Polynomials and Quadratics

Paper 1 Section A

Each correct answer in this section is worth two marks.

1. A parabola has equation $y = -2x^2 + 4x + 5$.

   Which of the following are true?

   I. The parabola has a minimum turning point.

   II. The parabola has no real roots.

   A. Neither I nor II is true

   B. Only I is true

   C. Only II is true

   D. Both I and II are true

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.1	C	0.37	0.4	CN	A15, A17	HSN 070

Since the coefficient of x^2 is $-2 < 0$, the parabola has a maximum turning point.

The discriminant is $b^2 - 4ac$ where $a = -2$, $b = 4$, $c = 5$.

$= 4^2 - 4 \times (-2) \times 5$

$= 16 + 40$

> 0.

Since the discriminant is positive, the parabola has two distinct real roots.

Option A

2. Given $p(x) = x^2 + x - 6$, which of the following are true?

   I. $(x+3)$ is a factor of $p(x)$.

   II. $x = 2$ is a root of $p(x) = 0$.

   A. Neither I nor II is true

   B. Only I is true

   C. Only II is true

   D. Both I and II are true

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.1	C	0.78	0.67	NC	A21	HSN 170

$(x+3)$ is a factor $\Leftrightarrow p(-3) = 0$.

$p(-3) = (-3)^2 - 3 - 6 = 9 - 9 = 0$

So $(x+3)$ is a factor of $p(x)$.

$p(2) = 2^2 + 2 - 6 = 4 + 2 - 6 = 0$.

So $x = 2$ is a root of $p(x) = 0$. Option **D**

3. When $2ax^3 + (a+1)x - 6$ is divided by $x + 2$, the remainder is 2.

   What is the value of a?

   A. $\frac{5}{3}$

   B. $-\frac{4}{9}$

   C. $-\frac{5}{9}$

   D. $-\frac{5}{7}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	C	0.41	0.77	NC	A21	HSN 174

Let $f(x) = 2ax^3 + (a+1)x - 6$.

If the remainder is 2 then $f(-2) = 2$, ie

$$2a \times (-2)^3 + (a+1) \times (-2) - 6 = 2$$
$$-16a - 2a - 2 - 6 = 2$$
$$-18a = 10$$
$$a = -\frac{5}{9}$$

Option C

[END OF PAPER 1 SECTION A]

Paper 1 Section B

[SQA] 4. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(*b*)	4	C	NC	A3	sketch	
(*c*)	1	C	NC	A16, A6	$-1 < x < 5$	

•[1] pd: process, e.g. completing the square	•[1] $a = 2$
•[2] pd: process, e.g. completing the square	•[2] $b = 1$
•[3] ic: interpret minimum	•[3] *any two from:* parabola; min. t.p. $(2, 1)$; $(0, 5)$
•[4] ic: interpret y-intercept	•[4] the remaining one from above list
•[5] ss: reflect in x-axis	•[5] reflecting in x-axis
•[6] ss: translate parallel to y-axis	•[6] translating $+10$ units, parallel to y-axis
•[7] ic: interpret graph	•[7] $(-1, 5)$ i.e. $-1 < x < 5$

[SQA] 5. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
.	4	2.1	2	2					1.3.11	2.1.9	Source 1996 P1 qu.16

•[1] know to consider $f'(x) > 0$ — stated or implied by the evidence for •[4].

•[2] $\frac{dy}{dx} = 6x^2 - 6x - 36$

•[3] $6(x-3)(x+2) > 0$ — or by formula or completing the square

•[4] $x < -2,\ x > 3$

[SQA] 6. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers. 5

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1
.	5	2.1	1	4					2.1.6	Source 1991 P1 qu.18

- •1 for realising "$b^2 - 4ac$" ≥ 0
- •2 $kx^2 + 3x + (3-k) = 0$
- •3 $\Delta = 3^2 - 4k(3-k)$
- •4 $\Delta = (2k-3)^2$
- •5 for stating $(2k-3)^2$ is ≥ 0 for all real k

[SQA] 7. (*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. (2)

(ii) Find $f(g(x))$. (2)

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. (2)

(ii) Determine the nature of the roots of this equation when $k = 6$. (2)

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. (3)

