Higher Maths Paper 1 2018

① midpoint PQ $= \left(\frac{-2+4}{2}, \frac{4+0}{2}\right)$

$= (1,2)$

$m = \frac{6-2}{3-1}$

$= \frac{4}{2}$

$= 2$

Equation $\quad y-b = m(x-a) \quad m=2 \quad (a,b) = (1,2)$

$y-2 = 2(x-1)$

$\underline{\underline{y = 2x}}$

② $g(x) = \frac{1}{5}x - 4$

$x \xrightarrow{\div 5} \xrightarrow{-4} g(x)$

$g^{-1}(x) \xleftarrow[\times 5]{} \xleftarrow[+4]{} x$

$g^{-1}(x) = 5(x+4)$

$\underline{g^{-1}(x) = 5x+20}$.

③ $h(x) = 3\cos 2x$

$h'(x) = -6\sin 2x$

$h'\left(\frac{\pi}{6}\right) = -6\sin\frac{\pi}{3}$

$= -6 \times \frac{\sqrt{3}}{2}$

$= \underline{\underline{-3\sqrt{3}}}$.

④ $x^2+y^2-12x-6y-23=0$

$2g = -12 \qquad 2f = -6$

$g = -6 \qquad f = -3$

centre $(6,3)$

$m_{\text{radius}} = \frac{3-(-5)}{6-8}$

$= \frac{8}{-2}$

$= -4$

$m_{\text{tangent}} = \frac{1}{4}$

equation of tangent $y-b=m(x-a)$ $m=\frac{1}{4}$ $(a,b)=(8,-5)$

$y+5=\frac{1}{4}(x-8)$

$4y+20=x-8$

$\underline{\underline{4y=x-28}}$

(5) $\vec{AB}=\underline{b}-\underline{a}$

$=\begin{pmatrix}5\\t\\5\end{pmatrix}-\begin{pmatrix}-3\\4\\-7\end{pmatrix}$

$=\begin{pmatrix}8\\t-4\\12\end{pmatrix}$

$=4\begin{pmatrix}2\\\frac{1}{4}(t-4)\\3\end{pmatrix}$

$\vec{BC}=\underline{c}-\underline{b}$

$=\begin{pmatrix}7\\9\\8\end{pmatrix}-\begin{pmatrix}5\\t\\5\end{pmatrix}$

$=\begin{pmatrix}2\\9-t\\3\end{pmatrix}$

$\vec{AB}=4\vec{BC}$

ratio $\underline{\underline{4:1}}$

(b) $\frac{1}{4}(t-4)=9-t$

$t-4=36-4t$

$5t=40$

$\underline{\underline{t=8}}$

$\underline{\underline{a}}$ $-3\xrightarrow{+8}5\xrightarrow{+2}7$

ratio $8:2$ $\underline{\underline{4:1}}$

so $(t-4)=4(9-t)$

$t=8$

(6) $\log_5 250-\frac{1}{3}\log_5 8=$

$=\log_5 250-\log_5 8^{\frac{1}{3}}$

$=\log_5 \frac{250}{2}$

$=\log_5 125$

$=\log_5 5^3$

$=3\log_5 5$

$=\underline{\underline{3}}$

(7) Crosses y-axis $x=0$

$y = x^3 - 3x^2 + 2x + 5$

$y = 5$ $\quad$ $P(0,5)$

(b) $\frac{dy}{dx} = 3x^2 - 6x + 2$

When $x=0$ $\quad \frac{dy}{dx} = 2$

Equation $\quad y - b = m(x-a) \qquad m = 2 \quad (a,b) = (0,5)$

$y - 5 = 2(x - 0)$

$y = 2x + 5$

(c) Solve $\quad y = 2x+5 \quad$ --- (1)

$y = x^3 - 3x^2 + 2x + 5 \quad$ --- (2)

$x^3 - 3x^2 + 2x + 5 = 2x + 5$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or $x = 3$.

