X100/12/02

NATIONAL QUALIFICATIONS 2014

TUESDAY, 6 MAY
1.00 PM – 2.30 PM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.
What is the value of u_4?

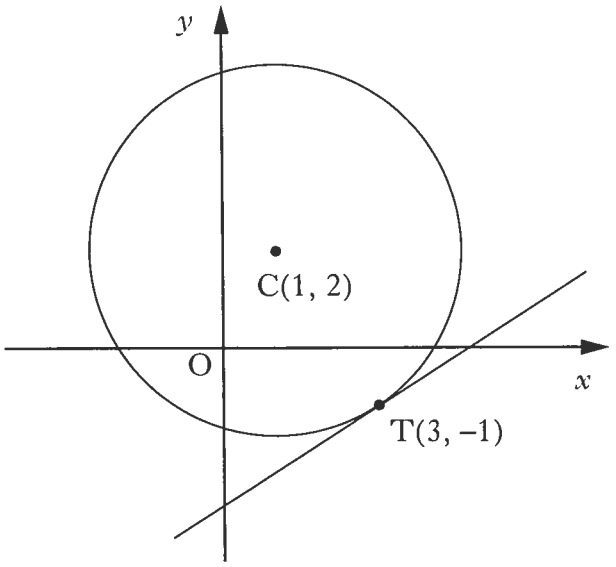
A $2\frac{1}{9}$

B $2\frac{1}{3}$

C 3

D 30

2. The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).



What is the gradient of this tangent?

A $\frac{1}{4}$

B $\frac{2}{3}$

C $\frac{3}{2}$

D 4

3. If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?

A 2

B 6

C 18

D 72

4. If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?

	$k\cos a$	$k\sin a$
A	-3	4
B	3	-4
C	4	-3
D	-4	3

5. Find $\int (2x+9)^5 \, dx$.

A $10\,(2x + 9)^4 + c$

B $\frac{1}{4}(2x + 9)^4 + c$

C $10\,(2x + 9)^6 + c$

D $\frac{1}{12}(2x + 9)^6 + c$

[Turn over

6. Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form.

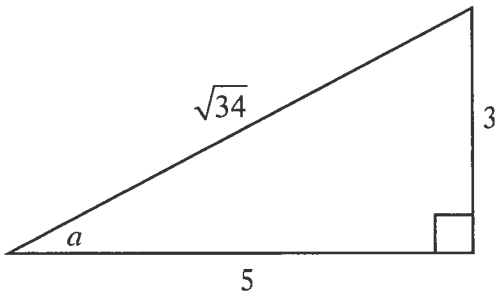
A $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

B $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$

C $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$

D $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$

7. A right-angled triangle has sides and angles as shown in the diagram.



What is the value of $\sin 2a$?

A $\frac{8}{17}$

B $\frac{3}{\sqrt{34}}$

C $\frac{15}{17}$

D $\frac{6}{\sqrt{34}}$

8. What is the derivative of $\left(4-9x^4\right)^{\frac{1}{2}}$?

A $-\frac{9}{2}\left(4-9x^4\right)^{-\frac{1}{2}}$

B $\frac{1}{2}\left(4-9x^{-4}\right)^{-\frac{1}{2}}$

C $2\left(4-9x^4\right)^{-\frac{1}{2}}$

D $-18x^3\left(4-9x^4\right)^{-\frac{1}{2}}$

9. $\sin x+\sqrt{3}\cos x$ can be written as $2\cos\left(x-\frac{\pi}{6}\right)$.

The maximum value of $\sin x+\sqrt{3}\cos x$ is 2.

What is the maximum value of $5\sin 2x+5\sqrt{3}\cos 2x$?

A 20

B 10

C 5

D 2

10. A sequence is defined by the recurrence relation

$u_{n+1}=(k-2)u_n+5$ with $u_0=3$.

For what values of k does this sequence have a limit as $n\to\infty$?

