Higher Maths Paper 2 2018

① area $= \int_{-1}^{3} (3 + 2x - x^2)\,dx$

$= \left[3x + x^2 - \frac{x^3}{3} \right]_{-1}^{3}$

$= \left(9 + 9 - \frac{27}{3}\right) - \left(-3 + 1 + \frac{1}{3}\right)$

$= \underline{\underline{\frac{32}{3}}}$

② (a) $\underline{u} \cdot \underline{v}$

$= \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$

$= 7 + 32 - 15$

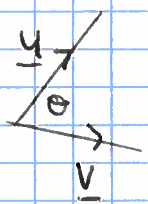
$= \underline{\underline{24}}$.

(b) $|\underline{u}| = \sqrt{1 + 16 + 9}$

$= \sqrt{26}$

$|\underline{v}| = \sqrt{49 + 64 + 25}$

$= \sqrt{138}$



$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|}$

$= \frac{24}{\sqrt{26} \times \sqrt{138}}$

$\cos\theta = 0.4006\ldots$

$\underline{\underline{\theta = 68.4^\circ}}$ (1 d.p)

③ $f(x) = x^3 - 7x - 6.$

$f'(x) = 3x^2 - 7$

$f'(2) = 3 \times 2^2 - 7$

$= 5 \quad > 0$ so $f(x)$ is increasing at $x = 2$

(4) $-3x^2-6x+7$

$= -3(x^2+2x)+7$

$= -3((x+1)^2-1^2)+7$

$= -3(x+1)^2+3+7$

$= \underline{\underline{-3(x+1)^2+10}}$.

(5) $m_{PQ} = \frac{-2-4}{9-3}$

$= \frac{-6}{6}$

$= -1$

midpoint $\left(\frac{3+9}{2}, \frac{4+(-2)}{2}\right)$

$= (6, 1)$

$m_{perp} = 1$.

Equation PQ

$y-b = m(x-a)$

$y-1 = 1(x-6)$

$\underline{\underline{y = x-5}}$

(b) Solve $y = x-5$ --- (1)

$3y+x = 25$ --- (2)

Substitute (1) in (2)

$3(x-5)+x = 25$

$3x-15+x = 25$

$4x = 40$

$x = 10$ $y=5$ $\underline{\underline{C(10,5)}}$

(c) radius $= \sqrt{(10-3)^2+(5-4)^2}$

$= \sqrt{49+1}$

$= \sqrt{50}$

Circle. $\underline{\underline{(x-10)^2+(y-5)^2 = 50}}$

(6). (a) (i) $f(g(x))$

$= f(2x)$

$= 3+\cos 2x$

(ii) $g(f(x))$

$= g(3+\cos x)$

$= 2(3+\cos x)$

$= 6+2\cos x$

(b) $f(g(x)) = g(f(x))$

$3 + \cos 2x = 6 + 2\cos x$

$3 + 2\cos^2 x - 1 = 6 + 2\cos x$

$2\cos^2 x - 2\cos x - 4 = 0$

$\cos^2 x - \cos x - 2 = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x = 2$ not possible.

$\cos x + 1 = 0$

$\cos x = -1$

$\underline{\underline{x = \pi}}$

(7) (a) (i)

$$\begin{array}{c|cccc}
2 & 2 & -3 & -3 & 2 \\
 & & 4 & 2 & -2 \\
\hline
 & 2 & 1 & -1 & \boxed{0}
\end{array}$$

remainder 0

so $x-2$ is a factor.

(ii) $2x^3 - 3x^2 - 3x + 2 = (x-2)(2x^2 + x - 1)$

$= (x-2)(2x-1)(x+1)$

(b) $u_{n+1} = au_n - 1$

$u_5 = 2a - 3$

$u_6 = au_5 - 1$

$= 2a^2 - 3a - 1$

$u_7 = au_6 - 1$

$= 2a^3 - 3a^2 - a - 1$ as required.

(c) $u_7 = u_5$

$\Rightarrow 2a^3 - 3a^2 - a - 1 = 2a - 3$

$$2a^3 - 3a^2 - 3a + 2 = 0$$

From (a) $(a-2)(2a-1)(a+1) = 0$

$a = 2 \quad a = +\frac{1}{2} \quad a = -1$

Since a limit exists $-1 < a < 1$

so $a = +\frac{1}{2}$.

