X100/12/02

NATIONAL QUALIFICATIONS 2013

WEDNESDAY, 22 MAY
1.00 PM – 2.30 PM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SECTION A

ALL questions should be attempted.

1. The functions f and g are defined by $f(x) = x^2 + 1$ and $g(x) = 3x - 4$, on the set of real numbers.

Find $g(f(x))$.

A $3x^2 - 1$

B $9x^2 - 15$

C $9x^2 + 17$

D $3x^3 - 4x^2 + 3x - 4$

2. The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

What is the gradient of the tangent to this curve at P?

A 2

B 6

C 12

D 13

3. Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A -32

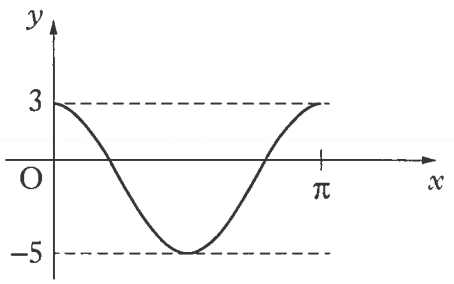
B -24

C 48

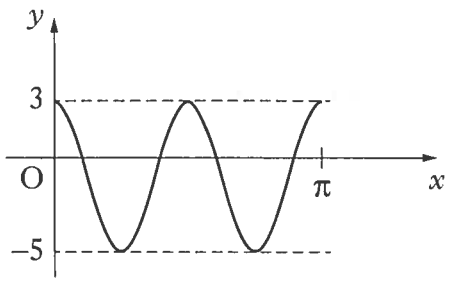
D 56

4. Which of the following shows the graph of $y = 4\cos 2x - 1$, for $0 \leq x \leq \pi$.

A



B



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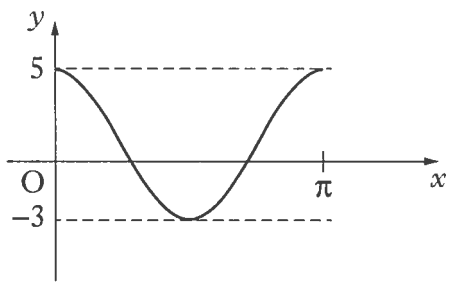

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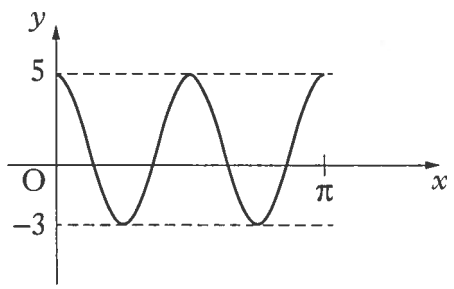


[Turn over

5. The line L passes through the point $(-2, -1)$ and is parallel to the line with equation $5x + 3y - 6 = 0$.

What is the equation of L?

A $3x + 5y - 11 = 0$

B $3x + 5y + 11 = 0$

C $5x + 3y - 13 = 0$

D $5x + 3y + 13 = 0$

6. What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

A 0

B 3

C 4

D 8

7. Find $\int x(3x + 2)\, dx$.

A $x^3 + c$

B $x^3 + x^2 + c$

C $\frac{1}{2}x^2\left(\frac{3}{2}x^2 + 2x\right) + c$

D $3x^2 + 2x + c$

8. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}1u_n + 8$, with $u_1 = 11$.

Here are two statements about this sequence:

(1) $u_0 = 9{\cdot}1$;

(2) The sequence has a limit as $n \longrightarrow \infty$.

Which of the following is true?

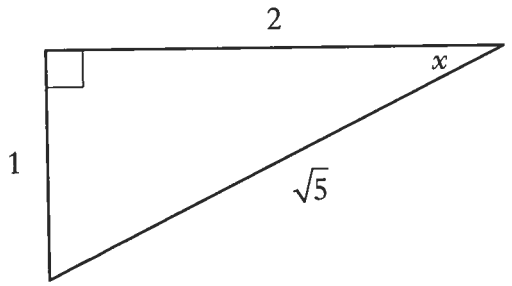
A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

9. The diagram shows a right-angled triangle with sides and angles as marked.



Find the value of $\sin 2x$.

