Higher Maths

Unit 1 Expressions and Functions

1.1 Applying algebraic skills to logarithms and exponentials.

1. a) $7^x = 24$

$x = \log_7 24$

$= 1.63$ (to 2dp)

b) $3^{-2x} = \frac{1}{50}$

$-2x = \log_3 0.02$

$-2x = -3.5608...$

$x = 1.78$ (to 2dp)

2. a) $C = 4e^{-0.025t}$

$t = 0 \quad C = 4$ mL.

b) $3 = 4e^{-0.025t}$

$e^{-0.025t} = 0.75$

$-0.025t = \ln 0.75$

$t = \frac{\ln 0.75}{-0.025}$

$= 11.5072829$

$=$ 11 minutes 30 seconds (nearest second)

c) $2 = 4e^{-0.025t}$

$e^{-0.025t} = 0.5$

$-0.025t = \ln 0.5$

$t = \frac{\ln 0.5}{-0.025}$

$= 27.72588722$

$=$ 27 minutes 44 seconds (nearest second)

3. $2\log_2 \frac{1}{x} + 3\log_2 x = 4$

$\log_2 \frac{1}{x^2} + \log_2 x^3 = 4\log_2 2$

$\log_2 x = \log_2 2^4$

$x = 2^4$

$= 16$

4. $M = M_0 e^{-kt}$ $\quad t = 3 \quad M = 0.8M_0$

a) $0.8M_0 = M_0 e^{-3k}$

$e^{-3k} = 0.8$

$-3k = \ln 0.8$

$k = \frac{\ln 0.8}{-3}$

$= 0.074381118377$

$= 0.074$ (to 3dp)

b) $M = M_0 e^{-0.074t}$ $\quad M = 0.5M_0$

$0.5M_0 = M_0 e^{-0.074t}$

$e^{-0.074t} = 0.5$

$-0.074t = \ln 0.5$

$t = \frac{\ln 0.5}{-0.074}$

$= 9.366853791$

$= 9$hrs 22 mins (nearest minute)

5. a) $\log_{10} Q = m \log_{10} P + c$

$\log_{10} Q = \log_{10} P^m + c \log_{10} 10$

$\log_{10} Q = \log_{10} P^m + \log_{10} 10^c$

$\log_{10} Q = \log_{10} (P^m \times 10^c)$

$Q = 10^c \times P^m$

or. $Q = k P^n$ where $k = 10^c$, $m = n$.

b) $n = m = \dfrac{2.60 - 1.32}{0.44 - 0.08}$ (0.08, 1.32) (0.44, 2.60)

$= \dfrac{1.28}{0.36}$

$= 3.5$

$\log_{10} Q = 3.5 \log_{10} P + c$ (0.08, 1.32)

$1.32 = 3.5 \times 0.08 + c$

$1.32 = 0.28 + c$

$c = 1.04 \Rightarrow k = 10^{1.04}$

$= 10.96478196$

$= 11.0$ (to 1dp)

6. a) $2 \log_x y = \log_x 2y + 2$

$\log_x y^2 = \log_x 2y + 2 \log_x x$

$\log_x y^2 = \log_x 2y + \log_x x^2$

$\log_x y^2 = \log_x 2x^2 y$

$y^2 = 2x^2 y$

$y = 2x^2$

b) $y = 2x^2$ $\quad x = \frac{1}{4}y$

$y = 2\left(\frac{1}{4}y\right)^2$

$y = 2 \times \frac{1}{16}y^2$

$y = \frac{1}{8}y^2$

$\frac{1}{8}y^2 - y = 0$

$y\left(\frac{1}{8}y - 1\right) = 0$

$y = 0$ or $y = 8$

$y > 0$ so $y = 8$

7. $\log_2(x+1) - \log_2 x = \log_2 8$

$\log_2\left(\frac{x+1}{x}\right) = \log_2 8$

$\frac{x+1}{x} = 8$

$x + 1 = 8x$

$1 = 7x$

$x = 1/7$

8. $8^{x+1} = 4^{2x-3}$

$\log_2(8^{x+1}) = \log_2(4^{2x-3})$

$(x+1)\log_2 8 = (2x-3)\log_2 4$

$(x+1)\log_2 2^3 = (2x-3)\log_2 2^2$

$3(x+1)\log_2 2 = 2(2x-3)\log_2 2$

$3(x+1) = 2(2x-3)$

$3x + 3 = 4x - 6$

$3 = x - 6$

$x = 9$

9. $M_t = M_0 e^{-0.04t}$ $\quad M_t = 0.2M_0$

$0.2M_0 = M_0 e^{-0.04t}$

$e^{-0.04t} = 0.2$

$-0.04t = \ln 0.2$

$t = \dfrac{\ln 0.2}{-0.04}$

$= 40.23594781$

$= 40$ seconds (nearest second)

10. $M_t = M_0 8^{-0.3t}$ $\quad M_t = 0.5M_0$

$0.5M_0 = M_0 8^{-0.3t}$

$0.5 = 8^{-0.3t}$

$-0.3t = \log_8 0.5$

$-0.3t = -\frac{1}{3}$

$t = \frac{10}{9}$

$=$ 1 year 41 days (nearest day)

11. $\log_2 y = \frac{1}{2}\log_2 x + 4$

$\log_2 y = \log_2 x^{1/2} + 4\log_2 2$

$\log_2 y = \log_2 x^{1/2} + \log_2 2^4$

$\log_2 y = \log_2 (2^4 x^{1/2})$

$y = 16x^{1/2}$

$y = 16\sqrt{x}$

12. $\log_2 \frac{1}{8} = p$

$\log_2 (2^{-3}) = p$

$-3 \log_2 2 = p$

$p = -3$

13. $\log_4 y = 2 - \log_4 5x$

$\log_4 y = 2\log_4 4 - \log_4 5x$

$\log_4 y = \log_4 4^2 - \log_4 5x$

$\log_4 y = \log_4 \left(\frac{4^2}{5x}\right)$

$y = \frac{4^2}{5x}$

$y = \frac{16}{5x}$

14. $\log_9 (x+2) = \frac{1}{2} + \log_9 (x-5) \qquad x > 5$

$\log_9 (x+2) = \frac{1}{2}\log_9 9 + \log_9 (x-5)$

$\log_9 (x+2) = \log_9 9^{1/2} + \log_9 (x-5)$

$\log_9 (x+2) = \log_9 9^{1/2}(x-5)$

$x+2 = 9^{1/2}(x-5)$

$x+2 = 3(x-5)$

$x+2 = 3x-15$

$2 = 2x-15$

$17 = 2x$

$x = 8.5$

19. $m_t = m_0 e^{kt}$ $\quad m_t = 0.5m_0 \quad t = 5$

a) $0.5m_0 = m_0 e^{5k}$

$e^{5k} = 0.5$

$5k = \ln 0.5$

$k = \frac{\ln 0.5}{5}$

$= -0.139$ (3sf)

b) $m_t = m_0 e^{-0.139t}$ $\quad t = 2 \quad m_0 = 1$ unit

$m_t = 1 \times e^{-0.139 \times 2}$

$= 0.757296\,8215$

$= 75.7\%$ left

ie. 24.3% decayed