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1
(*a*)	4	1.2	4						1.2.6	Source 1996 Paper 2 Qu.4
(*b*)	7	2.1	7						2.1.6, 2.1.7, 0.1	

(*a*)
- •1 $g(2x+1)$
- •2 $(2x+1)^2 + k$
- •3 $f(x^2+k)$
- •4 $2(x^2+k)+1$

(*b*)
- •5 $4x^2 + 4x + k + 1$ *AND* $2x^2 + 2k + 1$
- •6 $4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0$

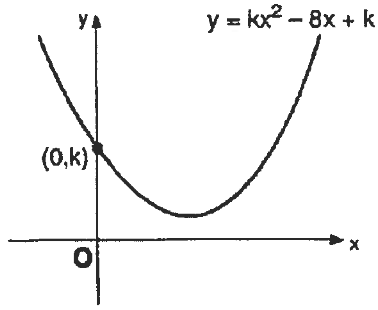
  so $2x^2 + 4x - k = 0$
- •7 $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$
- •8 so roots real & distinct
- •9 $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$
- •10 $b^2 - 4ac = 0$ for equal roots
- •11 $k = -2$

[SQA] 8. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A18	$k = \frac{1}{4}$	2001 P1 Q2

•[1] ss: know to set disc. to zero •[2] ic: substitute a, b and c into discriminant •[3] pd: process equation in k	•[1] $b^2 - 4ac = 0$ *stated or implied by* •[2] •[2] $(-5)^2 - 4 \times (k+6)$ •[3] $k = \frac{1}{4}$

[SQA] 9. **Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis.** 4



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1 Source
.	4	2.1	1	3					2.1.7	1992 P1 qu.17

•[1] *strat:* use discriminant
•[2] $b^2 - 4ac < 0$
•[3] $64 - 4k^2$
•[4] $k = 5$

[SQA] 10. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1 Source
.	2	2.1	2						2.1.7	1993 P1 qu.3

•[1] discriminant $= 16 - 4 \times 2 \times k$
•[2] $16 - 8k \geq 0$ for real roots $\Rightarrow k \leq 2$

[SQA] 11. For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots? **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1 Source
.	2	2.1	2						2.1.7	1996 P1 qu.2

•[1] $b^2 - 4ac = 0$

•[2] $a = 2\frac{1}{2}$

[SQA] 12. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1 Source
.	4	2.1	4						2.1.3	1989 P1 qu.2

•[1] *strat*: make 2 trial divisions or 2 trial evaluations

•[2] first linear factor

•[3] quadratic factor

•[4] other linear factors
$(x-1)(2x+1)(x+3)$

[SQA] 13. (*a*) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$. **1**

(*b*) Hence find the other roots. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.1 Source
(a)	1	2.1	1						2.1.1	1999 P1 qu.1
(b)	3	2.1	3						2.1.2	

•[1] $f(2) = 16 + 4 - 26 + 6 = 0$
or
the appearance of a '0' at the end of the 3rd line in the table below

•[2]

2	2	1	−13	6
		4	10	−6
	2	5	−3	0

•[3] $2x^2 + 5x - 3$

•[4] $-3, \frac{1}{2}$

[SQA] 14. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.

Find the value of p and the other roots. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
.	4	2.1	4						2.1.2		Source 1993 P1 qu.7

- •[1] $f(-3) = -54 - 27 - 3p + 30$ or synth. division e.g.
- •[2] $p = -17$
- •[3] $2x^2 - 9x + 10$
- •[4] $2, \frac{5}{2}$

$$\begin{array}{r|rrrr} -3 & 2 & -3 & p & 30 \\ & & -6 & 27 & -3p-81 \\ \hline & 2 & -9 & p+27 & -3p-51 \end{array}$$

and $-3p - 51 = 0$

[SQA] 15. (a) Show that $(x-3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**

(b) Hence express $f(x)$ in its fully factorised form. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	2	2.1	2						2.1.3		Source
(b)	2	2.1	2						2.1.3		1995 P1 qu.2

- •[1] $f(3) = 2 \times 3^3 + 3 \times 3^2 - 23 \times 3 - 12$ | or equivalent division
- •[2] $= 0$
- •[3] $2x^2 + 9x + 4$
- •[4] $(x-3)(2x+1)(x+4)$

