Higher Maths

Unit 1 Expressions and Functions

1.2. Applying trigonometric skills to manipulating expressions

1 a) $\sqrt{3}\cos x° + \sin x° = k\sin(x+\alpha)°$

$= k(\sin x° \cos\alpha° + \cos x° \sin\alpha°)$

$= (k\cos\alpha°)\sin x° + (k\sin\alpha°)\cos x°$

$k\cos\alpha° = 1$

$k\sin\alpha° = \sqrt{3}$ $\quad k = \sqrt{1+3}$ $\quad \tan\alpha° = \frac{\sqrt{3}}{1}$

$= 2$

$\alpha° = 60°$

$\sqrt{3}\cos x° + \sin x° = 2\sin(x+60)°$

b) minimum value of a fraction when denominater maximised.

$\sqrt{3}\cos x° + \sin x° + 4$

$= 2\sin(x+60)° + 4$

so max value $= 2+4$

$= 6$

when $\sin(x+60)° = 1$

$(x+60)° = 90°$

$x° = 30°$

2. $h = 3\sin 15t^\circ + 4\cos 15t^\circ + 25$

a) $3\sin 15t^\circ + 4\cos 15t^\circ = k\cos(15t - \alpha)^\circ$

$= k(\cos 15t^\circ \cos\alpha^\circ + \sin 15t^\circ \sin\alpha^\circ)$

$= (k\cos\alpha^\circ)\cos 15t^\circ + (k\sin\alpha^\circ)\sin 15t$

$k\cos\alpha^\circ = 4$

$k\sin\alpha^\circ = 3$ $\quad k = \sqrt{16+9}$ $\quad \tan\alpha^\circ = \frac{3}{4}$

