h(x) &= \frac{8}{12 \cos x^\circ + 5 \sin x^\circ + 3} \\
 &= \frac{8}{13 \sin(x - 292.6)^\circ + 3}
 \end{aligned}$$

max value of $13 \sin(x - 292.6)^\circ + 3$ is 16.

So min value of $h(x)$ is $\frac{8}{16} = \frac{1}{2}$.



$$\begin{aligned}
 \text{a) } \sin x^\circ &= \frac{RQ}{h} & \cos x^\circ &= \frac{PQ}{h} \\
 \text{so } RQ &= h \sin x^\circ & \text{so } PQ &= h \cos x^\circ
 \end{aligned}$$

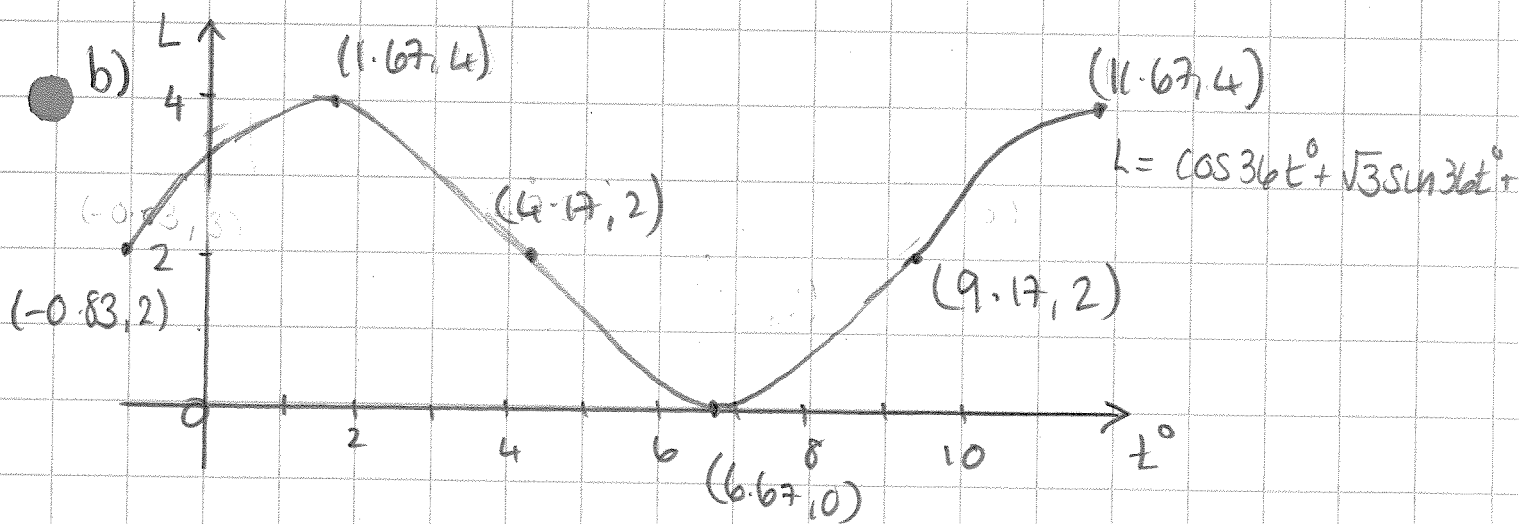
$$\begin{aligned}
 \text{b) } P &= h + h \sin x^\circ + h \cos x^\circ \\
 19.24 &= 8 + 8 \sin x^\circ + 8 \cos x^\circ \\
 8 \sin x^\circ + 8 \cos x^\circ &= 11.24 \\
 2 \sin x^\circ + 2 \cos x^\circ &= 2.81
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } 2\sin x^\circ + 2\cos x^\circ &= k\cos(x-\alpha)^\circ \\
 &= k(\cos x^\circ \cos \alpha^\circ + \sin x^\circ \sin \alpha^\circ) \\
 &= (k\cos \alpha^\circ)\cos x^\circ + (k\sin \alpha^\circ)\sin x^\circ \\
 k\cos \alpha^\circ &= 2 & k &= \sqrt{4+4} & \tan \alpha^\circ &= \frac{2}{2} \\
 k\sin \alpha^\circ &= 2 & &= 2\sqrt{2} & \alpha^\circ &= 45^\circ
 \end{aligned}$$

