X100/12/02

NATIONAL QUALIFICATIONS 2012

MONDAY, 21 MAY
1.00 PM – 2.30 PM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$.

Find the value of u_2.

A 7

B 10

C 25

D 35

2. What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?

A -24

B 3

C 5

D 6

3. If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?

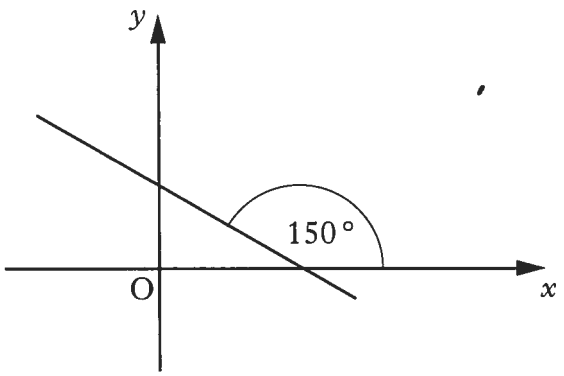
A -22

B 5

C 14

D 50

4. What is the gradient of the line shown in the diagram?



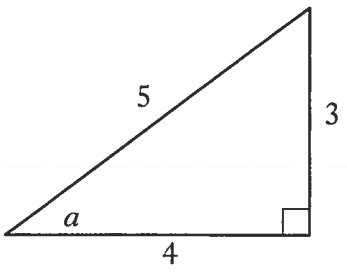
A $-\sqrt{3}$

B $-\frac{1}{\sqrt{3}}$

C $-\frac{1}{2}$

D $-\frac{\sqrt{3}}{2}$

5. The diagram shows a right-angled triangle with sides and angles as marked.



What is the value of $\cos 2a$?

A $\frac{7}{25}$

B $\frac{3}{5}$

C $\frac{24}{25}$

D $\frac{6}{5}$

6. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

A $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$

B $-3x^{-1} + 3x^{\frac{1}{2}}$

C $-6x^{-3} + 3x^{\frac{1}{2}}$

D $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

7. If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?

A -3

B -2

C $\frac{2}{3}$

D 1

[Turn over

8. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.
What is the rate of change of V with respect to r, at $r = 2$?

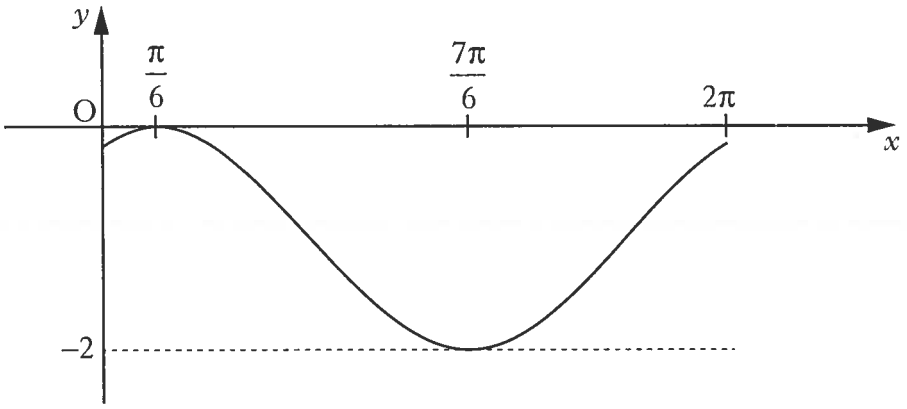
A $\frac{16\pi}{3}$

B $\frac{32\pi}{3}$

C 16π

D 32π

9. The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \leq x \leq 2\pi$.



What is the equation of this curve?

A $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

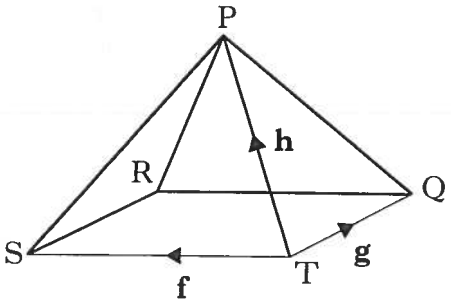
B $y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C $y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D $y = \cos\left(x + \frac{\pi}{6}\right) + 1$

10. The diagram shows a square-based pyramid P,QRST.

$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent **f**, **g** and **h** respectively.



