X100/12/11

NATIONAL QUALIFICATIONS 2015

WEDNESDAY, 20 MAY
9.00 AM – 10.30 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

Section B (30 marks)

Full credit will be given only where the solution contains appropriate working.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces in the answer booklet provided. Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SECTION A

ALL questions should be attempted.

1. Given $f(x) = 2x^3 - 7$ find the value of $f'(2)$.

A -6

B 9

C 24

D 137

2. The line with equation $2y = 3x + 5$ is perpendicular to the line with equation $y = kx$.
What is the value of k?

A $-\frac{3}{2}$

B $-\frac{2}{3}$

C $\frac{2}{3}$

D $\frac{3}{2}$

3. If $2x^3 + x^2 - 4x + 1$ is divided by $(x - 2)$, what is the remainder?

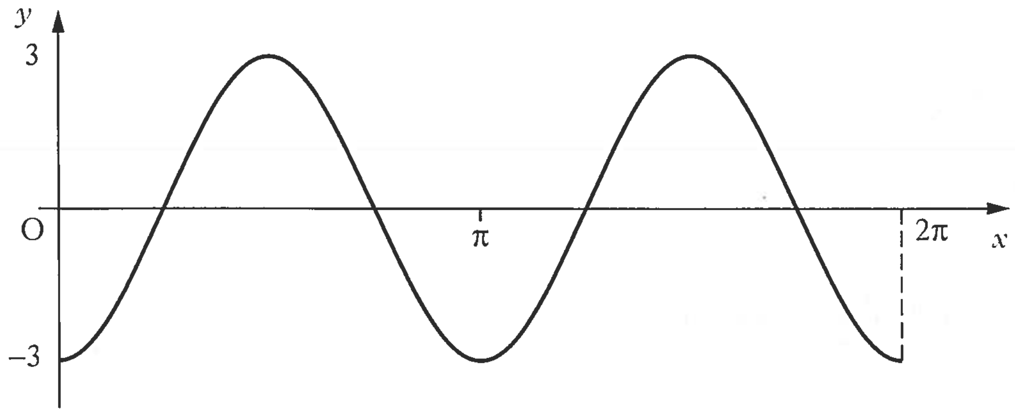
A -11

B 0

C 1

D 13

4. The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \leq x \leq 2\pi$.



What is the equation of this graph?

A $y = -3\cos 2x$

B $y = -3\cos 3x$

C $y = 3\cos 2x$

D $y = 3\cos 3x$

5. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}2u_n + 9$, $u_5 = 11$.

What is the value of u_3?

A 11·24

B 9·4

C 5

D 4

6. The points P, Q and R are collinear.

P is the point (−1, 6, 4), Q is the point (2, 0, 13) and $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$.

Calculate the ratio in which Q divides PR.

A 2 : 3

B 3 : 2

C 3 : 5

D 5 : 2

7. What is $\int (x+4)(x-4)\,dx$?

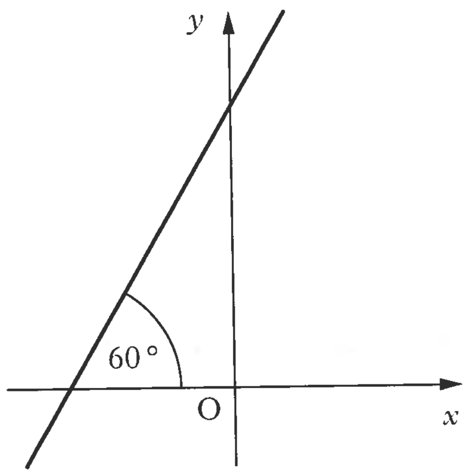
A $2x + c$

B $\frac{1}{3}x^3 + c$

C $\frac{1}{3}x^3 - 16x + c$

D $\left(\frac{1}{2}x^2 + 4x\right)\left(\frac{1}{2}x^2 - 4x\right) + c$

8. A straight line makes an angle of $60°$ with the x-axis as shown in the diagram.



What is the gradient of this line?