A $-3<k<-1$

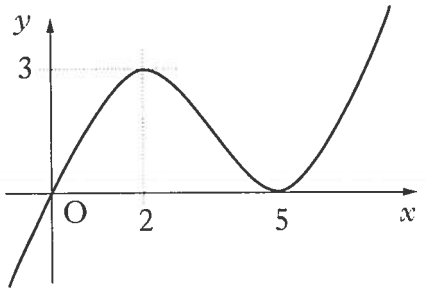
B $-1<k<1$

C $1<k<3$

D $k<3$

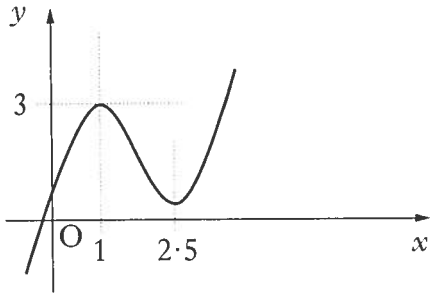
[Turn over

11. The diagram shows part of the graph of $y = f(x)$.



Which of the following diagrams could be the graph of $y = 2f(x) + 1$?

A



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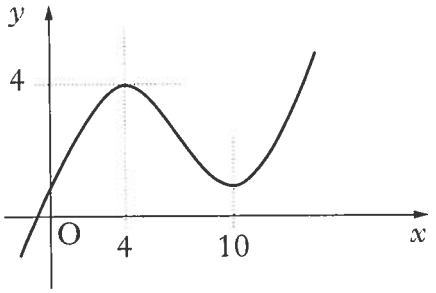

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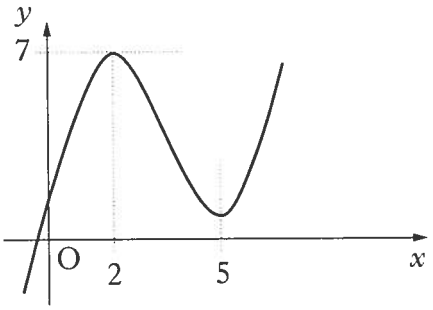
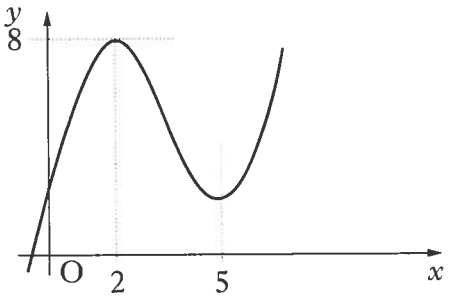


D



12. A function f, defined on a suitable domain, is given by $f(x) = \dfrac{6x}{x^2 + 6x - 16}$.

What restrictions are there on the domain of f?

A $x \neq -8$ or $x \neq 2$

B $x \neq -4$ or $x \neq 4$

C $x \neq 0$

D $x \neq 10$ or $x \neq 16$

13. What is the value of $\sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{4}\right)$?

A $\dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}$

B $\dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}}$

C $\dfrac{1}{2} - \dfrac{1}{\sqrt{2}}$

D $\dfrac{1}{2} + \dfrac{1}{\sqrt{2}}$

14. The vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular.

What is the value of k?

A $\dfrac{-6}{7}$

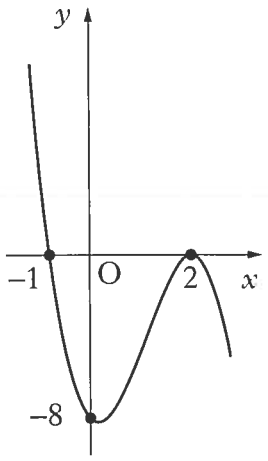
B -1

C 1

D $\dfrac{6}{7}$

[Turn over

15. The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).



What is the equation of the curve?

