X100/12/03

NATIONAL QUALIFICATIONS 2013

WEDNESDAY, 22 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

ALL questions should be attempted. *Marks*

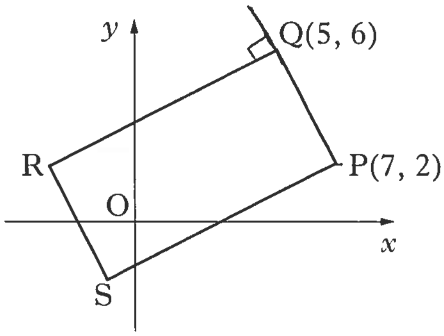
1. The first three terms of a sequence are 4, 7 and 16.

   The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

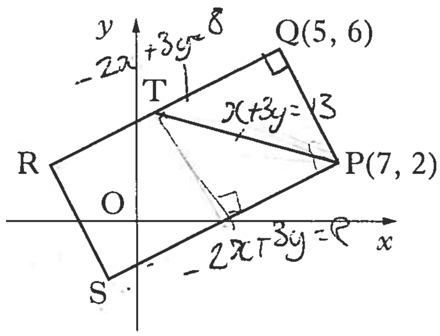
   Find the values of m and c. **4**

2. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



   (*a*) Find the equation of QR. **3**

   (*b*) The line from P with the equation $x + 3y = 13$ intersects QR at T.



   Find the coordinates of T. **3**

   (*c*) Given that T is the midpoint of QR, find the coordinates of R and S. **3**

[Turn over

Marks

3. (a) Given that $(x-1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. 4

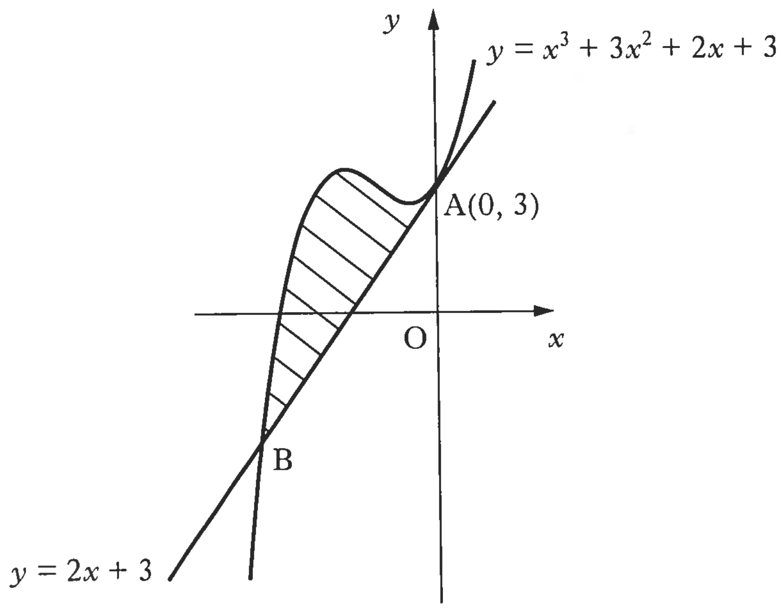
(b) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. 5

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

Show that B is the point (–3, –3) and find the area enclosed by the line and the curve. 6

5. Solve the equation

$$\log_5(3-2x) + \log_5(2+x) = 1, \text{ where } x \text{ is a real number.}$$ 4

Marks

6. Given that $\int_0^a 5\sin 3x\,dx = \frac{10}{3}$, $0 \le a < \pi$,

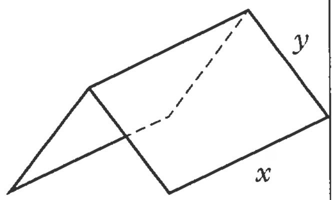
calculate the value of a. **5**

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is $24\,\text{m}^2$.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$ **3**

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame. **7**

8. Solve algebraically the equation

$$\sin 2x = 2\cos^2 x \qquad \text{for } 0 \le x < 2\pi$$ **6**

[Turn over for Question 9 on *Page six*

9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation *Marks*

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*? **3**

[*END OF QUESTION PAPER*]

Higher Paper 2 2013

① $u_0 = 4$ $u_1 = 7$ $u_2 = 16$

$u_1 = mu_0 + c$

$7 = 4m + c$ ... ①

$u_2 = mu_1 + c$

$16 = 7m + c$ ... ②

② − ① $9 = 3m$

$m = 3.$

in ② $7 = 12 + c$

$c = -5$

② (a) $m_{PQ} = \frac{2-6}{7-5}$

$= \frac{-4}{2}$

$= -2$

$m_{RQ} = \frac{1}{2}.$

$y - b = m(x - a)$

$y - 6 = \frac{1}{2}(x - 5)$

$2y - 12 = x - 5$

$2y = x + 7.$

(b) Solve. $x + 3y = 13.$

$x - 2y = -7$

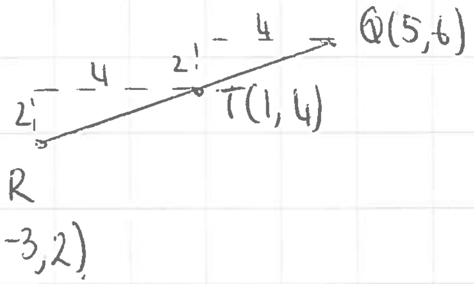
Subtract $5y = 20$

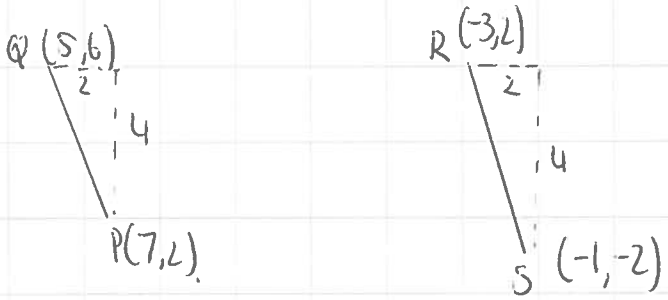
$y = 4 \Rightarrow x + 12 = 13$

$x = 1$

$T(1, 4)$

(c)