$= 5$ $\quad \alpha^\circ = 36.9^\circ$

$3\sin 15t^\circ + 4\cos 15t^\circ + 25$

$= 5\cos(15t - 36.9)^\circ + 25$

b) minimum $= -5 + 25$

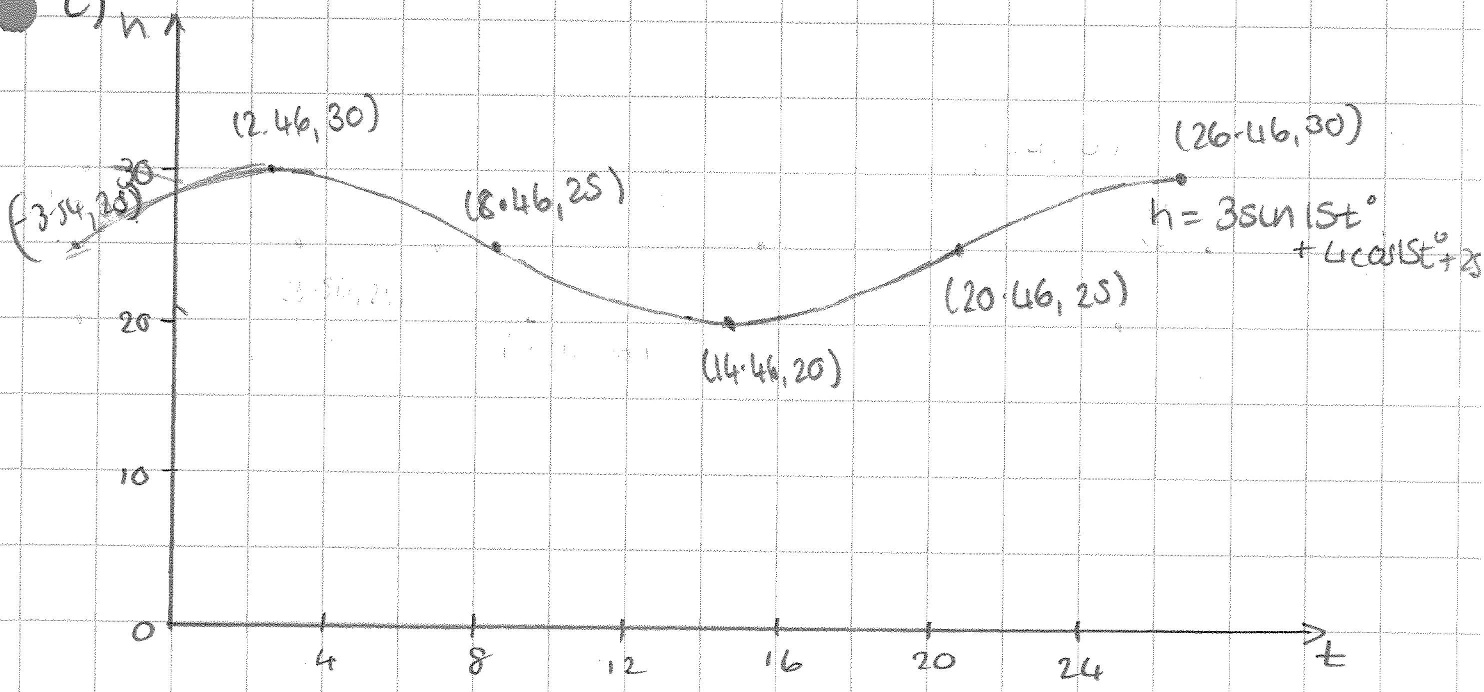
$= 20\text{ m}$

when $(15t - 36.9)^\circ = 180^\circ$

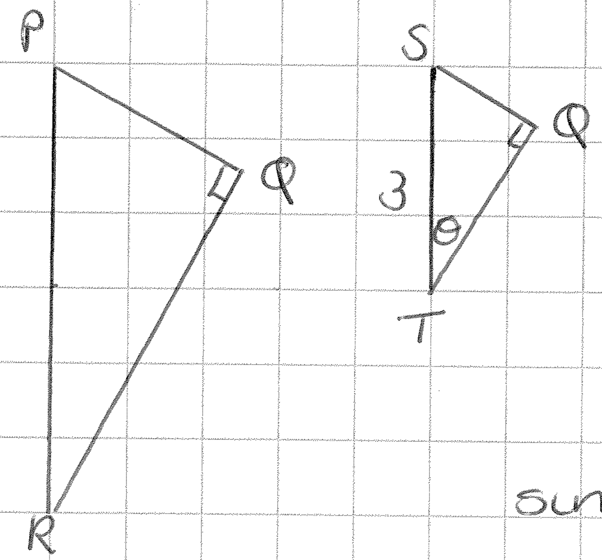
$15t^\circ = 216.9^\circ$

$t^\circ = 14.46$ ie. around 14:30.

c)



3. a)



$\cos\theta = \frac{TQ}{3}$

$TQ = 3\cos\theta$

So $RQ = 2 + 3\cos\theta$

$\sin\theta = \frac{SQ}{3}$

$SQ = 3\sin\theta$

So $PQ = 1 + 3\sin\theta$

b) $PR^2 = PQ^2 + RQ^2$

$= (1 + 3\sin\theta)^2 + (2 + 3\cos\theta)^2$

$= 1 + 6\sin\theta + 9\sin^2\theta + 4 + 12\cos\theta + 9\cos^2\theta$

$= 12\cos\theta + 6\sin\theta + 5 + 9(\sin^2\theta + \cos^2\theta)$

$= 12\cos\theta + 6\sin\theta + 14$

$a = 12 \quad b = 6 \quad c = 14$

c) $12\cos\theta + 6\sin\theta = k\cos(x - \alpha)$

$= k(\cos x\cos\alpha + \sin x\sin\alpha)$

$= (k\cos\alpha)\cos x + (k\sin\alpha)\sin x$

$k\cos\alpha = 12$

$k\sin\alpha = 6$

$k = \sqrt{144 + 36}$

$= \sqrt{180}$

$= 6\sqrt{5}$

$\tan\alpha = \frac{6}{12}$

$\alpha = 0.46$

$12\cos\theta + 6\sin\theta + 14 = 6\sqrt{5}\cos(x - 0.46) + 14$

max of $PR^2 = 6\sqrt{5} + 14$

max of $PR = 5.2$ (to 1dp).

4. a) $4\sin(x+30)^\circ - \cos x^\circ$

$= 4(\sin x^\circ \cos 30^\circ + \cos x^\circ \sin 30^\circ) - \cos x^\circ$

$= 4\sin x^\circ \times \frac{\sqrt{3}}{2} + 4\cos x^\circ \times \frac{1}{2} - \cos x^\circ$

$= 2\sqrt{3}\sin x^\circ + \cos x^\circ$

b) $2\sqrt{3}\sin x^\circ + \cos x^\circ = k\cos(x-\alpha)^\circ$

$= (k\cos\alpha^\circ)\cos x^\circ + (k\sin\alpha^\circ)\sin x^\circ$

$k\cos\alpha^\circ = 1$

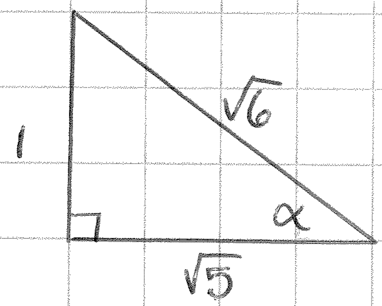
$k\sin\alpha^\circ = 2\sqrt{3}$ $\quad k = \sqrt{1+12}$ $\quad \tan\alpha^\circ = \frac{2\sqrt{3}}{1}$

$= \sqrt{13}$

$\alpha^\circ = 73.9^\circ$

$2\sqrt{3}\sin x^\circ + \cos x^\circ = \sqrt{13}\cos(x - 73.9)^\circ$

5.