A $y = -2(x+1)^2(x+2)$

B $y = -2(x+1)(x-2)^2$

C $y = 4(x+1)(x-2)$

D $y = -8(x+1)(x-2)^2$

16. The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$.

A $\frac{2}{3}$

B $\frac{4}{3}$

C $\frac{7}{3}$

D 3

17. $3x^2 + 12x + 17$ is expressed in the form $3(x+p)^2 + q$.

What is the value of q?

A 1

B 5

C 17

D –19

18. What is the value of $1 - 2\sin^2 15°$?

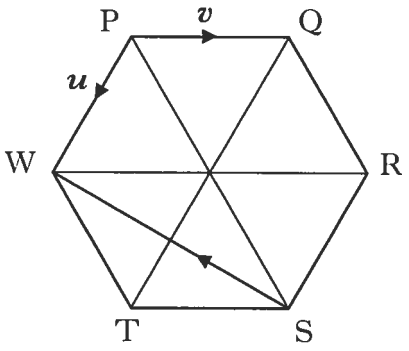
A $\frac{1}{2}$

B $\frac{3}{4}$

C $\frac{\sqrt{3}}{2}$

D $\frac{7}{8}$

19. The diagram shows a regular hexagon PQRSTW.
$\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.



What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?

A $-\boldsymbol{u} - 2\boldsymbol{v}$

B $-\boldsymbol{u} - \boldsymbol{v}$

C $\boldsymbol{u} - \boldsymbol{v}$

D $\boldsymbol{u} + 2\boldsymbol{v}$

20. Evaluate $2 - \log_5 \frac{1}{25}$.

A -3

B 0

C $\frac{3}{2}$

D 4

[END OF SECTION A]

SECTION B

Marks

ALL questions should be attempted.

21. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

23. (*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

(*c*) A second circle C_2 passes through P, Q and T.

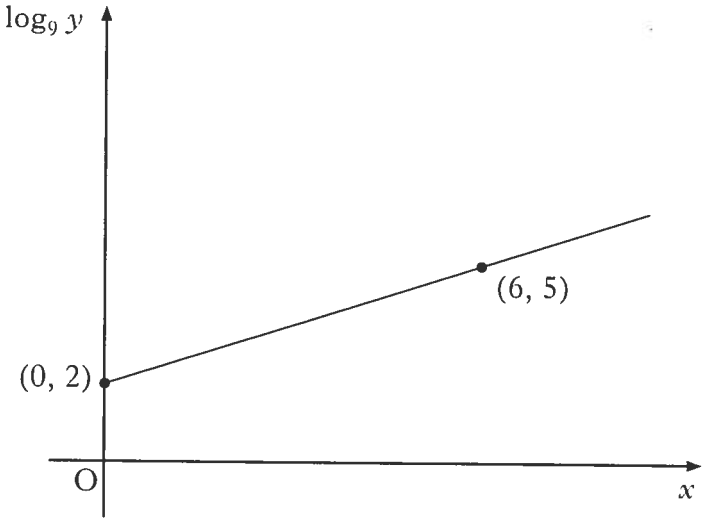
Find the equation of C_2. **3**

24. Two variables, x and y, are related by the equation

Marks

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a. **5**

[END OF SECTION B]

[END OF QUESTION PAPER]

Higher Maths 2014 Paper 1

① $u_{n+1} = \frac{1}{3}u_n + 1$

$u_3 = \frac{1}{3} \times 15 + 1$

$= 6$

$u_4 = \frac{1}{3} \times 6 + 1$

$= 3.$

(C)

② $m_{CT} = \frac{-1-2}{3-1}$

$= \frac{-3}{2}$

$m_{tangent} = \frac{2}{3}$ (B)

③ $\log_4 12 - \log_4 x = \log_4 6$

$\log_4 \frac{12}{x} = \log_4 6$

$\frac{12}{x} = 6$

$x = 2.$

(A)

④ $3\sin x - 4\cos x = k\cos(x-\alpha)$

$= k\cos x \cos\alpha + k\sin x \sin\alpha.$

$k\cos\alpha = -4$

$k\sin\alpha = 3.$

(D)