[SQA] 16. Express $x^4 - x$ in its fully factorised form. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
.	4	2.1	4						2.1.3		Source 1996 P1 qu.7

- •[1] $x(x^3 - 1)$
- •[2] synthetic division or eval. $f(k)$
- •[3] linear factor $= (x-1)$
- •[4] $x(x-1)(x^2 + x + 1)$

OR

- •[1] synthetic division or eval. $f(k)$
- •[2] linear factor $= (x-1)$
- •[3] cubic factor $= (x^3 + x^2 + x)$
- •[4] $x(x-1)(x^2 + x + 1)$

[SQA] 17. (*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **2**

(*b*) Show algebraically that there are no other real roots. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	2	2.1	2						2.1.2		Source
(b)	3	2.1	3						2.1.7		1997 P1 qu.5

- •[1] looking for $f(x) = \ldots\ldots = 0$
- •[2] $x = 2$ explicitly stated
- •[3] $2x^2 + x + 4$
- •[4] $b^2 - 4ac = 1 - 4 \times 2 \times 4$
- •[5] $b^2 - 4ac < 0$ means no real roots

[SQA] 18. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
.	4	2.1	4						2.1.3		Source 1998 P1 qu.2

- •[1] evaluating $f(k)$ for any integer by any method
- •[2] find 1 value of k s.t. $f(k) = 0$ *e.g.* $f(1)$ *or* $f(-2)$ *or* $f(5)$
- •[3] quad factor *e.g.* $x^2 - 3x - 10$
- •[4] $(x-1)(x+2)(x-5)$

[SQA] 19.

(a) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.
The function g is defined by $g(x) = x - 1$.
Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$. **4**

(b) Factorise fully $f(g(x))$. **3**

(c) The function k is such that $k(x) = \dfrac{1}{f(g(x))}$.
For what values of x is the function k not defined? **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	4	1.2	4						1.2.6		Source
(b)	3	2.1	3						2.1.3		1990 Paper 2
(c)	2	1.2	2						1.2.1		Qu. 6

(a)
- •1 $f(g(x)) = f(x-1)$
- •2 $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$
- •3 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$
- •4 $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

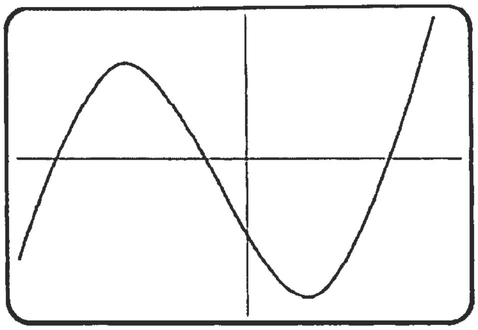
(b)
- •5 *first "0" e.g.*

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

- •6 $x^2 - 3x - 4 = (x+1)(x-4)$
- •7 $(x-2)(x+1)(x-4)$

(c)
- •8 denominator $(= (x-2)(x+1)(x-4)) \neq 0$
- •9 $-1,\ 2,\ 4$

[SQA] 20. The diagram shows part of the graph of the curve with equation
$f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.1
(*a*)	3	2.1	3						2.1.3		Source
(*b*)	2	1.2	2						1.2.9		1992 Paper 2
(*c*)	6	1.3	6						1.3.12		Qu.1

(*a*) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(*b*) •4 For all 3 points on x-axis

•5 $(0,-16)$

(*c*) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, and $x = -\frac{8}{3}$

•9 $y = -36$, and $y = \frac{400}{27}(14.8)$

•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	⋯	⋱	⋱	⋯	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2,-36)$

[SQA] 21. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(*a*) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(*b*) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	3	2.1	3						2.1.3		Source
(*b*)	4	2.1	4						2.1.3		1994 Paper 2 Qu.1