(ii) $\quad u = \frac{b}{1-a}$

$= \frac{-1}{1-(+\frac{1}{2})}$

$= -2$

(8) (a) $2\cos x - \sin x = k\cos(x-a)$

$= k\cos x\cos a + k\sin x\sin a$

$= k\cos a\cos x + k\sin a\sin x$

So

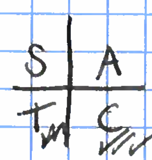
$$\left.\begin{aligned} k\cos a &= 2 \\ k\sin a &= -1 \end{aligned}\right\}$$

Square and add $\quad k^2 = (-1)^2 + 2^2$

$k^2 = 5$

$k = \sqrt{5}$

Divide. $\quad \tan a = -\frac{1}{2} \qquad$ ra $\quad 26.6^\circ$



$a = 360 - 26.6$

$a = 333.4^\circ$

So $\quad 2\cos x - \sin x = \sqrt{5}\cos(x - 333.4^\circ)$

(i) $\quad 6\cos x - 3\sin x = 3(2\cos x - \sin x)$

$= 3\sqrt{5}\cos(x - 333.4^\circ)$

minimum value $\quad -3\sqrt{5}$

(ii) angle $x - 333.4^\circ = 270$

$x = 603.4^\circ$ } subtract 360°

$x = 243.4^\circ$

(9) $P = 2x + \frac{128}{x}$

$P = 2x + 128x^{-1}$

$\frac{dP}{dx} = 2 - 128x^{-2}$

$= 2 - \frac{128}{x^2}$.

For minimum value $\frac{dP}{dx} = 0$

$2 - \frac{128}{x^2} = 0$

$\frac{128}{x^2} = 2$

$x^2 = 64$

$x = \pm 8$

Since $x > 0$ $x = 8cm$.

Nature

x	$\xrightarrow{1}$	8	$\xrightarrow{10}$
$\frac{dP}{dx} = 2 - \frac{128}{x^2}$	-126	0	1.72
	\	—	/

So $x = 8cm$ gives a minimum perimeter

(10) $x^2 + (m-3)x + m = 0$.

$a = 1$ $b = (m-3)$ $c = m$

$b^2 - 4ac > 0$

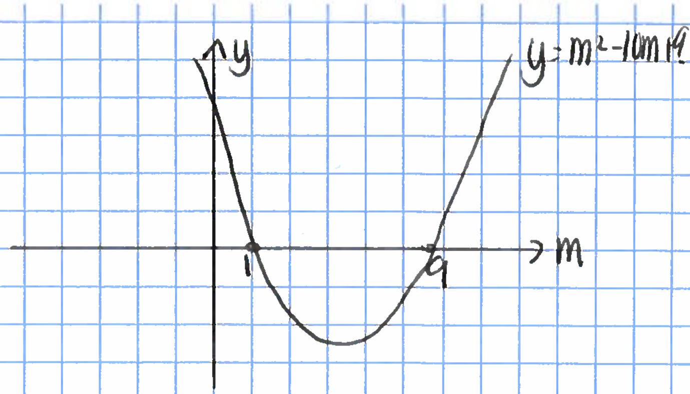
so $(m-3)^2 - 4m > 0$

$m^2 - 6m + 9 - 4m > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

$m < 1$ or $m > 9$.



(11) $P = 100(1 - e^{kt})$

(a) $P = 50 \quad t = 3.$

$50 = 100(1 - e^{3k})$

$0.5 = 1 - e^{3k}$

$e^{3k} = 0.5$

$3k = \ln 0.5$

$k = -0.231 \quad (3 s.f)$

(b) $t = 5 \quad P = ? \quad k = -0.231$

$P = 100(1 - e^{-0.231 \times 5})$

$P = 68.5\%$ (3 s.f)

So 68.5% wait for less than 5 minutes.

and 31.5% wait for ~~more~~ 5 minutes or more.

(12) (a) (i) $(x-13)^2 + (y+4)^2 = 100$

centre $(13, -4)$.

(ii) Substitute $(13, -4)$ into $x^2 + y^2 + 14x - 22y + c = 0$

$169 + 16 + 182 + 88 + c = 0$

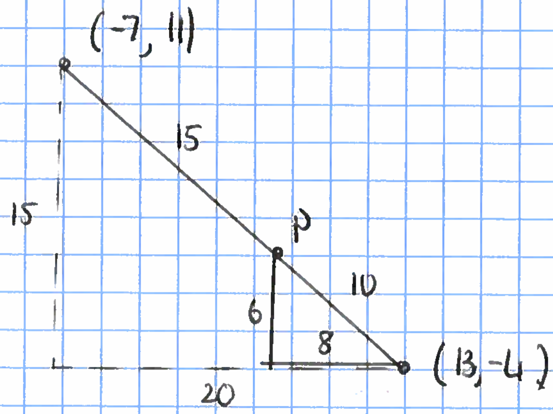
$c = -455$

as required.

(b) radius $C_1 = 10$

radius $C_2 = \sqrt{7^2 + (-11)^2 - (-455)}$

$= 25$



ratio 15 : 10
3 : 2

$\frac{2}{5}$ of 15 = 6
$\frac{2}{5}$ of 20 = 8.

P(5, 2)

(c) radius 15 + 25
= 40.

centre (5, 2)

equation $(x-5)^2 + (y-2)^2 = 1600$.