H

National Qualifications
SPECIMEN ONLY

S847/76/11

Mathematics
Paper 1 (Non-calculator)

Date — Not applicable

Duration — 1 hour 30 minutes

Total marks — 70

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

S847761 1

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Attempt ALL questions

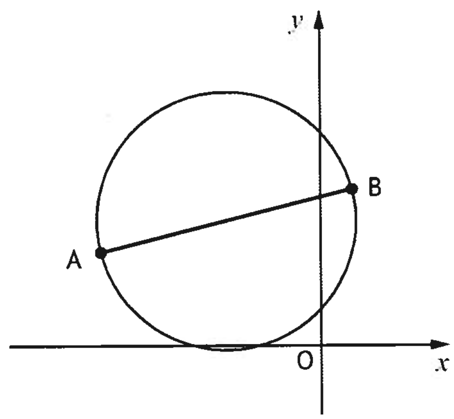
Total marks — 70

1. A curve has equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at the point where $x = 5$. 4

2. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. 3

3. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. 2

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. 2

MARKS

4. Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$. 3

5. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

Determine the value of x. 4

6. Determine the range of values of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots. 4

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. 5

8. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **5**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

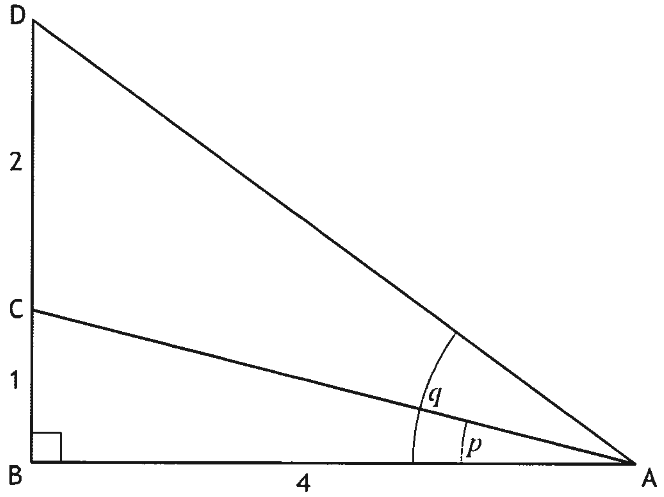
10. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x - 6) = \log_5 25$, where $x > 6$. **5**

11. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

MARKS

12. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q - p)$ is $\frac{19\sqrt{17}}{85}$. 5

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. 4

14. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. 5

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. 2

MARKS

15. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF SPECIMEN QUESTION PAPER]

Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•¹ differentiate •² calculate gradient •³ find the value of y •⁴ find equation of tangent	•¹ $2x-4$ •² 6 •³ 12 •⁴ $y=6x-18$	4
2.	•¹ find the centre •² calculate the radius •³ state equation of circle	•¹ $(-3,4)$ •² $\sqrt{17}$ •³ $(x+3)^2+(y-4)^2=17$ or equivalent	3
3. (a)	•¹ find gradient l_1 •² state gradient l_2	•¹ $\frac{1}{\sqrt{3}}$ •² $-\sqrt{3}$	2
3. (b)	•³ using $m=\tan\theta$ •⁴ calculating angle	•³ $\tan\theta=-\sqrt{3}$ •⁴ $\theta=\frac{2\pi}{3}$ or 120°	2
4.	•¹ complete integration •² substitute limits •³ evaluate	•¹ $-\frac{1}{6}x^{-1}$ •² $\left(-\frac{1}{6\times2}\right)-\left(-\frac{1}{6\times1}\right)$ •³ $\frac{1}{12}$	3

Question	Generic scheme	Illustrative scheme	Max mark
5.			4
	•[1] find $\overrightarrow{CD}$	•[1] $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$	
	•[2] find $\overrightarrow{AB}$	•[2] $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$	
	•[3] equate scalar product to zero	•[3] $5(x-4)+(-10)(-3)+(-5)(-1)=0$	
	•[4] calculate value of x	•[4] $x=-3$	
6.			4
	•[1] substitute into discriminant	•[1] $(p+1)^2-4\times1\times9$	
	•[2] apply condition for no real roots	•[2] $\ldots<0$	
	•[3] determine zeroes of quadratic expression	•[3] $-7, 5$	
	•[4] state range with justification	•[4] $-7<p<5$ with eg sketch or table of signs	
7.			5
	•[1] substitute for y in equation of circle	•[1] $x^2+(3x-5)^2+2x-4(3x-5)-5=0$	
	•[2] express in standard quadratic form	•[2] $10x^2-40x+40=0$	
	•[3] demonstrate tangency	•[3] $10(x-2)^2=0$ only one solution implies tangency	
	•[4] find x-coordinate	•[4] $x=2$	
	•[5] find y-coordinate	•[5] $y=1$	