(*a*) •¹ strategy

eg

2	2	1	−13	a
		4	10	−6
	2	5	−3	0

or $f(2) = 0 = 16 + 4 - 26 + a$

•² $a = 6$

•³ $(0, 6)$

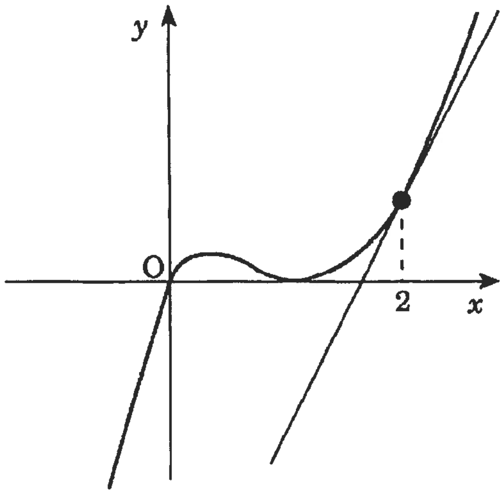
(*b*) •⁴ $2x^2 + 5x - 3$

•⁵ $(x+3)(2x-1)$

•⁶ $x = -3,\ \frac{1}{2}$

•⁷ $(-3, 0),\ \left(\frac{1}{2}, 0\right)$

[SQA] 22. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

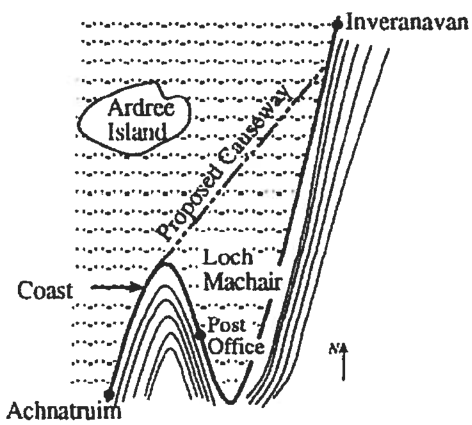
(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(a)	4	1.3	4						1.3.9,	1.1.7	Source
(b)	5	2.1	5						2.1.2,	2.1.8	1995 Paper 2 Qu.2

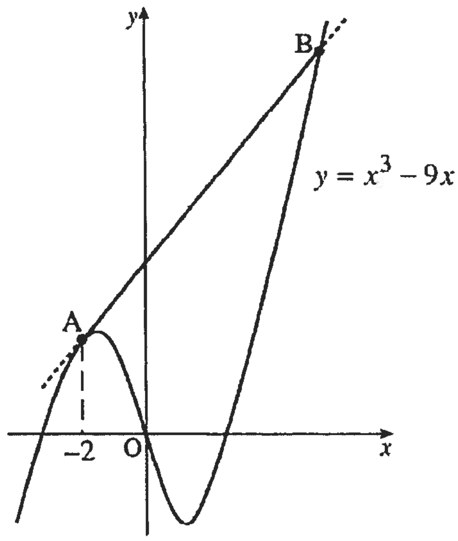
(a)
- •[1] $\frac{dy}{dx} = \ldots\ldots\ldots$
- •[2] $3x^2 - 4x + 1$
- •[3] $m_{x=2} = 5$
- •[4] $y - 2 = 5(x - 2)$

(b)
- •[5] equate 'y's
- •[6] $x^3 - 2x^2 - 4x + 8 = 0$
- •[7] e.g. synthetic division
- •[8] the appearance of:
  $x^2 - 4$
  or $x^2 - 4x + 4$
  or ± 2
  or $-2, 2, 2$
- •[9] $x = -2,\ y = -18$

[SQA] 23. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)i	1	0.1	1						0.1		Source
(*a*)ii	4	1.1	4						1.1.6, 4		**1998 Paper 2**
(*b*)	7	2.1	2	5					2.1.12 & 2.1.2		**Qu. 5**

(*a*)

•1 $y_{x=-2} = 10$

•2 $\frac{dy}{dx} = \ldots\ldots$

•3 $3x^2 - 9$

•4 $m_{x=-2} = 3$

•5 $y - 10 = 3(x + 2)$

(*b*)

•6 $y = 3x + 16$

•7 $3x + 16 = x^3 - 9x$

•8 $x^3 - 12x - 16 = 0$

•9 e.g.

