

National  
Qualifications  
2017

**X747/76/12**

**Mathematics  
Paper 2**

FRIDAY, 5 MAY

10:30 AM – 12:00 NOON

**Total marks — 70**

Attempt ALL questions.

**You may use a calculator.**

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer should not be taken as an indication of how much to write. It is not necessary to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space **you must clearly identify the question number** you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



\* X 7 4 7 7 6 1 2 \*

## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

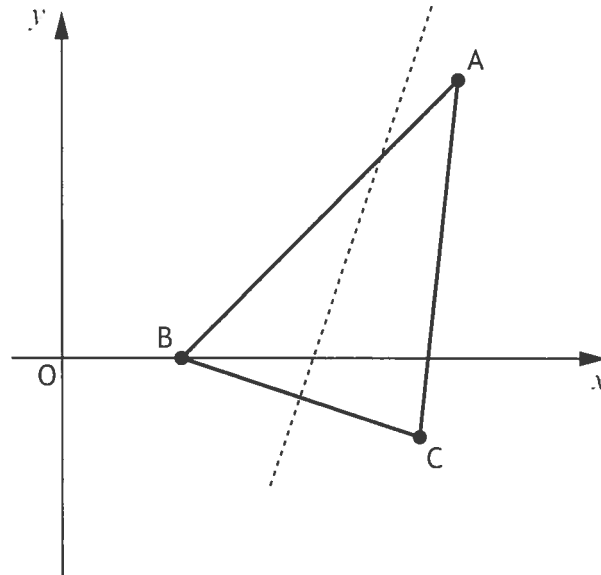
### Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## Attempt ALL questions

Total marks — 70

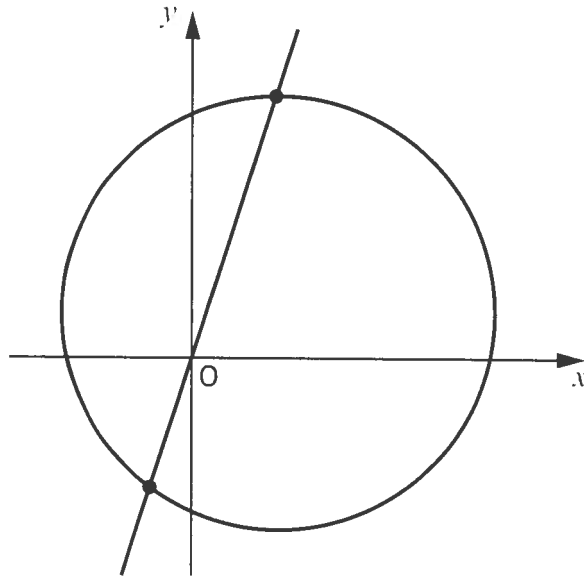
1. Triangle ABC is shown in the diagram below.  
 The coordinates of B are (3,0) and the coordinates of C are (9,-2).  
 The broken line is the perpendicular bisector of BC.



- (a) Find the equation of the perpendicular bisector of BC. 4
- (b) The line AB makes an angle of  $45^\circ$  with the positive direction of the  $x$ -axis.  
 Find the equation of AB. 2
- (c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC. 2
2. (a) Show that  $(x-1)$  is a factor of  $f(x) = 2x^3 - 5x^2 + x + 2$ . 2
- (b) Hence, or otherwise, solve  $f(x) = 0$ . 3

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3. The line  $y=3x$  intersects the circle with equation  $(x-2)^2 + (y-1)^2 = 25$ .

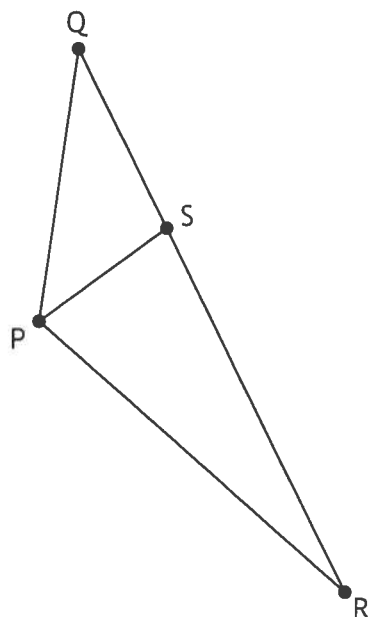


Find the coordinates of the points of intersection.