Express $\overrightarrow{RP}$ in terms of **f**, **g** and **h**.

A $-\mathbf{f}+\mathbf{g}-\mathbf{h}$

B $-\mathbf{f}-\mathbf{g}+\mathbf{h}$

C $\mathbf{f}-\mathbf{g}-\mathbf{h}$

D $\mathbf{f}+\mathbf{g}+\mathbf{h}$

11. Find $\int\left(\frac{1}{6x^2}\right)dx,\ x \neq 0.$

A $-12x^{-3}+c$

B $-6x^{-1}+c$

C $-\frac{1}{3}x^{-3}+c$

D $-\frac{1}{6}x^{-1}+c$

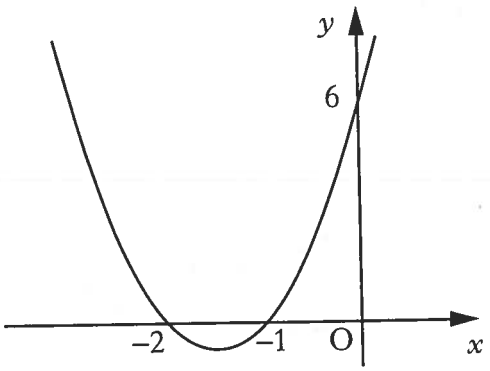
12. Find the maximum value of

$$2-3\sin\left(x-\frac{\pi}{3}\right)$$

and the value of x where this occurs in the interval $0 \leq x \leq 2\pi$.

	max value	x
A	-1	$\frac{11\pi}{6}$
B	5	$\frac{11\pi}{6}$
C	-1	$\frac{5\pi}{6}$
D	5	$\frac{5\pi}{6}$

13. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A $y = 6(x - 1)(x - 2)$

B $y = 6(x + 1)(x + 2)$

C $y = 3(x - 1)(x - 2)$

D $y = 3(x + 1)(x + 2)$

14. Find $\int (2x-1)^{\frac{1}{2}}\,dx$ where $x > \frac{1}{2}$.

A $\frac{1}{3}(2x-1)^{\frac{3}{2}} + c$

B $\frac{1}{2}(2x-1)^{-\frac{1}{2}} + c$

C $\frac{1}{2}(2x-1)^{\frac{3}{2}} + c$

D $\frac{1}{3}(2x-1)^{-\frac{1}{2}} + c$

15. If $\mathbf{u} = k\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$, where $k > 0$ and $\mathbf{u}$ is a unit vector, determine the value of k.

A $\frac{1}{2}$

B $\frac{1}{8}$

C $\frac{1}{\sqrt{2}}$

D $\frac{1}{\sqrt{10}}$

16. If $y = 3\cos^4 x$, find $\frac{dy}{dx}$.

A $12\cos^3 x \sin x$

B $12\cos^3 x$

C $-12\cos^3 x \sin x$

D $-12\sin^3 x$

17. Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{a}.(\mathbf{a}+\mathbf{b}) = 7$, what is the value of $\mathbf{a}.\mathbf{b}$?

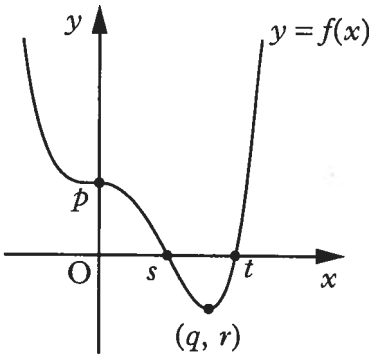
A $\frac{7}{25}$

B $-\frac{18}{5}$

C -6

D -18

18. The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r).



Here are two statements about $f(x)$:

(1) $f(x) < 0$ for $s < x < t$;

(2) $f'(x) < 0$ for $x < q$.