A $\frac{1}{2}$

B $\frac{1}{\sqrt{3}}$

C $\frac{\sqrt{3}}{2}$

D $\sqrt{3}$

9. Find the minimum value of $3\sin 2x + 5$ and the value of x where this occurs in the interval $0 \leq x < \pi$.

	min value	x
A	2	$\frac{3\pi}{2}$
B	2	$\frac{3\pi}{4}$
C	4	$\frac{3\pi}{2}$
D	4	$\frac{3\pi}{4}$

10. Solve $2\cos x + 1 = 0$ for x, where $\pi \leq x \leq \frac{3\pi}{2}$.

A $\frac{5\pi}{6}$

B $\frac{7\pi}{6}$

C $\frac{5\pi}{4}$

D $\frac{4\pi}{3}$

11. The curve $y = f(x)$ is such that $f'(x) = 4x - 1$.

The curve passes through the point (2, 9).

What is the equation of the curve?

A $y = 2x^2 - x - 5$

B $y = 2x^2 + 1$

C $y = 2x^2 - x + 3$

D $y = 2x^2 - x$

[Turn over

12. Given that the point R is $(3, -1, 2)$, $\overrightarrow{RS} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\overrightarrow{RT} = 3\overrightarrow{RS}$,

find the coordinates of T.

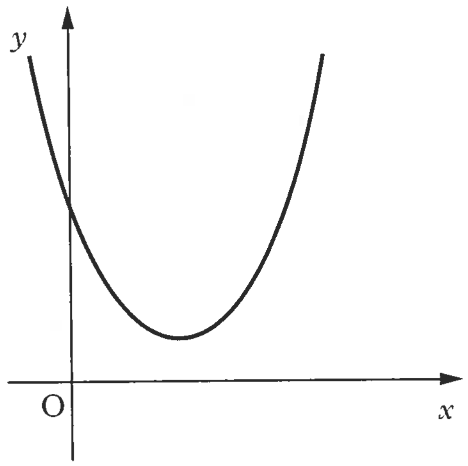
A $(3, 2, -11)$

B $(3, 4, -11)$

C $(9, 2, -7)$

D $(9, 4, -7)$

13. The diagram shows a curve with equation of the form $y = ax^2 + bx + c$.



Here are two statements about a, b and c:

(1) $a > 0$

(2) $b^2 - 4ac > 0$

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

14. If $\cos x = -\frac{2}{5}$, what is the value of $\cos 2x$?

A $\frac{33}{25}$

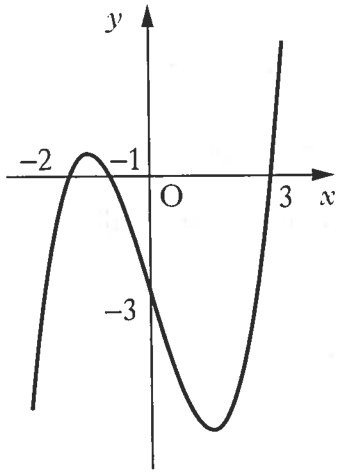
B $\frac{17}{25}$

C $-\frac{4}{5}$

D $-\frac{17}{25}$

15. The graph of a cubic function, $y = f(x)$, is shown below.

It passes through the points $(-2, 0)$, $(-1, 0)$, $(3, 0)$ and $(0, -3)$.



What is the equation of this curve?

A $y = \frac{1}{2}(x - 3)(x + 1)(x + 2)$

B $y = 2(x - 3)(x + 1)(x + 2)$

C $y = -\frac{1}{2}(x + 3)(x - 1)(x - 2)$

D $y = -2(x + 3)(x - 1)(x - 2)$

[Turn over

16. If $e^{4t} = 6$, find an expression for t.

A $t = \log_e \frac{3}{2}$

B $t = \frac{\log_e 6}{4}$

C $t = \frac{6}{\log_e 4}$

D $t = \frac{\log_e 6}{\log_e 4}$

17. Vectors **u** and **v** have components $\begin{pmatrix} \frac{3}{5} \\ 0 \\ t \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 0 \\ -10 \end{pmatrix}$ respectively.

