

CIE Higher Maths Homework 2

① a) $10^x = 300$

$$x = \log_{10} 300 \checkmark$$

$$= 2.477\dots$$

$$= 2.48 \text{ (to 3sf)}$$

b) $e^{2x} = 403$

$$2x = \ln 403 \checkmark$$

$$x = \frac{\ln 403}{2}$$

$$= 2.999\dots$$

$$= 3.00 \text{ (to 3sf)}$$

c) $\ln x = 1.78$

$$x = e^{1.78} \checkmark$$

$$= 5.929\dots$$

$$= 5.93 \text{ (to 3sf)}$$

d) $10^{0.75x} = 90$

$$0.75x = \log_{10} 90 \checkmark$$

$$x = \frac{\log_{10} 90}{0.75}$$

$$= 2.605\dots$$

$$= 2.61 \text{ (to 3sf)}$$

e) $e^{-0.032x} = 10$

$$-0.032x = \ln 10 \checkmark$$

$$x = \frac{\ln 10}{-0.032}$$

$$= -71.955\dots$$

$$= -72.0 \text{ (to 3sf)}$$

f) $\log_e \left(\frac{10}{3x} \right) = 1.3$

$$\frac{10}{3x} = e^{1.3} \checkmark$$

$$10 = 3x e^{1.3}$$

$$\frac{10}{e^{1.3}} = 3x$$

$$x = \frac{10}{3e^{1.3}} \quad \checkmark$$

$$= 0.9084\dots$$

$$= 0.908 \text{ (to 3sf)}$$

② a) $m = m_0 e^{-0.025t}$

$$= 1000 \times e^{-0.025 \times 25} \checkmark$$

$$= 535.261\dots$$

$$= 535 \text{ g (to 3sf)}$$

b) $m = 0.1M_0$ for 90% fall in mass

$$\therefore 0.1M_0 = M_0 e^{-0.025t}$$

$$0.1 \checkmark = e^{-0.025t}$$

$$\ln 0.1 \checkmark = -0.025t$$

$$t = \frac{\ln 0.1}{-0.025}$$

$$= 92.103\dots$$

$$= 92.1 \text{ years} \checkmark$$

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$$\textcircled{3} \text{ a) } P_t = P_0 e^{-kt}$$

$$t = 24, P_0 = 50, P_t = 10$$

$$\therefore \sqrt{10} = 50 e^{-k \times 24}$$

$$0.2 \sqrt{} = e^{-24k}$$

$$\ln 0.2 \sqrt{} = -24k$$

$$k = \frac{\ln 0.2 \sqrt{}}{-24}$$

$$= 0.06705 \dots$$

$$= 0.067 \text{ (to 3 dp)}$$

$$\text{b) } P_t = P_0 e^{-kt}$$

$$P_0 = 50, k = 0.067, t = 4 \text{ hrs}$$

$$\therefore P_t = 50 e^{-0.067 \times 4}$$

$$= 38.245 \dots$$

$$= 38.2 \text{ units (to 3 sf)}$$

No damage to tyre as pressure will remain above 30 units at 38.2 units.

✓

$$\textcircled{4} \text{ a) Straightline } y = mx + c$$

$$\therefore \log_{10} y = m \log_{10} x + c$$

$$\text{pt } (0, 5) \text{ and } \text{pt } (4, 0)$$

$$m = \frac{5-0}{0-4} \quad c = 5$$

$$= -\frac{5}{4} \sqrt{}$$

$$\therefore \log_{10} y = -\frac{5}{4} \log_{10} x + 5 \sqrt{}$$

$$\log_{10} y + \frac{5}{4} \log_{10} x = 5$$

$$\log_{10} y + \log_{10} x^{5/4} = 5$$

$$\sqrt{\log_{10} y x^{5/4}} = 5$$

$$y x^{5/4} = 10^5 \sqrt{}$$

$$y = \frac{10^5}{x^{5/4}}$$

$$= 10^5 x^{-5/4}$$

$$\therefore k = 10^5$$

$$n = -5/4 \sqrt{\text{both}}$$

$$\text{b) Straightline } y = mx + c$$

$$\therefore \ln y = m \ln x + c$$

$$\text{pt } (1, 2) \text{ and } \text{pt } (3, 3)$$

$$m = \frac{2-3}{1-3}$$

$$= \frac{-1}{-2}$$

$$= \frac{1}{2} \sqrt{}$$

$$\therefore \ln y = \frac{1}{2} \ln x + c$$

$$\text{sub pt } (1, 2)$$

$$2 = \frac{1}{2} \times 1 + c$$

$$2 = \frac{1}{2} + c$$

$$c = 3/2 \sqrt{}$$

$$\therefore \ln y = \frac{1}{2} \ln x + \frac{3}{2}$$

$$\ln y - \frac{1}{2} \ln x = \frac{3}{2}$$

$$\ln y - \ln x^{1/2} = \frac{3}{2}$$

$$\sqrt{\ln \left(\frac{y}{x^{1/2}} \right)} = \frac{3}{2}$$

$$\frac{y}{x^{1/2}} = e^{3/2} \sqrt{}$$

$$y = e^{3/2} x^{1/2} \quad /10$$

$$\therefore k = e^{3/2}$$

$$n = 1/2 \sqrt{\text{both}}$$

Total 35