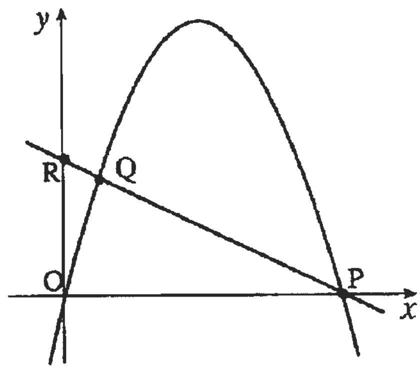
−2	1	0	−12	−16
		−2	4	16
	1	−2	−8	0

•10 e.g. $x^2 - 2x - 8$

•11 e.g. $(x + 2)(x - 4)$

•12 B is (4, 28)

[SQA] 24. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.



(*a*) Find the coordinates of the point P. 2

(*b*) R is the point (0, 2). Find the equation of PR. 2

(c) The line and the parabola also intersect at Q. Find the coordinates of Q. 4

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	2	1.2	2						1.2.9		Source
(*b*)	2	1.1	2						1.1.7		1999 Paper 2
(c)	4	2.1	4						2.1.8		Qu. 4

(*a*) •1 $4x - x^2 = 0$ *stated or implied by* •2

•2 $(4,0)$

(*b*) •3 $m = -\frac{1}{2}$

•4 $y = -\frac{1}{2}x + 2$

or $y - 2 = -\frac{1}{2}(x - 0)$

or $y - 0 = -\frac{1}{2}(x - 4)$

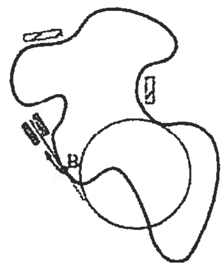
(c) •5 $4x - x^2 = 2 - \frac{1}{2}x$

•6 *e.g.* $2x^2 - 9x + 4 = 0$

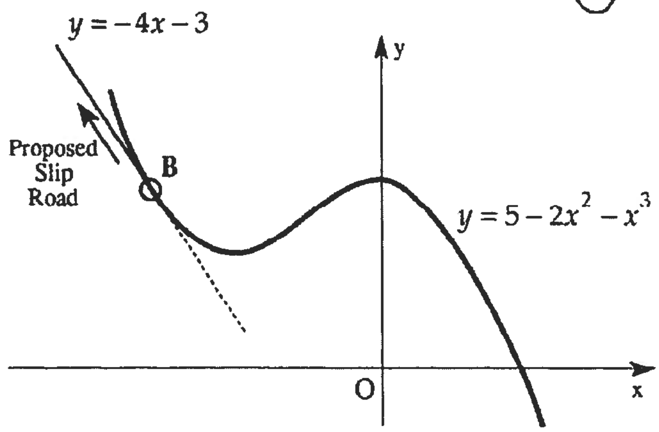
•7 $x = \frac{1}{2}, x = 4$

•8 Q is $\left(\frac{1}{2}, \frac{7}{4}\right)$

[SQA] 25. The diagram shows the plans for a proposed new racing circuit. The designer wishes to introduce a slip road at B for cars wishing to exit from the circuit to go into the pits. The designer needs to ensure that the two sections of road touch at B in order that drivers may drive straight on when they leave the circuit.



Relative to appropriate axes, the part of the circuit circled above is shown below. This part of the circuit is represented by a curve with equation $y = 5 - 2x^2 - x^3$ and the proposed slip road is represented by a straight line with equation $y = -4x - 3$.



(*a*) Calculate the coordinates of B. **(7)**

(*b*) Justify the designer's decision that this direction for the slip road does allow drivers to go straight on. **(1)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	7	2.1	7						2.1.8,	2.1.3	Source
(*b*)	1	2.1		1					2.1.8		1993 Paper 2 Qu.7

(*a*)
- •1 equating expressions for y
- •2 re-arranging cubic...... "..." = 0
- •3 strategy for solving cubic
- •4 first linear factor
- •5 quadratic factor
- •6 $x = -2,\ 2$
- •7 intersection at $(-2, 5)$

(*b*)
- •8 double root $\Rightarrow$ tangency or $y'(-2) = -4$ = gradient of line

[SQA] 26. (*a*) (i) Make a sketch of the graph of $y = x^3$, where $-3 \leq x \leq 3$, $x \in \mathbf{R}$.