A $\frac{4}{5}$

B $\frac{2}{5}$

C $\frac{2}{\sqrt{5}}$

D $\frac{1}{\sqrt{5}}$

10. If $0 < a < 90$, which of the following is equivalent to $\cos(270 - a)°$?

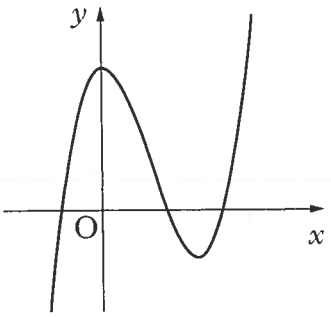
A $\cos a°$

B $\sin a°$

C $-\cos a°$

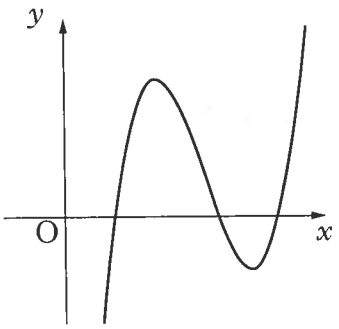
D $-\sin a°$

11. The diagram shows a cubic curve with equation $y = f(x)$.

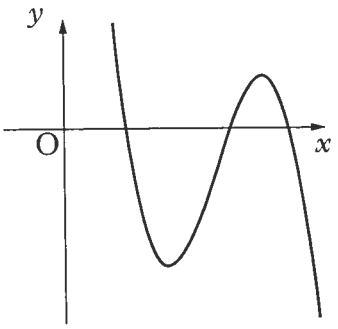


Which of the following diagrams could show the curve with equation $y = -f(x - k)$, $k > 0$?

A



B



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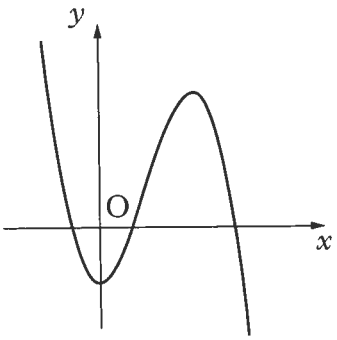

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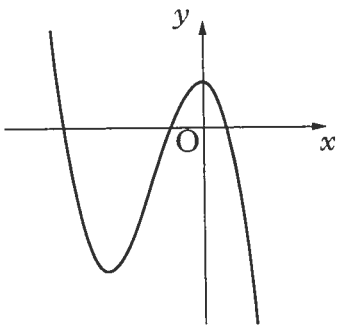


12. If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$.

A $\sqrt{14}$ units

B $\sqrt{42}$ units

C $\sqrt{66}$ units

D $\sqrt{70}$ units

13. A function f is defined on a suitable domain by $f(x) = \dfrac{x+2}{x^2-7x+12}$.

What value(s) of x cannot be in this domain?

A 3 and 4

B -3 and -4

C -2

D 0

14. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$?

A 11

B 14

C 15

D 21

15. Solve $\tan\left(\frac{x}{2}\right) = -1$ for $0 \leq x < 2\pi$.

A $\frac{\pi}{2}$

B $\frac{7\pi}{8}$

C $\frac{3\pi}{2}$

D $\frac{15\pi}{8}$

[Turn over

16. Find $\int (1-6x)^{-\frac{1}{2}} dx$ where $x < \frac{1}{6}$.

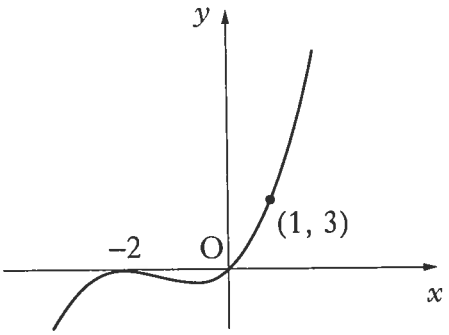
A $\frac{1}{9}(1-6x)^{-\frac{3}{2}} + c$

B $3(1-6x)^{-\frac{3}{2}} + c$

C $-\frac{1}{3}(1-6x)^{\frac{1}{2}} + c$

D $-3(1-6x)^{\frac{1}{2}} + c$

17. The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points (–2, 0), (0, 0) and (1, 3).



What are the values of a and k?