When $x = 3 \quad y = 2 \times 3 + 5$

$= 11$ $\quad$ $Q(3, 11)$

(8) $y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

$m = \sqrt{3}$

$m = \tan\theta = \sqrt{3}$

$\theta = 60°$

(9) (a) $\vec{BC} = \vec{BA} + \vec{AC}$

$= -\underline{t} + \underline{u}$

$\vec{BC} = \underline{u} - \underline{t}$

(b) $\vec{MD} = \vec{MC} + \vec{CA} + \vec{AD}$

$= \frac{1}{2}\vec{BC} + \vec{CA} + \vec{AD}$

$= \frac{1}{2}(\underline{u} - \underline{t}) - \underline{u} + \underline{v}$

$= \underline{v} - \frac{1}{2}\underline{u} - \frac{1}{2}\underline{t}$

(10) $y = \int (6x^2 - 3x + 4)\, dx$

$y = 2x^3 - \frac{3}{2}x^2 + 4x + c.$

When $x = 2$ $y = 14$

$14 = 2 \times 2^3 - \frac{3}{2} \times 2^2 + 4 \times 2 + c.$

$14 = 16 - 6 + 8 + c.$

$c = -4.$

So $\underline{\underline{y = 2x^3 - \frac{3}{2}x^2 + 4x - 4}}$

(11) (a) $y = 1 - \log_3 x$

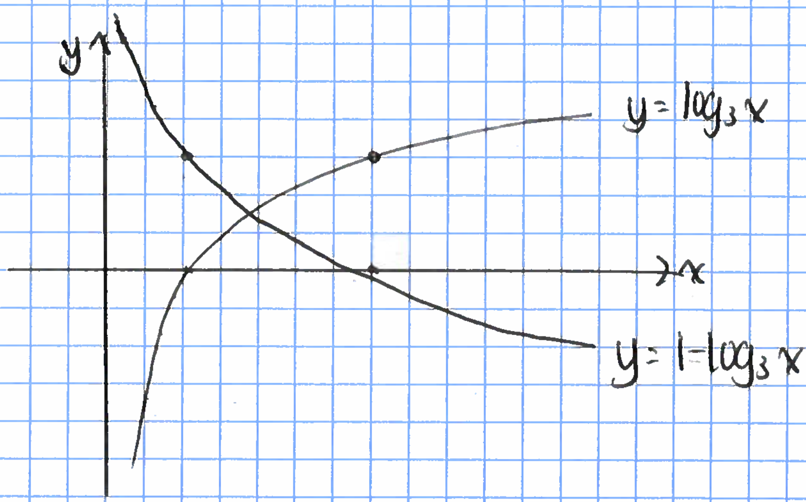
$= -\log_3 x + 1.$

reflection in x-axis

up 1

$(1,0) \to (1,0) \to (1,1)$

$(3,1) \to (3,-1) \to (3,0)$



(b) Point of intersection $\log_3 x = 1 - \log_3 x$

$$2\log_3 x = 1$$

$$\log_3 x = \frac{1}{2}$$

$$x = 3^{\frac{1}{2}}$$

$$\underline{\underline{x = \sqrt{3}}}$$

(12) (a) $2\underline{a} + \underline{b}$

$$= 2(4\underline{i} - 2\underline{j} + 2\underline{k}) + (-2\underline{i} + \underline{j} + p\underline{k})$$

$$= 8\underline{i} - 4\underline{j} + 4\underline{k} - 2\underline{i} + \underline{j} + p\underline{k}$$

$$= 6\underline{i} - 3\underline{j} + (4+p)\underline{k}$$

(b) $|2\underline{a} + b| = 7$

$$\sqrt{6^2 + (-3)^2 + (4+p)^2} = 7$$

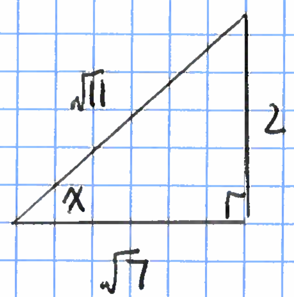
$$36 + 9 + 16 + 8p + p^2 = 49$$

$$p^2 + 8p + 12 = 0$$

$$(p+6)(p+2) = 0$$

$$\underline{\underline{p = -6 \text{ or } p = -2}}$$

(13) (a) (i)



(i) $\sin 2x = 2\sin x \cos x$

$$= 2 \times \frac{2}{\sqrt{11}} \times \frac{\sqrt{7}}{\sqrt{11}}$$

$$= \frac{4\sqrt{7}}{11}$$

(ii) $\cos 2x = \cos^2 x - \sin^2 x$

$$= \frac{7}{11} - \frac{4}{11}$$

$$= \frac{3}{11}$$

(b) $\sin 3x = \sin(2x + x)$

$$= \sin 2x \cos x + \cos 2x \sin x$$

$$= \frac{4\sqrt{7}}{11} \times \frac{\sqrt{7}}{\sqrt{11}} + \frac{3}{11} \times \frac{2}{\sqrt{11}}$$

$$= \frac{28+6}{11\sqrt{11}}$$

$$= \frac{34}{11\sqrt{11}}$$

$$= \underline{\underline{\frac{34\sqrt{11}}{121}}}$$

(14)

$$\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}} \, dx$$

$$= \int_{-4}^{9} (2x+9)^{-\frac{2}{3}} \, dx$$

$$= \left[\frac{(2x+9)^{\frac{1}{3}}}{\frac{1}{3} \times 2} \right]_{-4}^{9}$$

$$= \left[\frac{3}{2} (2x+9)^{\frac{1}{3}} \right]_{-4}^{9}$$

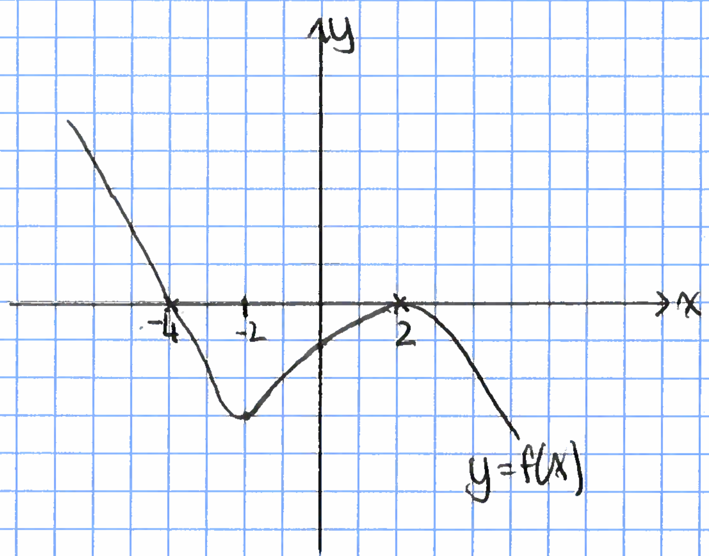
$$= \left(\frac{3}{2} \times 27^{\frac{1}{3}} \right) - \left(\frac{3}{2} \times 1^{\frac{1}{3}} \right)$$

$$= \frac{9}{2} - \frac{3}{2}$$

$$= \frac{6}{2}$$

$$= \underline{\underline{3}}.$$

(15)



$(x+4)$ is a factor
→ cuts x-axis at -4

$x=2$ repeated root
→ turning point at $(2,0)$

$f'(-2)=0$ → stationary point at $x=-2$

graph going up when crosses y-axis