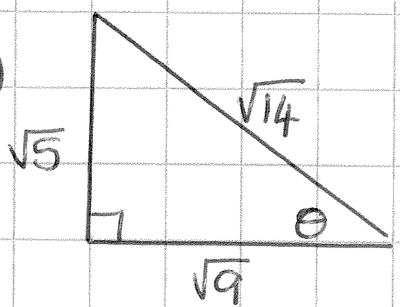
$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{1}{\sqrt{6}} \times \frac{\sqrt{5}}{\sqrt{6}}$

$= \frac{2\sqrt{5}}{6}$

$= \frac{\sqrt{5}}{3}$

6. a)



$\sin\theta = \frac{\sqrt{5}}{\sqrt{14}}$ $\quad \cos\theta = \frac{\sqrt{9}}{\sqrt{14}}$

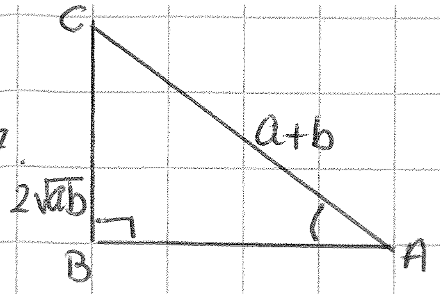
b) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{\sqrt{5}}{\sqrt{14}} \times \frac{\sqrt{9}}{\sqrt{14}}$

$= \frac{2\sqrt{45}}{14} = \frac{3\sqrt{5}}{7}$

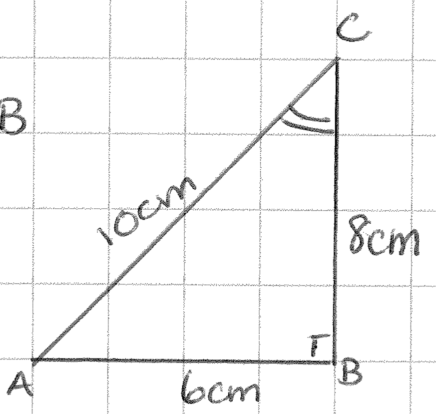
1-1.2-5

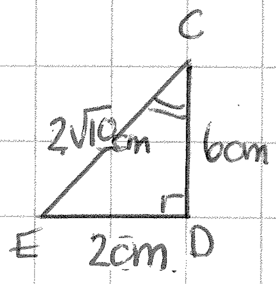
7.



$$AB^2 = (a+b)^2 - (2\sqrt{ab})^2$$
$$= a^2 + 2ab + b^2 - 4ab$$
$$= a^2 - 2ab + b^2$$
$$AB = a-b$$
$$P = (a+b) + (a-b) + 2\sqrt{ab}$$
$$= 2a + 2\sqrt{ab}$$
$$= 2(a + \sqrt{ab})$$

8. $ACE = ACB - ECB$





• using pyth.

$$\cos ACE = \cos(ACB - ECB)$$
$$= \cos ACB \cos ECB + \sin ACB \sin ECB$$
$$= \frac{8}{10} \times \frac{6}{2\sqrt{10}} + \frac{6}{10} \times \frac{2}{2\sqrt{10}}$$
$$= \frac{48 + 12}{20\sqrt{10}}$$
$$= \frac{60}{20\sqrt{10}}$$
$$= \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$
$$= \frac{3\sqrt{10}}{10}.$$

1.1.2-6

9. a) $12\cos x^\circ + 5\sin x^\circ = k\sin(x-\alpha)^\circ$

$= k(\sin x^\circ \cos\alpha^\circ - \cos x^\circ \sin\alpha^\circ)$

$= (k\cos\alpha^\circ)\sin x^\circ - (k\sin\alpha^\circ)\cos x^\circ$

$k\cos\alpha^\circ = 5$

$k\sin\alpha^\circ = -12$ $\quad k = \sqrt{144+25}$ $\quad \tan\alpha^\circ = -\frac{12}{5}$ $\quad$ S A ✓ T C ✓

