National
Qualifications
SPECIMEN ONLY

S847/76/12

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour 45 minutes

Total marks — 80

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

* S 8 4 7 7 6 1 2 *

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

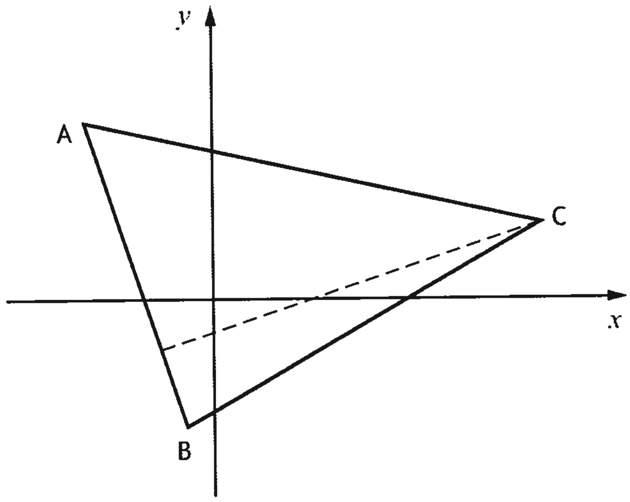
MARKS

Attempt ALL questions

Total marks — 80

1. The vertices of triangle ABC are A(−5, 7), B(−1, −5) and C(13, 3) as shown in the diagram.

   The broken line represents the altitude from C.



   (a) Find the equation of the altitude from C. 3

   (b) Find the equation of the median from B. 3

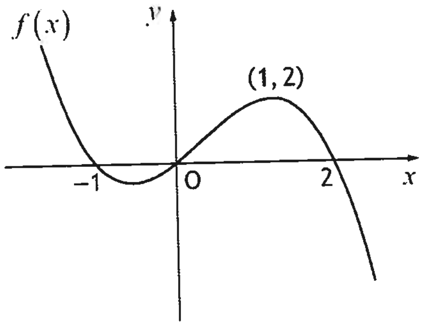
   (c) Find the coordinates of the point of intersection of the altitude from C and the median from B. 2

2. Find $\int \frac{4x^3+1}{x^2}\,dx,\ x \neq 0.$ 4

MARKS

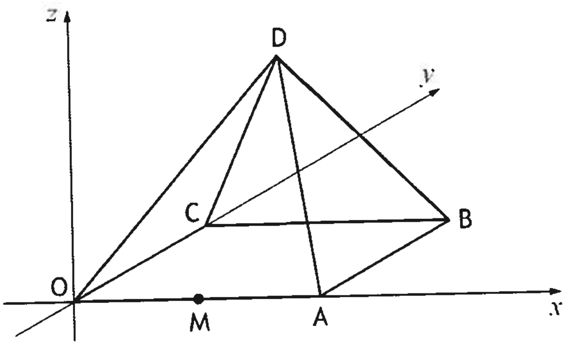
3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.



Find the values of a, b and k. 3

4. D,OABC is a square-based pyramid as shown.



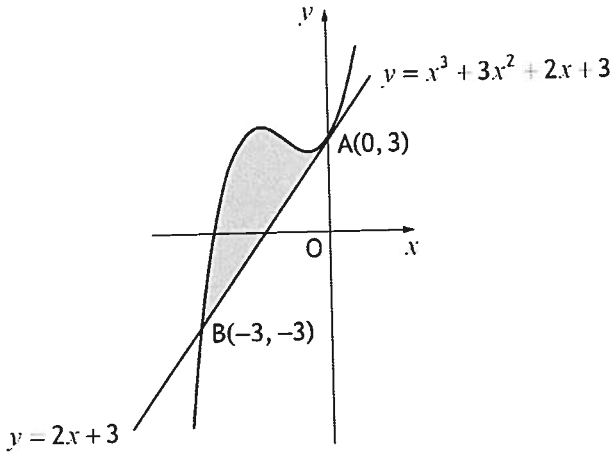
- O is the origin and OA = 4 units.
- M is the mid-point of OA.
- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. 3

(b) Find the size of angle BDM. 5

MARKS

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.



The line meets the curve again at B(−3, −3).

Find the area enclosed by the line and the curve. 5

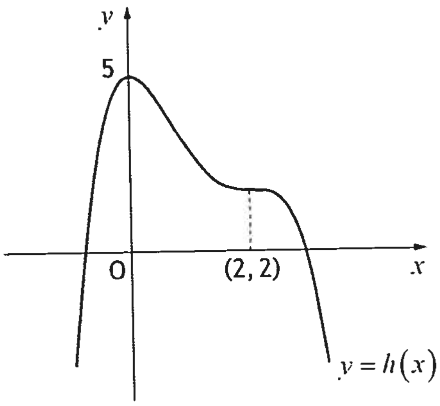
6. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. 3

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. 2

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. 2

MARKS

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $(0, 5)$ and $(2, 2)$.