Which of the following is true?

A Neither statement is corrrect.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

[Turn over

19. Solve $6 - x - x^2 < 0$.

A $-3 < x < 2$

B $x < -3, x > 2$

C $-2 < x < 3$

D $x < -2, x > 3$

20. Simplify $\dfrac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$.

A 2

B $3a$

C $\log_b 3a$

D $\log_b(9a^2 - 3a)$

[END OF SECTION A]

SECTION B

Marks

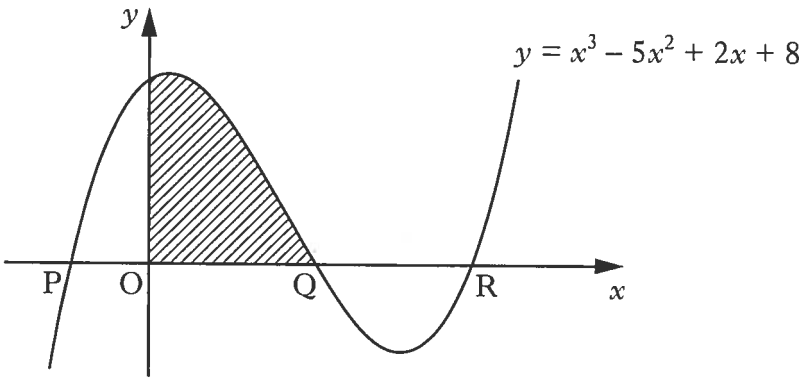
ALL questions should be attempted.

21. (*a*) (i) Show that $(x-4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. **6**

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area. **6**

22. (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x+a)$ where $k > 0$ and $0 \leq a < 2\pi$.

Calculate the values of k and a. **4**

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \leq x \leq 2\pi$. **3**

[Turn over for Question 23 on *Page twelve*

Marks

23. (*a*) Find the equation of ℓ_1, the perpendicular bisector of the line joining P(3, –3) to Q(–1, 9). **4**

(*b*) Find the equation of ℓ_2 which is parallel to PQ and passes through R(1, –2). **2**

(*c*) Find the point of intersection of ℓ_1 and ℓ_2. **3**

(*d*) Hence find the shortest distance between PQ and ℓ_2. **2**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

Higher Paper 1 2012

(1) $u_{n+1} = 3u_n + 4$.

$u_0 = 1$

$u_1 = 3 \times 1 + 4$

$= 7$

$u_2 = 3 \times 7 + 4$

$= 25$

(C)

(2) $y = x^3 - 6x + 1$

$\frac{dy}{dx} = 3x^2 - 6$

When $x = -2$

$\frac{dy}{dx} = 3 \times 4 - 6$

$= 6.$

(D)

(3) $x^2 - 6x + 14$

$= (x-3)^2 + 14 - 9$

$= (x-3)^2 + 5$

(B)

(4) $m = \tan 150$

$= -\tan 30$

$= -\frac{1}{\sqrt{3}}$

(B)

(5) $\cos 2a = \cos^2 a - \sin^2 a$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$ (A)

(6) $y = 3x^{-2} + 2x^{\frac{3}{2}}$

$\frac{dy}{dx} = -6x^{-3} + 3x^{\frac{1}{2}}$ (C)

(7) $\underline{u} \cdot \underline{v} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$

$0 = -3 + t - 2t$

$0 = -3 - t$

$t = -3$ (A)

(8) $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$ When $r = 2$

$\frac{dV}{dr} = 16\pi$ (C)

(9) $y = \cos\left(x - \frac{\pi}{6}\right) - 1$ (A)

(10) $\overrightarrow{RP} = \overrightarrow{RS} + \overrightarrow{ST} + \overrightarrow{TP}$

$= -\underline{g} - \underline{f} + \underline{h}$ (B)

(11) $\int \frac{1}{6x^2}\,dx$

$= \int \frac{1}{6} x^{-2}\,dx$

$= \frac{1}{6} \cdot \frac{x^{-1}}{-1} + C$

$= -\frac{1}{6} x^{-1} + C.$ (D)

