HOMEWORK SHEET 2

1)

1	1	0	-3	2
		1	1	-2
	1	1	-2	0

Remainder $= 0$ so $(x-1)$ is a factor

$$x^3 - 3x + 2 = (x-1)(x^2+x-2)$$
$$= (x-1)(x+2)(x-1)$$
$$= \underline{(x-1)^2(x+2)}$$

2) $2x^2+4x+7$

$$= 2\left(x^2+2x+\tfrac{7}{2}\right)$$
$$= 2\left(x^2+2x+1-1+\tfrac{7}{2}\right)$$
$$= 2\left((x+1)^2+\tfrac{5}{2}\right)$$
$$= 2(x+1)^2+5$$

so $\underline{q=5}$

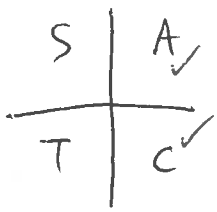
3) $\log_4 12 - \log_4 x = \log_4 6$

$$\log_4 \frac{12}{x} = \log_4 6$$
$$\therefore \frac{12}{x} = 6$$
$$6x = 12$$
$$\underline{x = 2}$$

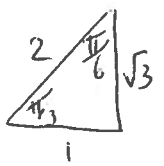
4) $2\cos x = \sqrt{3}$, $0 \le x \le 2\pi$

$\cos x = \frac{\sqrt{3}}{2}$

(exact value)



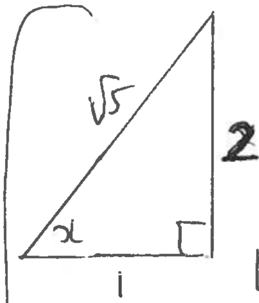
$\underline{x = \frac{\pi}{6}}$



or $x = 2\pi - \frac{\pi}{6}$

$$= \frac{12\pi}{6} - \frac{\pi}{6}$$
$$= \underline{\frac{11\pi}{6}}$$

5) $\cos x = \frac{1}{\sqrt{5}}$



$x^2 = (\sqrt{5})^2 - 1^2$
$= 5-1$
$= 4$
$x = \sqrt{4} = 2$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1$$
$$= 2\left(\frac{1}{5}\right) - 1$$
$$= \frac{2}{5} - 1$$
$$= \underline{-\frac{3}{5}}$$

6) $f(x) = (4-3x^2)^{-\frac{1}{2}}$ * CHAIN RULE *

$$f'(x) = -\frac{1}{2}(4-3x^2)^{-\frac{3}{2}} \cdot (-6x)$$
$$= 3x(4-3x^2)^{-\frac{3}{2}}$$
$$= \frac{3x}{(4-3x^2)^{3/2}}$$

(p2)

Q7) $f(x) = x^3 - 3x + 2$

$f'(x) = 0$ at st. pts

$f'(x) = 3x^2 - 3 = 0$

$3(x^2-1) = 0$

$3(x-1)(x+1) = 0$

$x = 1$ or $x = -1$

x	-2	-1	0	1	2
$3(x^2-1)$ $\frac{dy}{dx}$	$+$	0	$-$	0	$+$

$(-1, *)$ max t/pt $\quad$ $(1, *)$ min t/pt

$x = -1$:

