

# Homework sheet 1

①

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & 0 \end{array} \Rightarrow \text{remainder} = 0$$

so  $(x-1)$  a factor of  $f(x)$

$$\begin{aligned} \therefore f(x) &= 2x^3 + x^2 - 8x + 5 \\ &= (x-1)(2x^2 + 3x - 5) \\ &= (x-1)(2x+5)(x-1) \end{aligned}$$

②

$$\begin{aligned} &x^2 + 8x + 3 \\ &= (x^2 + 8x + 16) - 16 + 3 \\ &= (x+4)^2 - 13 \end{aligned}$$

$$\therefore \text{Mtp}(-4, -13)$$

③  $\log_5 2 + \log_5 50 - \log_5 4$

$$= \log_5 \frac{2 \times 50}{4}$$

$$= \log_5 \frac{100}{4}$$

$$= \log_5 25$$

$$= \log_5 5^2$$

$$= 2 \log_5 5$$

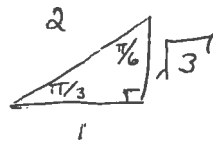
$$= 2 \times 1$$

$$= 2$$

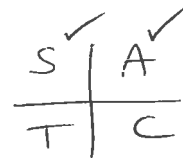
4.  $2\sin x - \sqrt{3} = 0$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2} \Rightarrow$



$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$   
 $= \frac{\pi}{3}$

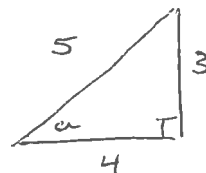


$x = \frac{\pi}{3}, \pi - \frac{\pi}{3}$   
 $= \frac{\pi}{3}, \frac{3\pi}{3} - \frac{\pi}{3}$   
 $= \frac{\pi}{3}, \frac{2\pi}{3}$

For  $\frac{\pi}{2} \leq x \leq \pi$

$x = \frac{2\pi}{3}$

5.  $\sin(x+a)$   
 $= \sin x \cos a + \cos x \sin a$   
 $= \sin x \cdot \frac{4}{5} + \cos x \cdot \frac{3}{5}$   
 $= \frac{4}{5} \sin x + \frac{3}{5} \cos x$



6.  $y = 4x^3 + 5x^2 - 3x + 2$

$\frac{dy}{dx} = 12x^2 + 10x - 3$

7.  $y = x^3 - 3x^2 - 9x + 12$  ①

$\frac{dy}{dx} = 3x^2 - 6x - 9$

SP when  $\frac{dy}{dx} = 0$

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$  or  $x = 3$

Sub into ①

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 12$

$= -1 - 3 + 9 + 12$

$= 17$

$y = 3^3 - 3 \times 3^2 - 9 \times 3 + 12$

$x$	-2	-1	1	3	4
$\frac{dy}{dx}$	+	0	-	0	+
	/	-	\	-	/

Max TP (-1, 17)

Min TP (3, -15)

$$\textcircled{8} \int (2x^{-4} + \cos 5x) dx$$

$$= -\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + C$$

$$\textcircled{9} \frac{dy}{dx} = 8x - 3$$

$$y = \int 8x - 3 dx$$

$$= \frac{8}{2}x^2 - 3x + C$$

$$= 4x^2 - 3x + C$$

Let  $y = 7$  and  $x = 2$

$$7 = 4(2)^2 - 3(2) + C$$

$$7 = 10 + C$$

$$C = -3$$

so  $y = 4x^2 - 3x - 3$

$$\textcircled{10} K \sin(x-a) = K \sin x \cos a - K \cos x \sin a$$

$$\sqrt{3} \sin x - \cos x = K \cos a \sin x - K \sin a \cos x$$

$$\text{so } K \cos a = \sqrt{3}$$

$$K \sin a = 1$$

$$\text{so } R^2 = 3 + 1$$

$$= 4$$

$$R = 2$$

so

$$\tan a = \frac{1}{\sqrt{3}}$$

$$a = 30^\circ$$

S	A ✓
T	C

$$\sqrt{3} \sin x - \cos x = 2 \sin(x - 30^\circ)$$

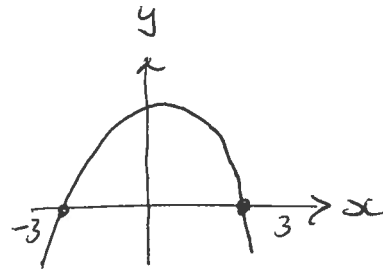
$$(11.) \quad 9 - x^2 \geq 0$$

$$\text{consider } 9 - x^2 = 0$$

$$(3 - x)(3 + x) = 0$$

$$x = 3 \quad \text{or} \quad x = -3$$

$$x \in \mathbb{R} : -3 \leq x \leq 3$$



$$(12.) \quad y = 2 \cos 3x$$

$$(13.) \quad \begin{aligned} \vec{EP} &= \underline{p} - \underline{e} \\ &= \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \vec{PF} &= \underline{f} - \underline{p} \\ &= \begin{pmatrix} 7 \\ 17 \\ 13 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 12 \\ 6 \end{pmatrix} \\ &= 2 \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \end{aligned}$$



$$\vec{PF} = 2\vec{EP}$$

P divides  $\vec{EF}$  in the ratio of 1:2

$$(14.) \quad 2 \cdot (p + q)$$

$$= 2 \cdot p + 2 \cdot q$$

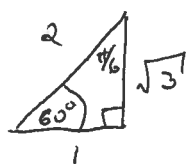
$$= p \cdot 2 + |q|^2$$

$$= 10 + 16$$

$$= 26$$

$$(15.) \quad \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$



$$\tan \frac{\pi}{6}$$

$$= \frac{1}{\sqrt{3}}$$