(ii) On the same diagram, draw the graph of $y = 6x + 1$. **(3)**

(*b*) State the number of roots which the equation $x^3 = 6x + 1$ has in the interval $-3 \leq x \leq 3$. **(1)**

(*c*) Calculate the value of the positive root, correct to 3 significant figures. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1
(*a*)	3	0.1	3						0.1		Source
(*b*)	1	0.1	1						0.1		1989 Paper 2
(*c*)	4	2.1	1	3					2.1.11		Qu. 3

(*a*)
- •1 suitable choice of scales
- •2 sketch of $y = x^3$ from $x = -3$ to $x = 3$
- •3 sketch of $y = 6x + 1$ from $x = -3$ to $x = 3$

(*b*)
- •4 3 roots

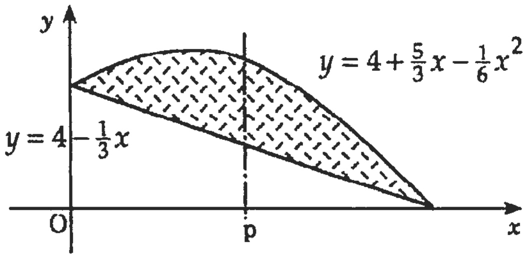
(*c*)
- •5 1st estimate: between 2 and 3
- •6 2nd estimate: between 2.5 and 2.6
- •7 3rd estimate: between 2.53 and 2.534
- •8 2.53

[SQA] 27. When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.

(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**



(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.

By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.

(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

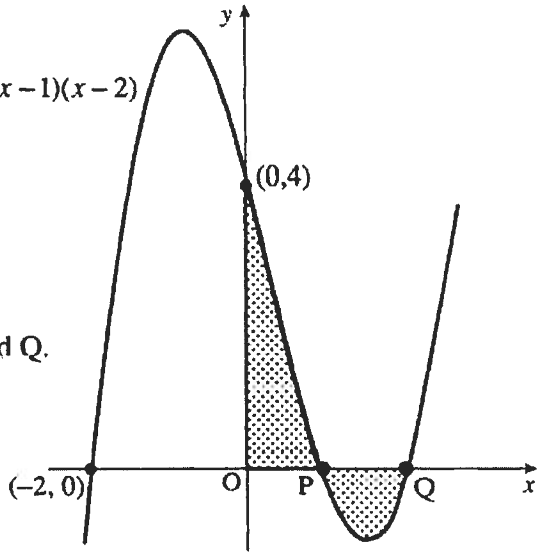
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	5	2.2	5						2.2.7		Source
(*b*)	3	2.2		3					2.2.5		1995 Paper 2
(*c*)	5	2.1	2	3					2.1.3		Qu.10

(*a*)
- •1 Area under curve - area under line
- •2 abscissae at intersection are 0 and 12
- •3 $\int_0^{12} \left(4 + \frac{5}{3}x - \frac{1}{6}x^2 - \left(4 - \frac{1}{3}x\right)\right) dx$
- •4 $\left[x^2 - \frac{1}{18}x^3\right]_0^{12}$ or equivalent
- •5 48

(*b*)
- •6 $\int \ldots\ldots\ldots \; dx = 24$
- •7 $\int_0^p \left(2x - \frac{1}{6}x^2\right) dx = 24$ or equivalent statement
- •8 $p^2 - \frac{1}{18}p^3 = 24$

(*c*)
- •9 "$f(6) = 0$" or equivalent
- •10 divide by $(p - 6)$
- •11 $p^2 - 12p - 72$
- •12 $p = 6 \pm \sqrt{108}$ or equivalent
- •13 outside range 0 – 12

[SQA] 28. **The diagram shows a sketch of the graph of $y = (x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.**