5

4. (a) Express  $3x^2 + 24x + 50$  in the form  $a(x+b)^2 + c$ . 3
- (b) Given that  $f(x) = x^3 + 12x^2 + 50x - 11$ , find  $f'(x)$ . 2
- (c) Hence, or otherwise, explain why the curve with equation  $y = f(x)$  is strictly increasing for all values of  $x$ . 2

5. In the diagram,  $\vec{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$  and  $\vec{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$ .



- (a) Express  $\vec{PQ}$  in terms of  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ . 2
- The point  $S$  divides  $QR$  in the ratio  $1:2$ .
- (b) Show that  $\vec{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$ . 2
- (c) Hence, find the size of angle  $QPS$ . 5
6. Solve  $5\sin x - 4 = 2\cos 2x$  for  $0 \leq x < 2\pi$ . 5
7. (a) Find the  $x$ -coordinate of the stationary point on the curve with equation  $y = 6x - 2\sqrt{x^3}$ . 4
- (b) Hence, determine the greatest and least values of  $y$  in the interval  $1 \leq x \leq 9$ . 3

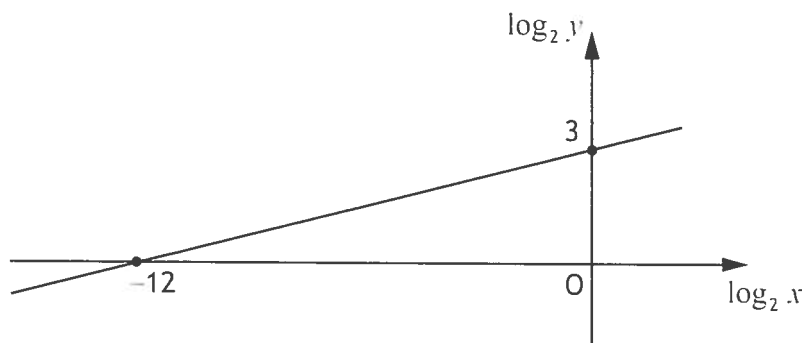
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8. Sequences may be generated by recurrence relations of the form  $u_{n+1} = k u_n - 20$ ,  $u_0 = 5$  where  $k \in \mathbb{R}$ .
- (a) Show that  $u_2 = 5k^2 - 20k - 20$ .
- (b) Determine the range of values of  $k$  for which  $u_2 < u_0$ .

2

4

9. Two variables,  $x$  and  $y$ , are connected by the equation  $y = kx^n$ .  
The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line as shown.

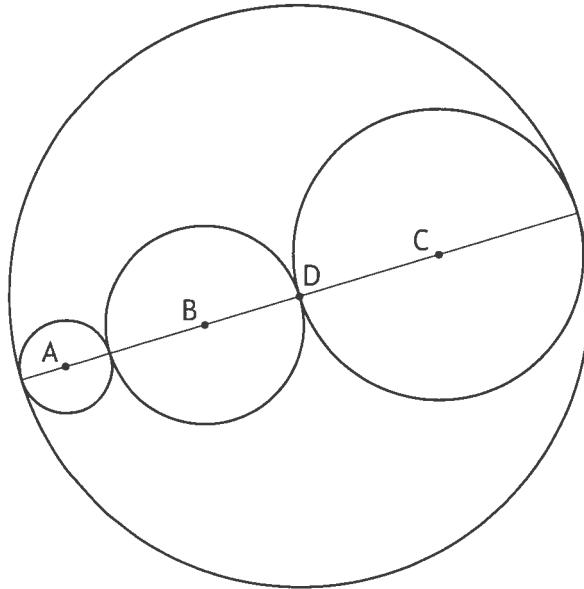


Find the values of  $k$  and  $n$ .

5

10. (a) Show that the points  $A(-7, -2)$ ,  $B(2, 1)$  and  $C(17, 6)$  are collinear.

Three circles with centres  $A$ ,  $B$  and  $C$  are drawn inside a circle with centre  $D$  as shown.



The circles with centres  $A$ ,  $B$  and  $C$  have radii  $r_A$ ,  $r_B$  and  $r_C$  respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

- (b) Determine the equation of the circle with centre  $D$ .

4

11. (a) Show that  $\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x = \sin^3 x$ , where  $0 < x < \frac{\pi}{2}$ .

3

- (b) Hence, differentiate  $\frac{\sin 2x}{2 \cos x} - \sin x \cos^2 x$ , where  $0 < x < \frac{\pi}{2}$ .

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