

# Higher Maths Homework 3

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1 a)  $\log_6 x + \log_6 (x+5) = 2$

$$\log_6 x(x+5) = 2 \quad \checkmark$$

$$x(x+5) = 6^2 \quad \checkmark$$

$$x^2 + 5x = 36$$

$$x^2 + 5x - 36 = 0$$

$$(x+9)(x-4) = 0$$

$$x = -9 \text{ or } x = 4 \quad \checkmark$$

NA

b)  $\log(3x-2) - \log x = 0$

$$\log \frac{3x-2}{x} = 0 \quad \checkmark$$

$$\frac{3x-2}{x} = 10^0 \quad \checkmark$$

$$\frac{3x-2}{x} = 1$$

$$3x-2 = x$$

$$2x = 2$$

$$x = 1 \quad \checkmark$$

c)  $\frac{1}{3} \log 64 + \frac{1}{2} \log 49 = \log 5 + \log x$

$$\log 64^{1/3} + \log 49^{1/2} = \log 5 + \log x$$

$$\log(64^{1/3} \times 49^{1/2}) = \log 5x \quad \checkmark$$

$$64^{1/3} \times 49^{1/2} = 5x \quad \checkmark$$

$$4 \times 7 = 5x$$

$$28 = 5x$$

$$x = \frac{28}{5}$$

$$= 5 \frac{3}{5} \quad \checkmark$$

d)  $\ln x + \ln x^2 - \ln x = 0$

$$\ln \frac{x \cdot x^2}{x} = 0 \quad \checkmark$$

$$\ln x^2 = 0$$

$$x^2 = e^0 \quad \checkmark$$

$$x^2 = 1$$

$$x = \pm \sqrt{1}$$

$$= 1 \text{ or } -1 \quad \checkmark$$

NA

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$$\begin{aligned}
 \textcircled{2} \quad & 2 \log_3 3 - \log_3 81 + \log_3 \frac{1}{9} \\
 &= 2 \log_3 3 - \log_3 3^4 + \log_3 9^{-1} \\
 &= 2 \log_3 3 - 4 \log_3 3 - \log_3 3^2 \\
 &= 2 \log_3 3 - 4 \log_3 3 - 2 \log_3 3 \\
 &= -4 \times 1 \\
 &= -4
 \end{aligned}$$

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$$\textcircled{3} \quad \text{a) } m = \log_n 4 \quad \text{b) } 3h - 1 = \log_a 8$$

$$n^m = 4$$

$$a^{(3h-1)} = 8$$

2

$$\begin{aligned}
 \textcircled{4} \quad \text{a) } & I_t = I_0 e^{-kt} \\
 & \checkmark 90 = 120 e^{-4k} \\
 & \frac{90}{120} = e^{-4k} \\
 & \checkmark 0.75 = e^{-4k}
 \end{aligned}$$

$$-4k = \ln 0.75$$

$$k = \frac{\ln 0.75}{-4}$$

$$= 0.0719 \dots$$

$$= 0.0719 \checkmark \text{ (to 3sf)}$$

$$\begin{aligned}
 \text{b) } & I_t = I_0 e^{-kt} \\
 & I_t = I_0 e^{-0.0719 \times 10} \\
 & I_t = I_0 \times 0.487 \\
 & \frac{I_t}{I_0} = 0.487 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{Reduction} &= 1 - 0.487 \\
 &= 0.512 \dots
 \end{aligned}$$

$$\begin{aligned}
 \% \text{ Reduction} &= 51.27 \dots \\
 &= 51.3 \% \checkmark
 \end{aligned}$$

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5a) straight-line  $y = mx + c$

$$\Rightarrow \ln y = mx + c$$

$$m = \frac{-2}{3} \checkmark$$

$$\Rightarrow \ln y = -\frac{2}{3}x + c$$

let  $x = 0$  when  $\ln y = 2$

$$\therefore 2 = -\frac{2}{3} \times 0 + c$$

$$c = 2$$

(or simply read  $c = 2$  from graph)

$$\therefore \ln y = -\frac{2}{3}x + 2 \checkmark$$

$$y = e^{-\frac{2}{3}x + 2} \checkmark$$

$$= e^{-\frac{2}{3}x} \cdot e^2 \checkmark$$

$$= (e^{-\frac{2}{3}})^x \cdot e^2$$

$$= 7.39 \times 0.513^x$$

$$e^{-\frac{2}{3}} = 0.5134\dots$$

$$= 0.513 \text{ (to 3sf)}$$

$$e^2 = 7.389\dots$$

$$= 7.39 \text{ (to 3sf)}$$

$$\therefore a = 7.39$$

$$b = 0.513 \checkmark$$

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b) Straight-line  $y = mx + c$

$$m = \frac{4-3}{5-1}$$
$$= \frac{1}{4} \checkmark$$

$$\Rightarrow \log_{10} y = \frac{1}{4}x + c$$

Let  $x = 1$  when  $\log_{10} y = 3$

$$\therefore 3 = \frac{1}{4} \times 1 + c$$

$$3 = \frac{1}{4} + c$$

$$\frac{12}{4} = \frac{1}{4} + c$$

$$c = \frac{11}{4} \checkmark$$

$$\Rightarrow \log_{10} y = \frac{1}{4}x + \frac{11}{4} \checkmark$$

$$y = 10^{\frac{1}{4}x + \frac{11}{4}} \checkmark$$

$$= 10^{\frac{1}{4}x} \cdot 10^{\frac{11}{4}}$$

$$= \left(10^{\frac{1}{4}}\right)^x \cdot 10^{\frac{11}{4}}$$

$$= 562.3 \times 1.8^x$$

$$\therefore a = 562.3$$

$$b = 1.8 \checkmark$$

$$\Rightarrow 10^{\frac{11}{4}} = 562.34\dots$$

$$= 562.3 \text{ (to 1dp)}$$

$$10^{\frac{1}{4}} = 1.778\dots$$

$$= 1.8 \text{ (to 1dp)}$$

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