⑤ $\int (2x+9)^5 \, dx$

$= \frac{(2x+9)^6}{6 \times 2} + C$

$= \frac{1}{12}(2x+9)^6 + C.$

(D)

⑥ $2\underline{u} - 3\underline{v} = \begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

(A)

(7) $\sin 2a = 2\sin a \cos a$

$= 2 \cdot \frac{3}{\sqrt{34}} \cdot \frac{5}{\sqrt{34}}$

$= \frac{30}{34}$

$= \frac{15}{17}$ (C)

(8) $y = (4 - 9x^4)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(4 - 9x^4)^{-\frac{1}{2}} \cdot -36x^3$

$= -18x^3 (4 - 9x^4)^{-\frac{1}{2}}$. (D)

(9) $5\sin 2x + 5\sqrt{3}\cos 2x$

$= 5(\sin 2x + \sqrt{3}\cos 2x)$

$= 5\left(2\cos\left(2x - \frac{\pi}{6}\right)\right)$ (B)

max 10.

(10) $u_{n+1} = (k-2)u_n + 5$

Limit exists if $-1 < k-2 < 1$

$1 < k < 3$ (C)

(11) $y = 2f(x) + 1$

$(0,0) \rightarrow (0,1)$

$(2,3) \rightarrow (2,7)$

(C)

(12) $x^2+6x-16 \neq 0$

$(x+8)(x-2) \neq 0$

$x \neq -8$, $x \neq 2$. (A)

(13) $\sin\frac{\pi}{3} - \cos\frac{5\pi}{4}$

$= \frac{\sqrt{3}}{2} + \cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}$. (B)

(14) $\underline{u} \cdot \underline{v} = 0$

$\begin{pmatrix} 1 \\ k \\ k \end{pmatrix} \cdot \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix} = 0$

$-6 + 2k + 5k = 0$

$7k = 6$

$k = \frac{6}{7}$ (D)

(15) roots $x=-1$ $x=2$ $x=2$

$y = k(x+1)(x-2)^2$

Point $(0,-8)$ $-8 = k(1)(-2)^2$

$k = -2$.

$y = -2(x+1)(x-2)^2$ (B)

(16) $\underline{a} \cdot (\underline{a} + 2\underline{b}) = \underline{a} \cdot \underline{a} + 2\underline{a} \cdot \underline{b}$

$= |\underline{a}|^2 + 2 \times \frac{2}{3}$

$= 1 + \frac{4}{3}$

$= \frac{7}{3}$. (C)

(17) $3x^2+12x+17$

$= 3(x^2+6x)+17$

$= 3[(x+2)^2-2^2]+17$

$= 3(x+2)^2-12+17$

$= 3(x+2)^2+5$

(B)

(18) $1-2\sin^2 15$

$= \cos 30$

$= \frac{\sqrt{3}}{2}$

(C)

(19) $\overrightarrow{SW} = \overrightarrow{ST} + \overrightarrow{TO} + \overrightarrow{OW}$

$= -\underline{v} - \underline{u} - \underline{v}$

$= -\underline{u} - 2\underline{v}$

(A)

(20) $2-\log_5 \frac{1}{25}$

$= 2-\log_5 \frac{1}{5^2}$

$= 2-\log_5 5^{-2}$

$= 2+2\log_5 5$

$= 4.$

(D).

(21) (a) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

For stationary points $\frac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0 \quad x = 2.$

$y = 0 \quad y = 12 - 8 = 4$

points $(0,0) \quad (2,4)$

Nature.

x	(-1) →	0	(1) →	2	3 →
$\frac{dy}{dx} = 3x(2-x)$	−	0	+	0	−
	\	—	/	—	\

$(0,0)$ is a minimum TP

$(2,4)$ is a maximum TP.