Here are two statements about **u** and **v**:

(1) when $t = \frac{4}{5}$, **u** is a unit vector

(2) when $t = 1$, **u** and **v** are parallel

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

18. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k?

A 5

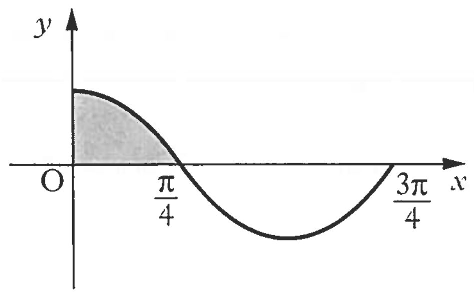
B 6

C 25

D 36

19. The diagram shows part of the graph of $y = a\cos bx$.

The shaded area is $\frac{1}{2}$ unit^2.



What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$?

A -1

B $-\frac{1}{2}$

C $\frac{1}{2}$

D $1\frac{1}{2}$

20. The only stationary point on the graph of $y = f(x)$ is the point (a, b).

What are the coordinates of the only stationary point on the graph of $y = -f(2x)$?

A $\left(\frac{1}{2}a, -b\right)$

B $(2a, -b)$

C $\left(-\frac{1}{2}a, b\right)$

D $(-2a, b)$

[*END OF SECTION A*]

[Turn over

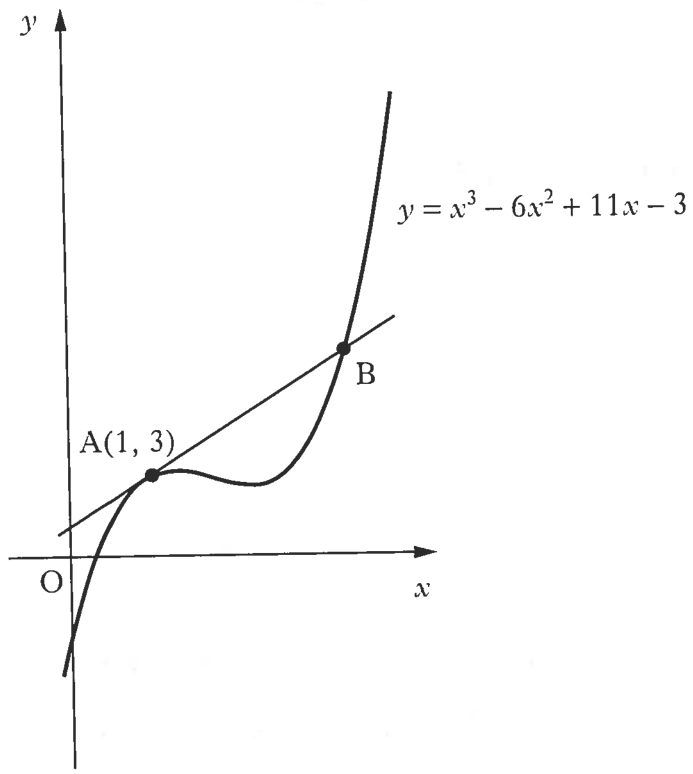
SECTION B

Marks

ALL questions should be attempted.

21. (*a*) Show that $(x-1)$ is a factor of $x^3 - 6x^2 + 9x - 4$ and hence factorise $x^3 - 6x^2 + 9x - 4$ fully. **4**

(*b*) The diagram shows the graph with equation $y = x^3 - 6x^2 + 11x - 3$.