On separate diagrams sketch the graphs of:

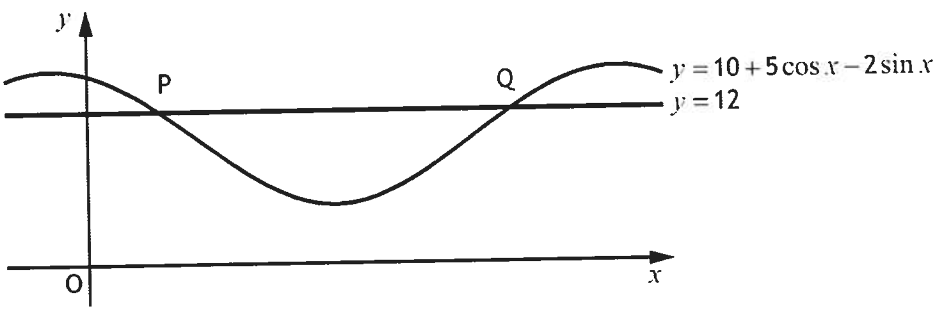
(a) $y = 2 - h(x)$. **2**

(b) $y = h'(x)$. **3**

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.

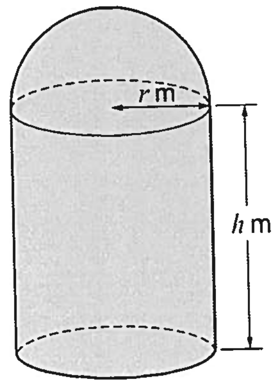


Find the x-coordinates of P and Q. **4**

MARKS

9. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ 3

(b) Determine the value of r which minimises the amount of metal needed to build the container. 6

10. Given that

$$\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \le a < \frac{\pi}{2},$$

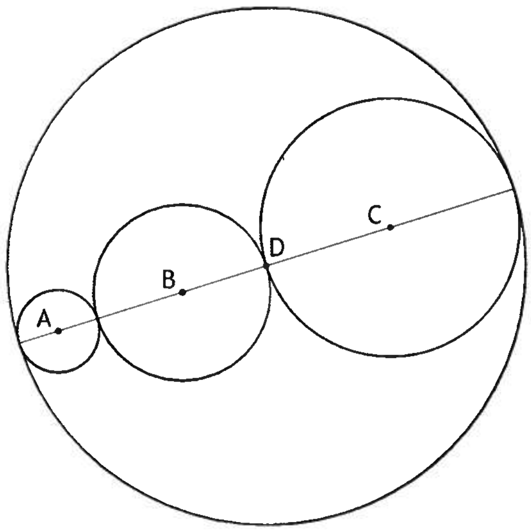
calculate the value of a. 6

MARKS

11. Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. 3

12. (a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear. 3

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. 4

MARKS

13. The concentration of a pesticide in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(a) It takes 25 days for the concentration of the pesticide to be reduced to one half of its initial concentration.

Calculate the value of k. 4

(b) Eighty days after the initial application, what is the percentage decrease in concentration of the pesticide? 3

[END OF SPECIMEN QUESTION PAPER]

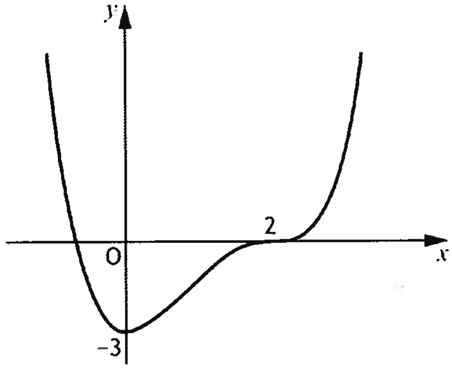
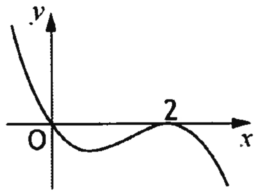
Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1. (a)	•1 calculate gradient of AB •2 use property of perpendicular lines •3 determine equation of altitude	•1 $m_{AB} = -3$ •2 $m_{alt} = \frac{1}{3}$ •3 $x - 3y = 4$	3
1. (b)	•4 calculate midpoint of AC •5 calculate gradient of median •6 determine equation of median	•4 (4,5) •5 $m_{BM} = 2$ •6 $y = 2x - 3$	3
1. (c)	•7 find x or y coordinate •8 find remaining coordinate	•7 $x = 1$ or $y = -1$ •8 $y = -1$ or $x = 1$	2
2.	•1 write in integrable form •2 integrate one term •3 integrate other term •4 complete integration and simplify	•1 $4x + x^{-2}$ •2 eg $\frac{4}{2}x^2 + \ldots$ •3 $\ldots \frac{x^{-1}}{-1}$ •4 $2x^2 - x^{-1} + c$	4
3.	•1 value of a •2 value of b •3 calculate k	•1 1 •2 −2 •3 −1	3