Question			Generic scheme	Illustrative scheme	Max mark
8.	(a)		•[1] use appropriate strategy •[2] obtain an expression for a and b •[3] obtain a second expression for a and b •[4] find the value of a or b •[5] find the second value	•[1] $(1)^3-4(1)^2+a(1)+b=0$ •[2] $a+b=3$ •[3] $2a+b=-4$ •[4] $a=-7$ or $b=10$ •[5] $b=10$ or $a=-7$	5
8.	(b)		•[6] obtain quadratic factor •[7] complete factorisation •[8] state solutions	•[6] $(x^2-3x-10)$ •[7] $(x-1)(x-5)(x+2)$ •[8] $x=1$, $x=5$, $x=-2$	3
9.	(a)		•[1] interpret information •[2] solve to find m	•[1] $13=28m+6$ •[2] $m=\frac{1}{4}$	2
9.	(b)	(i)	•[3] state condition	•[3] a limit exists as $-1<\frac{1}{4}<1$	1
9.	(b)	(ii)	•[4] know how to calculate limit •[5] calculate limit	•[4] $L=\frac{1}{4}L+6$ •[5] $L=8$	2

Question	Generic scheme	Illustrative scheme	Max mark
10. (a)	•¹ state value	•¹ 2	1
10. (b)	•¹ use laws of logarithms •² link to part (a) •³ use laws of logarithms •⁴ write in standard quadratic form •⁵ solve for x and identify appropriate solution	•¹ $\log_4 x(x-6)$ •² $\log_4 x(x-6)=2$ •³ $x(x-6)=4^2$ •⁴ $x^2-6x-16=0$ •⁵ 8	5
11.	•¹ start to differentiate •² complete differentiation •³ evaluate derivative	•¹ $3\times 4\sin^2 x\ldots$ •² $\ldots\times\cos x$ •³ $\dfrac{-3\sqrt{3}}{2}$	3
12.	•¹ calculate lengths AC and AD •² select appropriate formula and express in terms of p and q •³ calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$ •⁴ calculate other two and substitute into formula •⁵ arrange into required form	•¹ $\text{AC}=\sqrt{17}$ and $\text{AD}=5$ stated or implied by •³ •² $\cos q\cos p+\sin q\sin p$ stated or implied by •⁴ •³ $\cos p=\dfrac{4}{\sqrt{17}}$, $\cos q=\dfrac{4}{5}$ $\sin p=\dfrac{1}{\sqrt{17}}$, $\sin q=\dfrac{3}{5}$ •⁴ $\dfrac{4}{5}\times\dfrac{4}{\sqrt{17}}+\dfrac{3}{5}\times\dfrac{1}{\sqrt{17}}$ •⁵ $\dfrac{19}{5\sqrt{17}}\times\dfrac{\sqrt{17}}{\sqrt{17}}=\dfrac{19\sqrt{17}}{85}$ or $\dfrac{19}{5\sqrt{17}}=\dfrac{19\sqrt{17}}{5\times 17}=\dfrac{19\sqrt{17}}{85}$	5

Question	Generic scheme	Illustrative scheme	Max mark
13.	•¹ know to and start to integrate •² complete integration •³ substitute for x and y •⁴ state expression for y	•¹ eg $y=\frac{4}{2}x^2\ldots$ •² $y=\frac{4}{2}x^2-\frac{6}{3}x^3+c$ •³ $9=2(-1)^2-2(-1)^3+c$ •⁴ $y=2x^2-2x^3+5$	4
14. (a)	•¹ use double angle formula •² express as a quadratic in $\cos x°$ •³ start to solve •⁴ reduce to equations in $\cos x°$ only •⁵ process solutions in given domain	**Method 1:** Using factorisation •¹ $2\cos^2 x°-1\ldots$ **stated or implied by •²** •² $2\cos^2 x°-3\cos x°+1=0$ •³ $(2\cos x°-1)(\cos x°-1)$ (= 0 must appear at either of these lines to gain •²) **Method 2:** Using quadratic formula •¹ $2\cos^2 x°-1\ldots$ **stated or implied by •²** •² $2\cos^2 x°-3\cos x°+1=0$ **stated explicitly** •³ $\frac{-(-3)\pm\sqrt{(-3)^2-4\times2\times1}}{2\times2}$ **In both methods:** •⁴ $\cos x°=\frac{1}{2}$ and $\cos x°=1$ •⁵ 0, 60, 300 Candidates who include 360 lose •⁵. **or** •⁴ $\cos x=1$ and $x=0$ •⁵ $\cos x°=\frac{1}{2}$ and $x=60$ or 300 Candidates who include 360 lose •⁵.	5
14. (b)	•⁶ interpret relationship with (a) •⁷ state valid values	•⁶ $2x=0$ and 60 and 300 •⁷ 0, 30, 150, 180, 210 and 330	2

Question	Generic scheme	Illustrative scheme	Max mark
15. (a)	•¹ interpret notation •² complete process	•¹ $g\left(x^3-1\right)$ •² $3x^3-2$	2
15. (b)	•³ start to rearrange for x •⁴ rearrange •⁵ state expression for $h(x)$	•³ $3x^3=y+2$ •⁴ $x=\sqrt[3]{\dfrac{y+2}{3}}$ •⁵ $h(x)=\sqrt[3]{\dfrac{x+2}{3}}$	3

[END OF SPECIMEN MARKING INSTRUCTIONS]