(i) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 11x - 3$ at the point A(1, 3). **3**

(ii) Hence find the coordinates of B, the point of intersection of this tangent with the curve. **3**

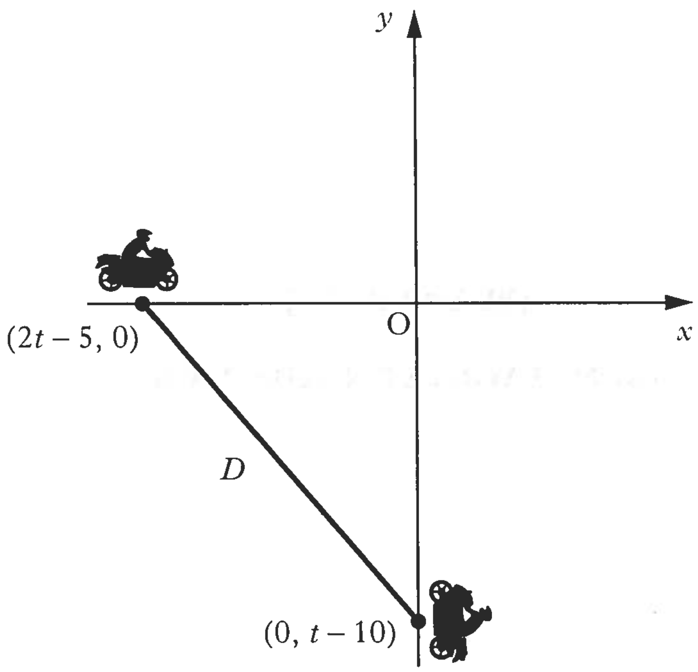
22. The function $f(x) = \frac{4}{x^2} + x$ is defined on the domain $x > 0$, $x \in \mathbb{R}$, the set of real numbers.

Find the maximum and minimum values of $f(x)$ on the closed interval $1 \leq x \leq 4$. **6**

23. Solve $\log_2(3x + 7) = 3 + \log_2(x - 1)$, $x > 1$. **4**

24. Find the range of values for k such that $kx^2 + 3x + 9k = 0$ has real roots. **4**

25. A stunt performed by two members of a motorcycle display team requires them to travel, at speed, at right angles to each other across the arena.



The positions of the motorcyclists, relative to suitable axes, t seconds after the stunt begins, are $(2t - 5, 0)$ and $(0, t - 10)$.

(*a*) Show that, at any given moment, the distance, D, between them is given by

$$D = \sqrt{5t^2 - 40t + 125}.$$ **2**

(*b*) Determine whether the distance between the motorcyclists is increasing or decreasing 5 seconds after the start of the stunt. **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

Higher Maths 2015 Paper 1

(1) $f(x) = 2x^3 - 7$

$f'(x) = 6x^2$

$f'(2) = 6 \times 2^2$

$= 24$ (C)

(2) $2y = 3x + 5$

$y = \frac{3}{2}x + \frac{5}{2}$ $m = \frac{3}{2}$ $m_{perp} = -\frac{2}{3}$

$k = -\frac{2}{3}$ (B)

(3) $2x^3 + x^2 - 4x + 1$

2	2	1	−4	1
		4	10	12
	2	5	6	13

(D)

(4) $y = -3\cos 2x$ (A)

(5) $u_{n+1} = 0.2u_n + 9$

$u_5 = 0.2u_4 + 9$

$11 = 0.2u_4 + 9$

$2 = 0.2u_4$

$u_4 = 10$

$u_4 = 0.2u_3 + 9$

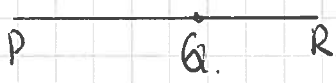
$10 = 0.2u_3 + 9$

$1 = 0.2u_3$

$u_3 = 5.$

(C)

(6)



$\vec{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$

$= \begin{pmatrix} 2 \\ 0 \\ 13 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$\vec{QR} = 2\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

so $PQ : QR$

$3 : 2$ (B)

⑦ $\int (x+4)(x-4)\,dx$

$= \int (x^2 - 16)\,dx$

$= \frac{x^3}{3} - 16x + c.$ (C)

⑧ $m = \tan 60$

$= \sqrt{3}$ (D)

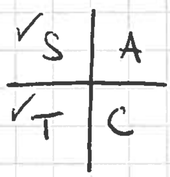
⑨ min value $-3+5$
$= 2$

when $2x = \frac{3\pi}{2}$

$x = \frac{3\pi}{4}.$ (B).