(*a*) **Write down the coordinates of P and Q.** **(2)**

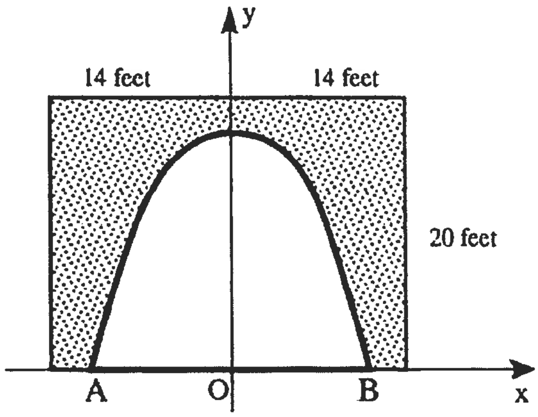
(*b*) **Find the total shaded area.** **(7)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	2.1	2						2.1.2		Source
(*b*)	7	2.2	6	1					2.2.6		**1997 Paper 2 Qu.4**

(*a*)
- •¹ (1, 0)
- •² (2, 0)

(*b*)
- •³ $\int f(x)dx$
- •⁴ $\int_0^1 - \int_1^2$
- •⁵ $(x+2)\left(x^2-3x+2\right)$ or equiv.
- •⁶ x^3-x^2-4x+4
- •⁷ $\frac{1}{4}x^4-\frac{1}{3}x^3-2x^2+4x$
- •⁸ $1\frac{11}{12}$ or $-\frac{7}{12}$
- •⁹ $2\frac{1}{2}$

[SQA] 29. The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.1 Source
(*a*)	2	0.1	2						0.1		1993 Paper 2 Qu.2
(*b*)	4	2.2	4						2.2.6		

(*a*) •1 $18 - \frac{1}{8}x^2 = 0$

•2 $x = \pm 12$

(*b*) •3 $Area = 2\int_0^{12} y \, dx$

•4 integrating

•5 288

•6 for knowing to subtract area of parabola from area of rectangle and multiply by 3.

[SQA] 30. A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

(*a*) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

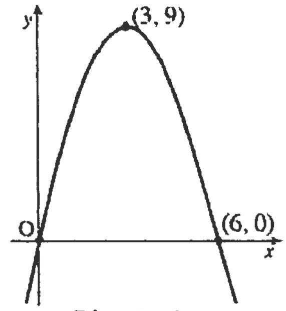


Diagram 1

(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

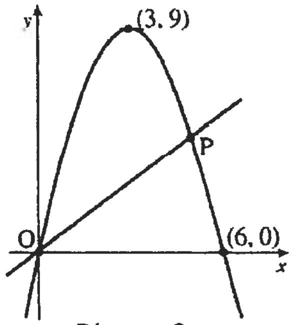


Diagram 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
(*a*)	2	1.2	2						1.2.7		Source
(*b*)	4	2.2	4						2.2.6		**1998 Paper 2**
(c)i	2	2.1	2						2.1.8		**Qu. 4**
(c)ii	3	2.2		3					2.2.7		

(*a*)
- •1 $a = 1$
- •2 $b = 6$

(*b*)
- •3 $\int_0^6 x(6 - x)\,dx$
- •4 $\int \left(6x - x^2\right)dx$
- •5 $3x^2 - \frac{1}{3}x^3$
- •6 36

(*c*)
- •7 $x = 6x - x^2$
- •8 $x_P = 5$
- •9 $\int_0^5 \left(6x - x^2 - x\right)\,dx$ or equiv.
- •10 $\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ or equiv.
- •11 $\frac{125}{6}$ or equiv.

[SQA] 31.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(a)	2	0.1	2						0.1		Source
(b)	3	2.2	3						2.2.7		1990 P1 qu.13

•[1] $2x^2 = 4x - 2x^2$ *or* $y = 4 - y$	•[3] $\int_{-1}^{1}\left(4 - 2x^2 - 2x^2\right)dx$
•[2] $x = 1$ *and* $x = -1$	•[4] $4x - \frac{2}{3}x^3 - \frac{2}{3}x^3$
	•[5] $5\frac{1}{3}$

[SQA] 32. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

•[1] ss: know to examine radius	•[1] $g = 2k, f = -k, c = -k - 2$ *stated or implied by* •[2]
•[2] pd: process	•[2] $r^2 = 5k^2 + k + 2$
•[3] pd: process	•[3] (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$)
•[4] ic: interpret quadratic inequation	•[4] use discr. **or** complete sq. **or** diff.
•[5] ic: interpret quadratic inequation	•[5] true for all k

[END OF PAPER 1 SECTION B]