$R(-3, 2)$

$S(-1, -2)$

③ (a)

1	1	3	1	−5
		1	4	5
	1	4	5	0

remainder 0

so $(x-1)$ is a factor

$x^3 + 3x^2 + x - 5 = (x-1)(x^2 + 4x + 5)$

← doesn't factorise

$b^2 - 4ac = 16 - 4 \times 5$

$= -4$

(b) $y = x^4 + 4x^3 + 2x^2 - 20x + 3$

$\frac{dy}{dx} = 4x^3 + 12x^2 + 4x - 20$

$= 4(x^3 + 3x^2 + x - 5)$

$$= 4(x-1)(x^2+6x+5)$$

For stationary points $\frac{dy}{dx} = 0$

$$4(x-1)(x^2+6x+5) = 0$$

$$x = 1$$

only one stationary point.

<u>Nature</u>

x	$0 \rightarrow$	1	$2 \rightarrow$
$\frac{dy}{dx} = 4(x-1)(x^2+6x+5)$	-20	0	68
	\	—	/

so $x=1$ gives a minimum turning point.

④ Solve $y = x^3 + 3x^2 + 2x + 3$ ... ①

$y = 2x + 3$ ... ②

Substitute ② in ①

$$x^3 + 3x^2 + 2x + 3 = 2x + 3.$$

$$x^3 + 3x^2 = 0$$

$$x^2(x+3) = 0$$

$x^2 = 0$ $\quad$ $x + 3 = 0$

$x = 0$ $\quad$ $x = -3.$

A $(0, 3)$ $\quad$ B $(-3, -3)$ $\quad$ as required.

$$\text{area} = \int_{-3}^{0} \left((x^3 + 3x^2 + 2x + 3) - (2x+3) \right) dx$$

$$= \int_{-3}^{0} (x^3 + 3x^2)\, dx$$

$$= \left[\frac{x^4}{4} + x^3 \right]_{-3}^{0}$$

$$= 0 - \left(\frac{(-3)^4}{4} + (-3)^3 \right)$$

$$= - \left(\frac{81}{4} - 27 \right)$$

$$= 6\frac{3}{4} \text{ square units.}$$

⑤

$$\log_5 (3-2x) + \log_5 (2+x) = 1$$

$$\log_5 (3-2x)(2+x) = 1$$

$$(3-2x)(2+x) = 5$$

$$6 + 3x - 4x - 2x^2 = 5$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1)$$

$$x = \frac{1}{2} \quad \text{or} \quad x = -1$$

⑥

$$\int_0^a 5 \sin 3x \, dx = \frac{10}{3}$$

$$\left[-\frac{5}{3} \cos 3x \right]_0^a = \frac{10}{3}.$$

$$-\frac{5}{3} \cos 3a + \frac{5}{3} \cos 0 = \frac{10}{3}$$

$$-\frac{5}{3} \cos 3a = \frac{5}{3}$$

$$\cos 3a = -1$$

$$3a = \pi,$$

$$a = \frac{\pi}{3}.$$

(7) $L = 3x + 4y$ ... (1)

$\text{Area} = 2xy$

$24 = 2xy$

$y = \frac{12}{x}$

In (1) $L = 3x + 4 \cdot \frac{12}{x}$

$L = 3x + \frac{48}{x}$ as required.

(b) $\frac{dL}{dx} = 3 - 48x^{-2}$

$= 3 - \frac{48}{x^2}$.

For minimum $\frac{dL}{dx} = 0$

$3 - \frac{48}{x^2} = 0$

$3x^2 = 48$

$x^2 = 16$

$x = 4$

Nature

x	(1) →	4	(10) →
$\frac{dL}{dx} = 3 - \frac{48}{x^2}$	−45	0	3.52.
	−	0	+
	\	—	/

So $x = 4$ gives a minimum length

(ii) $b = 3 \times 4 + \frac{48}{4}$

$= 12 + 12$

$= 24m$

minimum cost $= 24 \times 8.25$

$= £198$

(8) $\sin 2x = 2\cos^2 x$

$2\sin x \cos x = 2\cos^2 x$

$2\sin x \cos x - 2\cos^2 x = 0$

$2\cos x (\sin x - \cos x) = 0$

$\cos x = 0$ $\sin x = \cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\tan x = 1$ (assuming $\cos x \neq 0$ for this solution)

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Solutions $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$.

(9) $P_t = P_0 e^{-kt}$

$P_t = \frac{1}{2} P_0$ $t = 25$

$\frac{1}{2} P_0 = P_0 e^{-25k}$.

$e^{-25k} = \frac{1}{2}$

$-25k = \ln \frac{1}{2}$.

$k = -\frac{1}{25} \ln \frac{1}{2}$.

$k = 0.028$ (2 s.f)

(b) $t = 80$ $k = 0.028$

$$P_t = P_0 e^{-kt}$$

$$P_t = P_0 e^{-0.028 \times 80}$$

$$P_t = 0.11 \; P_0 \qquad (2sf)$$

11% left $\Rightarrow$ 89% decrease.