Question	Generic scheme	Illustrative scheme	Max mark
4. (a)	•¹ state components of $\overrightarrow{DB}$ •² state coordinates of M •³ state components of $\overrightarrow{DM}$	•¹ $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ •² $(2,0,0)$ stated or implied by •³ •³ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	3
4. (b)	•⁴ evaluate $\overrightarrow{DB}.\overrightarrow{DM}$ •⁵ evaluate $\lvert\overrightarrow{DB}\rvert$ •⁶ evaluate $\lvert\overrightarrow{DM}\rvert$ •⁷ use scalar product •⁸ calculate angle	•⁴ 32 •⁵ $\sqrt{44}$ •⁶ $\sqrt{40}$ •⁷ $\cos BDM = \frac{32}{\sqrt{44}\sqrt{40}}$ •⁸ $40 \cdot 3°$ or 0.703 rads	5

Question	Generic scheme	Illustrative scheme	Max mark
5.	•[1] know to integrate and interpret limits	•[1] $\int_{-3}^{0} \ldots dx$	5
	•[2] use 'upper − lower'	•[2] $\int_{-3}^{0} (x^3 + 3x^2 + 2x + 3) - (2x + 3)\, dx$	
	•[3] integrate	•[3] $\frac{1}{4}x^4 + x^3$	
	•[4] substitute limits	•[4] $0 - \left(\frac{1}{4}(-3)^4 + (-3)^3\right)$	
	•[5] evaluate area	•[5] $\frac{27}{4}$ units^2	

Question	Generic scheme	Illustrative scheme	Max mark
6. (a)	**Method 1**	**Method 1**	3
	•¹ identify common factor	•¹ $3(x^2+8x......$ stated or implied by •²	
	•² complete the square	•² $3(x+4)^2......$	
	•³ process for c and write in required form	•³ $3(x+4)^2+2$	
	Method 2	**Method 2**	3
	•¹ expand completed square form	•¹ $ax^2+2abx+ab^2+c$	
	•² equate coefficients	•² $a=3,\ 2ab=24,\ ab^2+c=50$	
	•³ process for b and c and write in required form	•³ $3(x+4)^2+2$	
6. (b)	•⁴ differentiate two terms	•⁴ $3x^2+24x....$	2
	•⁵ complete differentiation	•⁵ $....+50$	
6. (c)	**Method 1**	**Method 1**	2
	•⁶ link with (a) and identify sign of $(x+4)^2$	•⁶ $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \forall\ x$	
	•⁷ communicate reason	•⁷ $\therefore 3(x+4)^2+2>0 \Rightarrow$ always strictly increasing	
	Method 2	**Method 2**	2
	•⁶ identify minimum value of $f'(x)$	•⁶ eg minimum value $=2$ or annotated sketch	
	•⁷ communicate reason	•⁷ $2>0 \therefore (f'(x)>0) \Rightarrow$ always strictly increasing	

Question	Generic scheme	Illustrative scheme	Max mark
7. (a)	•[1] evidence of reflecting in x-axis •[2] vertical translation of 2 units identifiable from graph	•[1] reflection of graph in x-axis •[2] graph moves parallel to y-axis by 2 units upwards 	2
7. (b)	•[3] identify roots •[4] interpret point of inflexion •[5] complete cubic curve	•[3] 0 **and** 2 only •[4] turning point at $(2,0)$ •[5] cubic passing through origin with negative gradient 	3

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	•[1] use compound angle formula	•[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly**	4
	•[2] compare coefficients	•[2] $k\cos a = 5, k\sin a = 2$ **stated explicitly**	
	•[3] process for k	•[3] $k = \sqrt{29}$	
	•[4] process for a and express in required form	•[4] $\sqrt{29}\cos(x + 0 \cdot 38)$	
8. (b)	•[5] equate to 12 and simplify constant terms	•[5] $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$	4
	•[6] use result of part (a) and rearrange	•[6] $\cos(x + 0 \cdot 3805...) = \frac{2}{\sqrt{29}}$	
	•[7] solve for $x + a$	•[7] $1 \cdot 1902...,$ (•[7]) | $5 \cdot 0928...$ (•[8])	
	•[8] solve for x	•[8] $0 \cdot 8097...,$ | $4 \cdot 712...$	

