Higher Maths 2017 Paper 1

① $f(x) = 5x$ $\quad g(x) = 2\cos x$

(a) $f(g(0))$

$= f(2\cos 0)$

$= f(2)$

$= 10$

(b) $g(f(x))$

$= g(5x)$

$= 2\cos 5x$

② $x^2 + y^2 - 8x - 6y - 15 = 0$

$2f = -8$ $\quad 2g = -6$

$f = -4$ $\quad g = -3$ $\quad$ centre $(4, 3)$

$m_{radius} = \frac{3-1}{4-(-2)}$

$= \frac{2}{6}$

$= \frac{1}{3}$ $\quad m_{perp} = -3$

Equation of tangent $\quad y - b = m(x - a)$ $\quad m = -3$, $(a, b) = (-2, 1)$

$y - 1 = -3(x + 2)$

$y - 1 = -3x - 6$

$y = -3x - 5$

③ $y = (4x - 1)^{12}$

$\frac{dy}{dx} = 12(4x - 1)^{11} \times 4$

$= 48(4x - 1)^{11}$

④ $x^2 + 4x + (k - 5) = 0$

$a = 1 \quad b = 4 \quad c = (k - 5)$

Equal roots $\Rightarrow$ $b^2 - 4ac = 0$

$16 - 4 \times 1 \times (k-5) = 0$

$16 - 4k + 20 = 0$

$-4k = -36$

$k = 9$

(5) (a) $\underline{u} \cdot \underline{v} = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 8 \\ 6 \end{pmatrix}$

$= 15 - 8 - 6$

$= 1$

(b) $\underline{u} \cdot \underline{w} = |\underline{u}||\underline{w}| \cos\frac{\pi}{3}$

$= \sqrt{27} \times \sqrt{3} \times \frac{1}{2}$

$= \frac{1}{2}\sqrt{81}$

$= \frac{9}{2}$

$|\underline{u}| = \sqrt{25+1+1}$

$= \sqrt{27}$

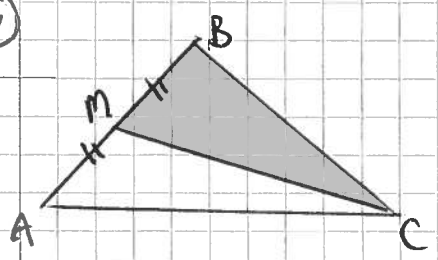
(6) $h(x) = x^3 + 7$.

x → cube → $+7$

$\sqrt[3]{\ }$ ← ← x

$h^{-1}(x) = \sqrt[3]{x-7}$

(7)



midpoint $M = \left(\frac{-3+7}{2}, \frac{5+9}{2}\right)$

$= (2, 7)$

$m_{MC} = \frac{11-7}{2-2}$

$= \frac{4}{0}$

= undefined. vertical line.

equation $x = 2$

(8) $d(t) = \frac{1}{2t}$

$= \frac{1}{2}t^{-1}$

$d'(t) = -\frac{1}{2}t^{-2}$

$= \frac{-1}{2t^2}$

When $t = 5$ $\quad d'(5) = \frac{-1}{2 \times 5^2}$

$= -\frac{1}{50}$.

(9) $u_{n+1} = mu_n + 6$

(a) (i) $u_2 = mu_1 + 6$

$13 = m \times 28 + 6$

$28m = 7$

$m = \frac{1}{4}$

(b) (i) Limit exists since $-1 < \frac{1}{4} < 1$

(ii) $b = \frac{b}{1-a}$

$= \frac{6}{1-\frac{1}{4}}$

$= \frac{6}{\frac{3}{4}}$

$= 6 \times \frac{4}{3}$

$= 8$.