	a	k
A	–2	$\frac{1}{3}$
B	–2	3
C	2	$\frac{1}{3}$
D	2	3

18. Given that $y = \sin(x^2 - 3)$, find $\frac{dy}{dx}$.

A $\sin 2x$

B $\cos 2x$

C $2x \sin(x^2 - 3)$

D $2x \cos(x^2 - 3)$

19. Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

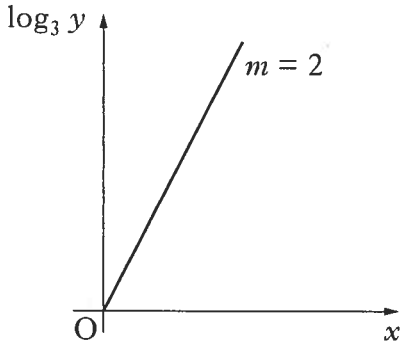
A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

20. The graph of $\log_3 y$ plotted against x is a line through the origin with gradient 2, as shown.



Express y in terms of x.

A $y = 2x$

B $y = 9x$

C $y = 6^x$

D $y = 9^x$

[END OF SECTION A]

[Turn over for SECTION B on *Page twelve*]

SECTION B

Marks

ALL questions should be attempted.

21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. 3

22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(*a*) Write down the centre and calculate the radius of C_1. 2

(*b*) The point P(3, 2) lies on the circle C_1.
Find the equation of the tangent at P. 3

(*c*) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1.
Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. 3

(*d*) Show that the tangent found in part (*b*) is also a tangent to circle C_2. 4

23. (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k \sin(x - a)°$, where $k > 0$ and $0 \leq a < 360$.
Calculate the values of k and a. 4

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \leq x < 360$. 2

24. (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB. 4

(*b*) The point C lies on the x-axis.
If TB and TC are perpendicular, find the co-ordinates of C. 5

[END OF SECTION B]

[END OF QUESTION PAPER]

Higher Maths Paper 1 2013.

① $g(f(x))$
$= g(x^2+1)$
$= 3(x^2+1)-4$
$= 3x^2-1$ (A)

② $y = x^2 - 4x + 7$
$\frac{dy}{dx} = 2x - 4$
When $x = 5$ $\frac{dy}{dx} = 10 - 4$
$= 6$ (B)

③ $2x^2 + 4x + 5 = 0$
discriminant $b^2 - 4ac$
$= 4^2 - 4 \times 2 \times 5$
$= 16 - 40$
$= -24$ (B)

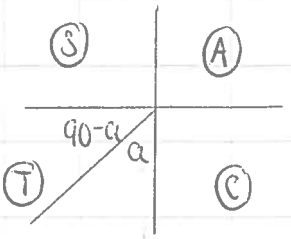
④ (A)

⑤ $5x + 3y - 6 = 0$
$3y = -5x + 6$
$y = -\frac{5}{3}x + 2$ $m = -\frac{5}{3}$

(10) $\cos(270-a)$

$= -\cos(90-a)$

$= -\sin a.$



(D)

(11) $y = -f(x-k)$

↑ move k to right then reflected in x-axis

(B).

(12) $\underline{f} + \underline{g} = 3\underline{i} + 2\underline{k} + 2\underline{i} + 4\underline{j} + 3\underline{k}$

$= 5\underline{i} + 4\underline{j} + 5\underline{k}$

$|\underline{f} + \underline{g}| = \sqrt{25 + 16 + 25}$

$= \sqrt{66}$

(C)

(13) $x^2 - 7x + 12 \neq 0$

$(x-4)(x-3) \neq 0$

$x \neq 4$ or $x \neq 3.$

(A)

(14) $\underline{a} \cdot (\underline{a} + \underline{b})$

$= \underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b}$

$= |a|^2 + 5$

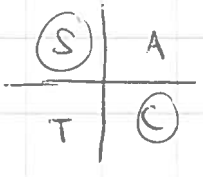
$= 9 + 5$

$= 14$

(B)