$= 13$

$\alpha^\circ = 292.6^\circ$

$12\cos x^\circ + 5\sin x^\circ = 13\sin(x - 292.6)^\circ$

b) minimum of a fraction occurs when the denominator is maximised

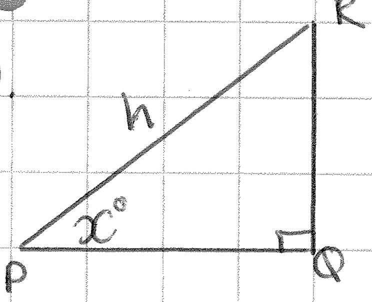
ie $h(x) = \dfrac{8}{12\cos x^\circ + 5\sin x^\circ + 3}$

$= \dfrac{8}{13\sin(x-292.6)^\circ + 3}$

max value of $13\sin(x-292.6)^\circ + 3$ is 16.

So min value of $h(x)$ is $\frac{8}{16} = \frac{1}{2}$

10.



a) $\sin x^\circ = \frac{RQ}{h}$ $\qquad \cos x^\circ = \frac{PQ}{h}$

So $RQ = h\sin x^\circ$ $\qquad$ So $PQ = h\cos x^\circ$

b) $P = h + h\sin x^\circ + h\cos x^\circ$

$19.24 = 8 + 8\sin x^\circ + 8\cos x^\circ$

$8\sin x^\circ + 8\cos x^\circ = 11.24$

$2\sin x^\circ + 2\cos x^\circ = 2.81$

c) $2\sin x^\circ + 2\cos x^\circ = k\cos(x-\alpha)^\circ$

$= k(\cos x^\circ \cos\alpha^\circ + \sin x^\circ \sin\alpha^\circ)$

$= (k\cos\alpha^\circ)\cos x^\circ + (k\sin\alpha^\circ)\sin x^\circ$

$k\cos\alpha^\circ = 2 \qquad k = \sqrt{4+4} \qquad \tan\alpha^\circ = \frac{2}{2}$

$k\sin\alpha^\circ = 2 \qquad\quad = 2\sqrt{2} \qquad\quad \alpha^\circ = 45^\circ$

$2\sin x^\circ + 2\cos x^\circ = 2\sqrt{2}\cos(x-45)^\circ$

11. a) $\cos 36t^\circ + \sqrt{3}\sin 36t^\circ = R\cos(36t-\alpha)^\circ$

$= (R\cos\alpha^\circ)\cos 36t^\circ + (R\sin\alpha^\circ)\sin 36t$

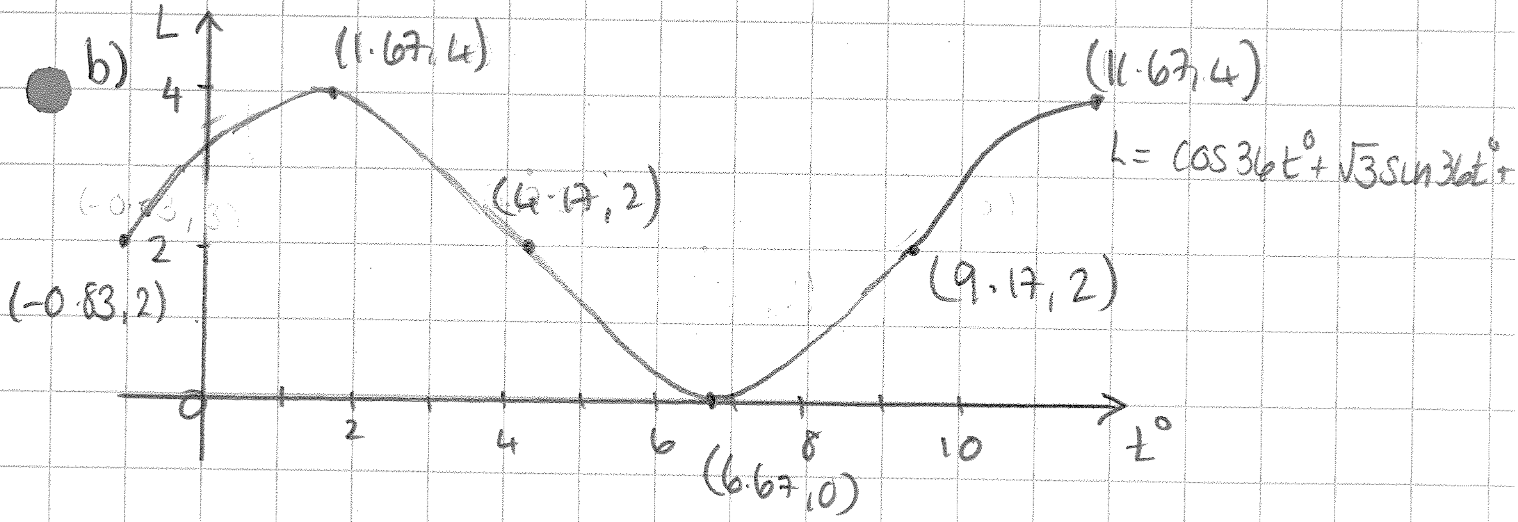
$R\cos\alpha^\circ = 1 \qquad R = \sqrt{1+3} \qquad \tan\alpha^\circ = \frac{\sqrt{3}}{1}$