⑩ $2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$



ra $\frac{\pi}{3}$

$x = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3},\ \frac{4\pi}{3}$ (D)

⑪ $f'(x) = 4x - 1$

$f(x) = \int (4x-1)\,dx$

$y = 2x^2 - x + c.$

Point $(2,9)$

$9 = 2{\times}4 - 2 + c$

$c = 3.$ (C)

$y = 2x^2 - x + 3$

(12) $\vec{RT} = 3\vec{RS}$

$\underline{t} - \underline{r} = 3\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

$\underline{t} - \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ -9 \end{pmatrix}$

$\underline{t} = \begin{pmatrix} 9 \\ 2 \\ -7 \end{pmatrix}$ T(9, 2, -7) (C)

(13) $y = ax^2 + bx + c$ $a > 0$ ∪ ✓

$b^2 - 4ac > 0$ two real roots. ✗

(B)

(14) $\cos x = -\frac{2}{5}$

$\cos 2x = 2\cos^2 x - 1$

$= 2\left(-\frac{2}{5}\right)^2 - 1$

$= 2 \times \frac{4}{25} - 1$

$= -\frac{17}{25}$ (D)

(15) $y = k(x+2)(x+1)(x-3)$

Point $(0, -3)$ $-3 = k(2)(1)(-3)$

$-3 = k(-6)$

$k = \frac{1}{2}$ (A).

(16)

$e^{4t} = 6$

$4t = \ln 6$

$t = \frac{1}{4}\ln 6$

(B)

(17)

$t = \frac{4}{5}$

$|u| = \sqrt{\left(\frac{3}{5}\right)^2 + 0^2 + \left(\frac{4}{5}\right)^2}$

$= \sqrt{\frac{9}{25} + \frac{16}{25}}$

$= 1$

(1) ✓

$t = 1$

$\underline{u} = \begin{pmatrix} \frac{3}{5} \\ 0 \\ 1 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} -6 \\ 0 \\ -10 \end{pmatrix} = -\frac{1}{10}\begin{pmatrix} \frac{6}{10} \\ 0 \\ 1 \end{pmatrix}$

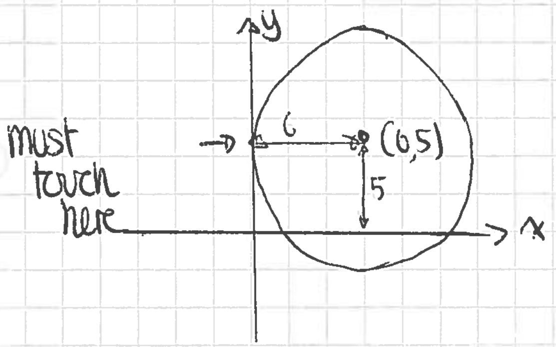
$= -\frac{1}{10}\begin{pmatrix} \frac{3}{5} \\ 0 \\ 1 \end{pmatrix}$

so $\underline{u}$ and $\underline{v}$ are parallel (2) ✓

(D)

(18)

centre $(6,5)$



ie radius $= 6$.