(15) $\tan \frac{x}{2} = -1$

$\frac{x}{2} = \frac{3\pi}{4}, \frac{7\pi}{4}$



(20) $y = mx + c$

$\log_3 y = 2x + 0$

$\log_3 y = 2x$

$y = 3^{2x}$

$y = 9^x$

(D)

(21) $2x^2 + 12x + 1 = 2(x^2 + 12x) + 1$

$= 2((x+6)^2 - 36) + 1$

$= 2(x+6)^2 - 71$

(22) (a) $x^2 + y^2 + 2x + 6y - 27 = 0$

$2g = 2$ $\quad 2f = 6$

$g = 1$ $\quad f = 3$

centre $(-1, -3)$

radius $\sqrt{1^2 + 3^2 - (-27)}$

$= \sqrt{32}$

$= 4\sqrt{2}$

(b) $m_{radius} = \frac{2+3}{3+1}$

$= 1$ $\quad m_{tangent} = -1$

equation $y - b = m(x - a)$

$y - 2 = -1(x - 3)$

$y - 2 = -x + 3$

$y = -x + 5$

(c) $(a, b) = (10, -1)$

$r = 2\sqrt{2}$

equation $(x-10)^2 + (y+1)^2 = (2\sqrt{2})^2$

$x^2 - 20x + 100 + y^2 + 2y + 1 = 8$

$x^2 + y^2 - 20x + 2y + 93 = 0$

as required

(a) $y = -x + 5$

Substitute into

$x^2 + y^2 - 20x + 2(-x+8) + 93 = 0$ (2y)

$x^2 + (-x+5)^2 - 20x + 2(-x+5) + 93 = 0$

$x^2 + x^2 - 10x + 25 - 20x - 2x + 10 + 93 = 0$

$2x^2 - 32x + 128 = 0$

$x^2 - 16x + 64 = 0$

$(x-8)(x-8) = 0$

$x = 8$

equal roots

so line is a tangent

(23) (a) $\sqrt{3}\sin x - \cos x = k\sin(x-a)$

$= k\sin x\cos a - k\cos x\sin a.$

$k\cos a = \sqrt{3}$

$k\sin a = -1$

Square and add $k^2 = 3+1$

$k = 2$

Divide $\tan a = \frac{-1}{\sqrt{3}}$

r.a $30°$

$a = 330°$

$\sqrt{3}\sin x - \cos x = 2(\sin(x-330))$

(b) max $4 + 5\cos x - 5\sqrt{3}\sin x$

$= 4 + 5(\cos x - \sqrt{3}\sin x)$

$= 4 - 5(\sqrt{3}\sin x - \cos x)$

$= 4 - 10(\sin(x-330))$

max 14.

(24) (a) (i) $\vec{AT} = \underline{t} - \underline{a}$

$= \begin{pmatrix}3\\2\\5\end{pmatrix} - \begin{pmatrix}-7\\-8\\1\end{pmatrix}$

$= \begin{pmatrix}10\\10\\4\end{pmatrix} = 2\begin{pmatrix}5\\5\\2\end{pmatrix}$

$\vec{TB} = \underline{b} - \underline{t}$

$= \begin{pmatrix}18\\17\\11\end{pmatrix} - \begin{pmatrix}3\\2\\5\end{pmatrix}$

$= \begin{pmatrix}15\\15\\6\end{pmatrix} = 3\begin{pmatrix}5\\5\\2\end{pmatrix}$

So $3\vec{AT} = 2\vec{TB}$

so AT and TB are parallel and since T is a common point they must be collinear.

$3AT = 2TB$

$\frac{AT}{TB} = \frac{2}{3}$ ratio 2:3.

(b) $C(x, 0, 0)$.

$\vec{TB} = \begin{pmatrix} 15 \\ 15 \\ 6 \end{pmatrix}$

$\vec{TC} = \underline{c} - \underline{t}$

$= \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} x-3 \\ -2 \\ -5 \end{pmatrix}$

$\vec{TB} \cdot \vec{TC} = 0$

$\begin{pmatrix} 15 \\ 15 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} x-3 \\ -2 \\ -5 \end{pmatrix} = 0$

$15x - 45 - 30 - 30 = 0$

$15x = 105$

$x = 7$.