(10) area $= \int_0^2 (\text{upper} - \text{lower})\,dx$

$= \int_0^2 \left((x^3 - 4x^2 + 3x + 1) - (x^2 - 3x + 1)\right) dx$

$= \int_0^2 (x^3 - 5x^2 + 6x)\,dx$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2 \right]_0^2$$

$$= \left(\frac{2^4}{4} - 5 \times \frac{2^3}{3} + 3 \times 2^2 \right) - 0$$

$$= 4 - \frac{40}{3} + 12)$$

$$= 16 - \frac{40}{3}$$

$$= \frac{48}{3} - \frac{40}{3}$$

$$= \frac{8}{3} \quad \text{square units.}$$

(b) area below line $= \int_0^2 ((1-x) - (x^2 - 3x + 1))\,dx$

$$= \int_0^2 (2x - x^2)\,dx$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= \left(4 - \frac{8}{3}\right) - 0$$

$$= \frac{4}{3}.$$

fraction below line $= \frac{4}{3} \div \frac{8}{3}$

$$= \frac{1}{2}.$$

(11) $m_{AB} = \frac{a-2}{5-(-7)}$

$$= \frac{a-2}{12}.$$

$3y - 2x = 4$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$

$m = \frac{2}{3}$

So $\frac{a-2}{12} = \frac{2}{3}$

$a-2 = \frac{24}{3}$

$a-2 = 8$

$a = 10$.

(12) $\log_a 36 - \log_a 4 = \frac{1}{2}$.

$\log_a 9 = \frac{1}{2}$

$9 = a^{\frac{1}{2}}$

$a = 81$

(13) $\int \frac{1}{(5-4x)^{\frac{1}{2}}}$

$= \int (5-4x)^{-\frac{1}{2}} dx$

$= \frac{(5-4x)^{\frac{1}{2}}}{\frac{1}{2} \times (-4)} + C$

$= -\frac{1}{2}(5-4x)^{\frac{1}{2}} + C$.

(14) (a) $\sqrt{3}\sin x - \cos x = k\sin(x-a)$

$= k\sin x \cos a - k\cos x \sin a$

$= k\cos a \sin x - k\sin a \cos x$

$k\cos a = \sqrt{3}$

$k\sin a = 1$

Square and add $k^2 = \sqrt{3}^2 + 1^2$

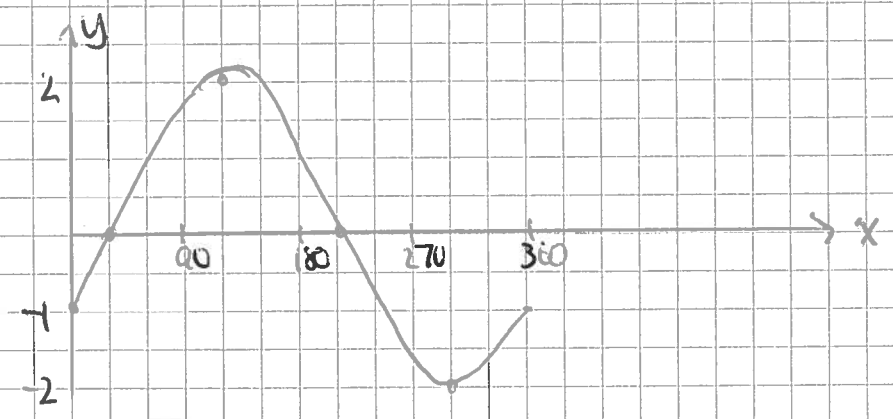
$k^2 = 4$

$k = 2$

Divide $\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30°$

$\sqrt{3}\sin x - \cos x = 2\sin(x - 30°)$



cuts y-axis
$y = 2\sin(-30)$
$= -1$

(15) (a) $h(x) = f(x+a) + b$

↑ shift to right a is negative

↑ up

$a = -5 \quad b = 3.$

(b) $\int_1^3 f(x)\,dx = 4.$

area of rectangle $= 2 \times 3$

$= 6$ square units.

$\int_6^8 h(x)\,dx = 6 + 4$

$= 10$ square units.

(c) $f'(1) = 6$

↑ gradient at $x = 1$ is 6.

$h'(8) = -6$ (by symmetry)