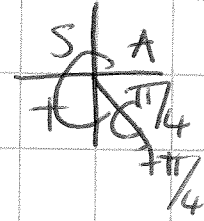
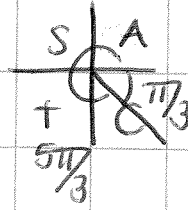
$$2\sin x^\circ + 2\cos x^\circ = 2\sqrt{2}\cos(x-45)^\circ$$

$$\begin{aligned}
 \text{II. a) } \cos 36t^\circ + \sqrt{3}\sin 36t^\circ &= R\cos(36t-\alpha)^\circ \\
 &= (R\cos \alpha^\circ)\cos 36t^\circ + (R\sin \alpha^\circ)\sin 36t^\circ \\
 R\cos \alpha^\circ &= 1 & R &= \sqrt{1+3} & \tan \alpha^\circ &= \frac{\sqrt{3}}{1} \\
 R\sin \alpha^\circ &= \sqrt{3} & &= 2 & \alpha^\circ &= 60^\circ
 \end{aligned}$$

$$\begin{aligned}
 \cos 36t^\circ + \sqrt{3}\sin 36t^\circ + 2 \\
 = 2\cos(36t-60)^\circ + 2
 \end{aligned}$$



$$\begin{aligned}
 12. \quad & \cos \frac{5\pi}{3} - \tan \frac{7\pi}{4} \\
 &= \cos \frac{\pi}{3} - \left(-\tan \frac{\pi}{4}\right) \\
 &= \cos \frac{\pi}{3} + \tan \frac{\pi}{4} \\
 &= \frac{1}{2} + 1 \\
 &= 1\frac{1}{2}
 \end{aligned}$$



$$13. a) \quad 2\sin(x + \pi/6) - 2\cos x = \sqrt{3}\sin x - \cos x$$

$$\begin{aligned}
 \bullet \quad \text{LHS} &= 2\sin(x + \pi/6) - 2\cos x \\
 &= 2(\sin x \cos \pi/6 + \cos x \sin \pi/6) - 2\cos x \\
 &= 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) - 2\cos x \\
 &= \sqrt{3}\sin x + \cos x - 2\cos x \\
 &= \sqrt{3}\sin x - \cos x \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \sqrt{3}\sin x - \cos x &= k\sin(x - \alpha) \\
 &= k(\sin x \cos \alpha - \cos x \sin \alpha) \\
 &= (k\cos \alpha)\sin x - (k\sin \alpha)\cos x
 \end{aligned}$$

$$k\cos \alpha = \sqrt{3}$$

$$k\sin \alpha = 1$$

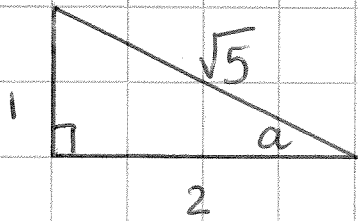
$$\begin{aligned}
 k &= \sqrt{1+3} \\
 &= 2
 \end{aligned}$$

$$\tan \alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \pi/6$$

$$\sqrt{3}\sin x - \cos x = 2\sin(x - \pi/6)$$

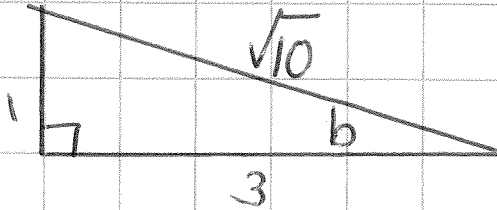
14.



$$a) \quad i) \quad \cos a = \frac{2}{\sqrt{5}}$$

$$\begin{aligned}
 ii) \quad \cos 2a &= 2\cos^2 a - 1 \\
 &= 2 \times \left(\frac{2}{\sqrt{5}}\right)^2 - 1 \\
 &= 2 \times \frac{4}{5} - 1 \\
 &= \frac{8}{5} - \frac{5}{5} \\
 &= \frac{3}{5}
 \end{aligned}$$

b)



$$\begin{aligned}
 \sin(2a+b) &= \sin 2a \cos b + \cos 2a \sin b \\
 &= 2\sin a \cos a \cos b + \cos 2a \sin b \\
 &= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{3}{5} \times \frac{1}{\sqrt{10}} \\
 &= \frac{12}{5\sqrt{10}} + \frac{3}{5\sqrt{10}} \\
 &= \frac{15}{5\sqrt{10}} \\
 &= \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\
 &= \frac{3\sqrt{10}}{10}
 \end{aligned}$$