$f(-1) = (-1)^3 - 3(-1) + 2$

$= -1 + 3 + 2$

$= 4$

$(-1, 4)$ max t/pt

$f(1) = (1)^3 - 3(1) + 2$

$= 1 - 3 + 2$

$= 0$

$(1, 0)$ min t/pt

8) $\int \left(4x^{\frac{1}{2}} + x^{-3}\right) dx$

$= 4\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{x^{-2}}{-2} + C$

$= 4 \times \frac{2}{3} x^{\frac{3}{2}} - \frac{x^{-2}}{2} + c$

$= \frac{8}{3} x^{\frac{3}{2}} - \frac{1}{2x^2} + c$

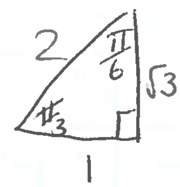
9) $f'(x) = \sin 3x$

$f(x) = \int \sin 3x \, dx$

$y = \dfrac{-\cos 3x}{3} + c$ $\qquad (\frac{\pi}{9}, 1)$

$1 = \dfrac{-\cos 3(\frac{\pi}{9})}{3} + c$

$1 = \dfrac{-\cos \frac{\pi}{3}}{3} + c$



$1 = -\frac{1}{2} \div 3 + c$

$1 = -\frac{1}{6} + c$

$c = 1\frac{1}{6}$

$y = -\dfrac{\cos 3x}{3} + 1\frac{1}{6}$

$\left\{ \text{or } y = -\frac{1}{3}\cos 3x + \frac{7}{6} \right\}$

10) $\sin x - \cos x = k \sin(x - a)$

$= k \sin x \cos a - k \sin a \cos x$

so $-k\sin a = -1 \qquad k\sin a = 1 \sim (1)$

$k \cos a = 1 \sim (2)$

$k^2 \sin^2 a + k^2 \cos^2 a = 1^2 + 1^2$

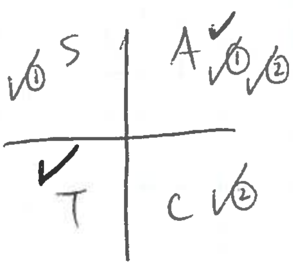
$k^2 = 2$

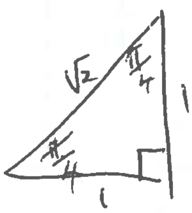
$k = \sqrt{2}$

$\dfrac{k \sin a}{k \cos a} = \dfrac{1}{1}$

$\tan a = 1$

$a = \frac{\pi}{4}$





so $\boxed{\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)}$

(11) $f(x) = 3x+1 \qquad g(x) = x^2 - 2$

$f(g(x)) = f(x^2-2)$
$= 3(x^2-2)+1$
$= 3x^2 - 6 + 1$
$= \underline{3x^2 - 5}$

$g(f(x)) = g(3x+1)$
$= (3x+1)^2 - 2$
$= 9x^2 + 6x + 1 - 2$
$= \underline{9x^2 + 6x - 1}$

(12) $y = \log_a(x-b)$

Should pass through $(1,0) \longrightarrow (4,0)$ (3 to right) $\Big\}$ $\underline{b = 3}$

$(3,1) \longleftarrow (6,1) \qquad \underline{a = 3}$

$\{ y = \log_3(x-3) \}$

(13) $\underline{u} \cdot \underline{v} = 0$ when perp.

so $\begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix} = 0$

$0 - 4 + k = 0$

$\underline{k = 4}$

(14) $\overrightarrow{DE} = \begin{array}{cc} e & d \\ \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} & - \begin{pmatrix} 10 \\ -8 \\ -15 \end{pmatrix} \end{array}$

$= \begin{pmatrix} -9 \\ 6 \\ 12 \end{pmatrix}$

$= 3\begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

$= 3\overrightarrow{EF}$

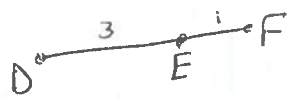
$\overrightarrow{EF} = \begin{array}{cc} f & e \\ \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} & - \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \end{array}$

$= \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$

so $\overrightarrow{DE} = 3\overrightarrow{EF}$

so vectors are parallel, and have a common point E

so are collinear.



$\overrightarrow{DE} = 3\overrightarrow{EF}$

so $\boxed{3:1}$

(15) (LHS) $\dfrac{\cos^3 x}{1 - \sin^2 x}$ $\left[\begin{array}{l} \sin^2 x + \cos^2 x = 1 \\ \text{so } \cos^2 x = 1 - \sin^2 x \end{array} \right]$

$= \dfrac{\cos x(\cos^2 x)}{\cos^2 x}$

$= \cos x$ (RHS)

(16) $5x + 3y - 6 = 0$

$3y = -5x + 6$

$y = \boxed{-\frac{5}{3}}x + 2 \qquad m = -\frac{5}{3}$

parallel $\rightarrow$ same gradient

so $m = -\frac{5}{3}$, $(-2,-1)$

$y - (-1) = -\frac{5}{3}(x - (-2))$

$y + 1 = -\frac{5}{3}(x+2)$

$3y + 3 = -5x - 10$

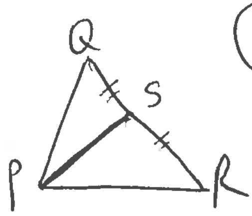
$3y = -5x - 13$

$\boxed{y = -\frac{5}{3}x - \frac{13}{3}}$

(p.4)

Q.17

median → to middle



midpt of QR : $S\left(\frac{-1+3}{2}, \frac{4+6}{2}\right)$

$P(-3,-2)$ $\quad$ $S(1,5)$

$$m_{PS} = \frac{5-(-2)}{1-(-3)}$$

$$= \frac{7}{4}$$

(18) $m_{radius} = \frac{9-5}{7-2} = \frac{4}{5}$

$m_{tangent}$ is perpendicular

so $m_{tangent} = -\frac{5}{4}$ $\quad (7,9)$

$$y-9 = -\frac{5}{4}(x-7)$$

$$4y - 36 = -5x + 35$$

$$4y = -5x + 71$$

$$y = -\frac{5}{4}x + \frac{71}{4}$$

(19) $u_{n+1} = \underbrace{0.4}_{a}u_n \underbrace{-240}_{b}$

$-1 < 0.4 < 1$ so a limit exists

$$L = \frac{b}{1-a}$$

$$= \frac{-240}{1-0.4}$$

$$= \frac{-240}{0.6}$$

$$= -400$$

(20) $A = \int_0^3 x^3(3-x)\,dx$

$$= \int_0^3 3x^3 - x^4\,dx$$

$$= \left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$$

$$= \left(\frac{3(3)^4}{4} - \frac{(3)^5}{5}\right) - \left(\frac{3(0)^4}{4} - \frac{(0)^5}{5}\right)$$

$$= \frac{243}{4} - \frac{243}{5}$$

$$= \frac{1215}{20} - \frac{972}{20}$$

$$= \frac{243}{20}$$

$$= 12\tfrac{3}{20} \text{ sq units}$$