$\sqrt{6^2 + 5^2 - k} = 6$

$36 + 25 - k = 36$

$k = 25$

(C)

(19)

$\frac{1}{2} - 2 \times \frac{1}{2} = -\frac{1}{2}$

(B)

(20)

$y = -f(2x)$

half x co-ord

mai change sign of y coord

$\left(\frac{1}{2}a, -b\right)$

(A)

(21)

Synthetic division by 1:

1	1	−6	9	−4
		1	−5	4
	1	−5	4	0

remainder 0
so $(x-1)$ is a factor

$$x^3-6x^2+9x-4 = (x-1)(x^2-5x+4)$$
$$= (x-1)(x-4)(x-1)$$
$$= (x-1)^2(x-4)$$

(b) (i) $y = x^3-6x^2+11x-3$

$$\frac{dy}{dx} = 3x^2-12x+11$$

When $x=1$ $\frac{dy}{dx} = 3-12+11$
$= 2$

Equation
$$y-b = m(x-a)$$
$$y-3 = 2(x-1)$$
$$y-3 = 2x-2$$
$$y = 2x+1$$

(ii) Solve $y = 2x+1$
$y = x^3-6x^2+11x-3$

$$x^3-6x^2+11x-3 = 2x+1$$
$$x^3-6x^2+9x-4 = 0$$
$$(x-1)^2(x-4) = 0$$
$$x=1 \text{ or } x=4$$

When $x=4$ $y = 8+1$
$= 9$

B(4, 9)

(22)

$f(x) = \frac{4}{x^2} + x$

$= 4x^{-2} + x$

$f'(x) = -8x^{-3} + 1$

$= -\frac{8}{x^3} + 1$

For max/min $f'(x) = 0$

$-\frac{8}{x^3} + 1 = 0$

$\frac{8}{x^3} = 1$

$x^3 = 8$

$x = 2$ $y = 3$

Nature

x	$\rightarrow$	2	$\rightarrow$
$f'(x) = -\frac{8}{x^3} + 1$	$-$	0	$+$
	\	—	/

(2,3) is a minimum T.P.

End points $x = 1$ $f(1) = 4 + 1$ $= 5$

$x = 4$ $f(4) = 1 + 4$ $= 5$

minimum value 3 maximum value 5

(23)

$\log_2(3x+7) = 3 + \log_2(x-1)$

$\log_2(3x+7) - \log_2(x-1) = 3$

$\log_2 \frac{3x+7}{x-1} = 3$

$$\frac{3x+7}{x-1} = 2^3$$

$$3x+7 = 8x-8$$

$$-5x = -15$$

$$x = 3$$

(24) $kx^2 + 3x + 9k = 0$

$a = k \quad b = 3 \quad c = 9k.$

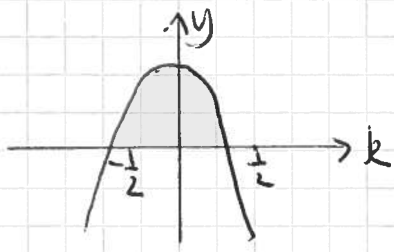
Real roots $\quad b^2 - 4ac \geq 0$

$$9 - 4 \times k \times 9k \geq 0$$

$$9 - 36k^2 \geq 0$$

Graph $\quad y = 9 - 36k^2$

$$= 9(1-2k)(1+2k)$$



$$-\frac{1}{2} \leq k \leq \frac{1}{2}$$

(25) $(2t-5, 0) \qquad (0, t-10).$

(a) $D = \sqrt{(2t-5-0)^2 + (0-(t-10))^2}$

$$= \sqrt{4t^2 - 20t + 25 + t^2 - 20t + 100}$$

$$= \sqrt{5t^2 - 40t + 125}$$ as required.

(b) $\frac{dD}{dt} = \frac{1}{2}(5t^2 - 40t + 125)^{-\frac{1}{2}} \cdot (10t - 40)$

$$= \frac{10t - 40}{2\sqrt{5t^2 - 40t + 125}}$$

When $t = 5$

$$\frac{dD}{dt} = \frac{50-40}{2\sqrt{125-200+125}}$$

$$= \frac{10}{2\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{5}{5\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}$$

$$> 0$$

so the distance is increasing