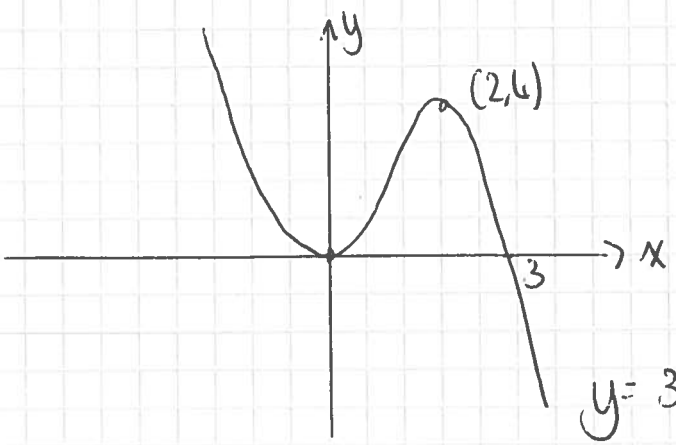
(b) cuts x-axis $y = 0$

$3x^2 - x^3 = 0$

$x^2(3 - x) = 0$

$x = 0 \quad x = 3 \qquad (0,0) \quad (3,0)$

cuts y-axis $x = 0 \quad y = 0 \qquad (0,0)$



(22) $6x^3 + 7x^2 + ax + b$

(a) $x+1$ factor

-1	6	7	a	b
		-6	-1	$-a+1$
	6	1	$a-1$	$b-a+1$

so $b - a + 1 = 0$ ... (1)

division by $x-2$

2	6	7	a	b
		12	38	$2a+76$
	6	19	$a+38$	$b+2a+76$

$b + 2a + 76 = 72$

$b + 2a + 4 = 0$ ... (2)

(2) − (1) gives $3a + 3 = 0$

$a = -1$

in (1) $b + 1 + 1 = 0$

$b = -2$

So $\underline{a = -1 \text{ and } b = -2}$

Using first table.

$6x^3 + 7x^2 - x - 2 = (x+1)(6x^2 + x - 2)$

$= (x+1)(3x+2)(2x-1)$

(23) (a) $x^2 + y^2 + 2x - 4y - 15 = 0$ ... ①

$y = 3x - 5$ ... ②

Substitute ② in ①.

$x^2 + (3x-5)^2 + 2x - 4(3x-5) - 15 = 0$

$x^2 + 9x^2 - 30x + 25 + 2x - 12x + 20 - 15 = 0$

$10x^2 - 40x + 30 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ or $x = 1$

$y = 4$ $\quad$ $y = -2$

$\underline{P(1,-2) \quad Q(3,4)}$

(b) $T = (-1, 2)$

$m_{PT} = \frac{-2-2}{1-(-1)}$

$= -2$

$m_{QT} = \frac{4-2}{3-(-1)}$

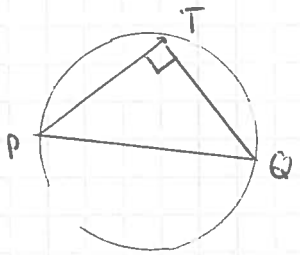
$= \frac{2}{4}$

$= \frac{1}{2}$

$m_{PT} \times m_{QT} = -2 \times \frac{1}{2}$

$= -1$ so PT and QT are perpendicular

(c)



PQ is diameter

midpt = (2, 1) = centre.

$PQ = \sqrt{(3-1)^2 + (4-(-2))^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$

radius = $\sqrt{10}$

equation $(x-2)^2 + (y-1)^2 = 10$.

(24) Straight line.

$$\log_q y = mx + c.$$

$$m = \frac{5-2}{6-0} = \frac{1}{2} \qquad c = 2$$

$$\log_q y = \frac{1}{2}x + 2$$

$$y = q^{(\frac{1}{2}x+2)}$$

$$y = (q^{\frac{1}{2}})^x \cdot q^2$$

$$y = 81(3^x)$$

$R = 81$ and $a = 3$.