$R\sin\alpha^\circ = \sqrt{3} \qquad\quad = 2 \qquad\quad \alpha^\circ = 60^\circ$

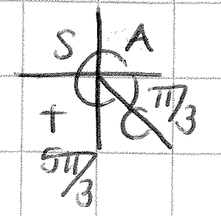
$\cos 36t^\circ + \sqrt{3}\sin 36t^\circ + 2$

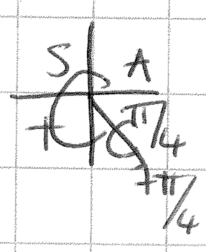
$= 2\cos(36t-60)^\circ + 2$

b)



12. $\cos\frac{5\pi}{3} - \tan\frac{7\pi}{4}$





$= \cos\frac{\pi}{3} - \left(-\tan\frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3} + \tan\frac{\pi}{4}$

$= \frac{1}{2} + 1$

$= 1\frac{1}{2}$

13. a) $2\sin(x+\pi/6) - 2\cos x = \sqrt{3}\sin x - \cos x$

$LHS = 2\sin(x+\pi/6) - 2\cos x$

$= 2(\sin x\cos\pi/6 + \cos x\sin\pi/6) - 2\cos x$

$= 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) - 2\cos x$

$= \sqrt{3}\sin x + \cos x - 2\cos x$

$= \sqrt{3}\sin x - \cos x$

$= RHS$

b) $\sqrt{3}\sin x - \cos x = k\sin(x-\alpha)$

$= k(\sin x\cos\alpha - \cos x\sin\alpha)$

$= (k\cos\alpha)\sin x - (k\sin\alpha)\cos x$

$k\cos\alpha = \sqrt{3}$

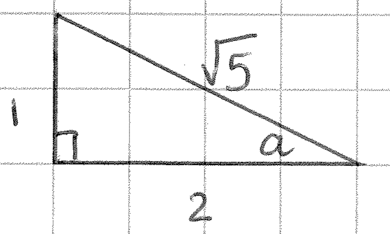
$k\sin\alpha = 1$ $\qquad k = \sqrt{1+3}$ $\qquad \tan\alpha = \frac{1}{\sqrt{3}}$

$= 2$

$\alpha = \pi/6$

$\sqrt{3}\sin x - \cos x = 2\sin(x-\pi/6)$

14.



a) i) $\cos a = \frac{2}{\sqrt{5}}$

ii) $\cos 2a = 2\cos^2 a - 1$

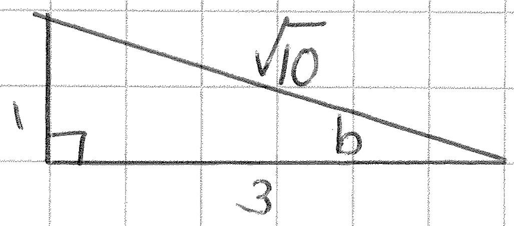
$= 2 \times \left(\frac{2}{\sqrt{5}}\right)^2 - 1$

$= 2 \times \frac{4}{5} - 1$

$= \frac{8}{5} - \frac{5}{5}$

$= \frac{3}{5}$

b)



$\sin(2a+b) = \sin 2a \cos b + \cos 2a \sin b$

$= 2\sin a \cos a \cos b + \cos 2a \sin b$

$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{3}{5} \times \frac{1}{\sqrt{10}}$

$= \frac{12}{5\sqrt{10}} + \frac{3}{5\sqrt{10}}$

$= \frac{15}{5\sqrt{10}}$

$= \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{3\sqrt{10}}{10}.$