Question	Generic scheme	Illustrative scheme	Max mark
9. (a)	•[1] equate volume to 100 •[2] obtain an expression for h •[3] demonstrate result	•[1] $V = \pi r^2 h = 100$ •[2] $h = \frac{100}{\pi r^2}$ •[3] $A = \pi r^2 + 2\pi r^2 + 2\pi r \times \frac{100}{\pi r^2}$ leading to $A = \frac{200}{r} + 3\pi r^2$	3
9. (b)	•[4] start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $A'(r) = 6\pi r \ldots$ •[5] $A'(r) = 6\pi r - \frac{200}{r^2}$ •[6] $6\pi r - \frac{200}{r^2} = 0$ •[7] $r = \sqrt[3]{\frac{100}{3\pi}}$ $(\approx 2\cdot 20)$ metres •[8] table of signs for a derivative when $r = 2\cdot 1974\ldots$ •[9] minimum when $r \approx 2\cdot 20$ (m) or •[8] $A''(r) = 6\pi + \frac{400}{r^3}$ •[9] $A''(2\cdot 1974\ldots) > 0$ $\therefore$ minimum when $r \approx 2\cdot 20$ (m)	6

Question	Generic scheme	Illustrative scheme	Max mark
10.	•[1] start to integrate •[2] complete integration •[3] process limits •[4] simplify numeric term and equate to $\frac{1}{2}$ •[5] start to solve equation •[6] solve for a	•[1] $-\frac{1}{4}\cos\ldots$ •[2] $-\frac{1}{4}\cos\left(4x-\frac{\pi}{2}\right)$ •[3] $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$ •[4] $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}=\frac{1}{2}$ •[5] $\cos\left(4a-\frac{\pi}{2}\right)=-1$ •[6] $a=\frac{3\pi}{8}$	6
11.	**Method 1** •[1] substitute for $\sin 2x$ •[2] simplify and factorise •[3] substitute for $1-\cos^2 x$ and simplify	**Method 1** •[1] $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •[2] $\sin x\left(1-\cos^2 x\right)$ •[3] $\sin x\times\sin^2 x$ leading to $\sin^3 x$	3
	Method 2 •[1] substitute for $\sin 2x$ •[2] simplify and substitute for $\cos^2 x$ •[3] expand and simplify	**Method 2** •[1] $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •[2] $\sin x-\sin x\left(1-\sin^2 x\right)$ •[3] $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	3

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	**Method 1** •1 calculate m_{AB} •2 calculate m_{BC} •3 interpret result and state conclusion	**Method 1** •1 eg $m_{AB}=\frac{3}{9}=\frac{1}{3}$ •2 eg $m_{BC}=\frac{5}{15}=\frac{1}{3}$ •3 ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 2 •1 calculate an appropriate vector, eg $\overrightarrow{AB}$ •2 calculate a second vector, eg $\overrightarrow{BC}$ and compare •3 interpret result and state conclusion	**Method 2** •1 eg $\overrightarrow{AB}=\begin{pmatrix}9\\3\end{pmatrix}$ •2 eg $\overrightarrow{BC}=\begin{pmatrix}15\\5\end{pmatrix}\therefore\ \overrightarrow{AB}=\frac{3}{5}\overrightarrow{BC}$ •3 ... $\Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	3
	Method 3 •1 calculate m_{AB} •2 find equation of line and substitute point •3 communication	**Method 3** •1 $m_{AB}=\frac{3}{9}=\frac{1}{3}$ •2 eg, $y-1=\frac{1}{3}(x-2)$ leading to $6-1=\frac{1}{3}(17-2)$ •3 since C lies on line A, B and C are collinear	3
12. (b)	•4 find radius •5 determine an appropriate ratio •6 find centre •7 state equation of circle	•4 $6\sqrt{10}$ •5 eg $2:3$ or $\frac{2}{5}$ (using B and C) or $3:5$ or $\frac{8}{5}$ (using A and C) •6 $(8,3)$ •7 $(x-8)^2+(y-3)^2=360$	4

Question	Generic scheme	Illustrative scheme	Max mark
13. (a)	•[1] interpret half-life •[2] process equation •[3] write in logarithmic form •[4] process for k	•[1] $\frac{1}{2}P_0 = P_0e^{-25k}$ **stated or implied by •[2]** •[2] $e^{-25k} = \frac{1}{2}$ •[3] $\log_e \frac{1}{2} = -25k$ •[4] $k \approx 0\cdot028$	4
13. (b)	•[5] interpret equation •[6] process •[7] state percentage decrease	•[5] $P_t = P_0e^{-80\times0\cdot028}$ •[6] $P_t \approx 0\cdot1065P_0$ •[7] 89%	3

[END OF SPECIMEN MARKING INSTRUCTIONS]