(12) $2 - 3\sin\left(x - \frac{\pi}{3}\right)$

max $2+3$ when $x - \frac{\pi}{3} = 3\frac{\pi}{2}$

$= 5$

$x = \frac{15\pi}{6}$ (B)

(13) $y = k(x+2)(x+1)$

$(0, 6) \Rightarrow 6 = k(2)(1)$

$k = 3$

$y = 3(x+2)(x+1)$ (D)

(14) $\int (2x-1)^{\frac{1}{2}}\,dx$

$= \frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + C = \frac{1}{3}(2x-1)^{\frac{3}{2}} + C.$ (A)

(15) $\underline{u} = k\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

$\sqrt{3^2+(-1)^2+0}$

$= \sqrt{10}$

$k = \frac{1}{\sqrt{10}}$ (D)

(16) $y = 3\cos^4 x$

$= 3(\cos x)^4$

$\frac{dy}{dx} = 12(\cos x)^3 \cdot -\sin x$

$= -12\cos^3 x \sin x$ (C)

(17) $\underline{a}.(\underline{a}+\underline{b}) = 7$

$\underline{a}.\underline{a} + \underline{a}.\underline{b} = 7$

$|a| = \sqrt{3^2+6^2}$

$= 5$

$\underline{a}.\underline{b} = 7 - |\underline{a}|^2$

$= 7-5^2$

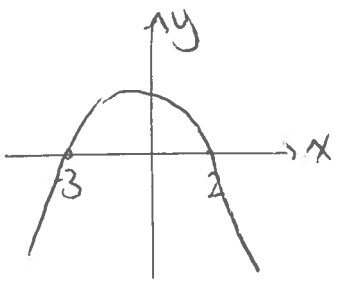
$= -18$ (D)

(18) (1) ✓

(2) x (B)

(19) $6 - x - x^2 < 0$

$(3+x)(2-x) < 0$



$x < -3$ or $x > 2$ (B)

(20) $\dfrac{\log_b 9a^2}{\log_b 3a}$

$= \dfrac{\log_b (3a)^2}{\log_b 3a}$

$= 2$

(A)

(21) (a) (i) 4

1	-5	2	8
	4	-4	-8
1	-1	-2	0

remainder 0 so $(x-4)$ is a factor

(ii) $x^3 - 5x^2 + 2x + 8 = (x-4)(x^2 - x - 2)$

$= (x-4)(x-2)(x+1)$

(iii) $(x-4)(x-2)(x+1) = 0$

$x = 4$, $x = 2$ or $x = -1$

(b) area $= \int_0^2 x^3 - 5x^2 + 2x + 8 \, dx$

$= \left[\frac{x^4}{4} - \frac{5x^3}{3} + x^2 + 8x \right]_0^2$

$= \left(\frac{16}{4} - \frac{40}{3} + 4 + 16 \right) - 0$

$= 24 - 13\frac{1}{3}$

$= 10\frac{2}{3}$ square units.

(22) (a) $\cos x - \sqrt{3}\sin x = k\cos(x+a)$

$= k\cos x \cos a - k\sin x \sin a$

$k\cos a = 1$

$k\sin a = \sqrt{3}$

Square and add $k^2 = 1 + 3$

$k^2 = 4$

$k = 2$

Divide $\tan a = \sqrt{3}$

$a = \frac{\pi}{3}$

$\cos x - \sqrt{3}\sin x = 2\cos\left(x + \frac{\pi}{3}\right)$

(b) Cuts x-axis $2\cos\left(x + \frac{\pi}{3}\right) = 0$

$x + \frac{\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}$

$\left(\frac{\pi}{6}, 0\right) \quad \left(\frac{7\pi}{6}, 0\right)$

Cuts y-axis $x = 0$ $y = 2\cos\frac{\pi}{3}$

$y = 2 \times \frac{1}{2}$

$= 1$ $(0, 1)$

(23) (a) midpt = $(1, 3)$

$m_{PQ} = \frac{9 - (-3)}{-1 - 3}$

$= \frac{12}{-4}$

$= -3$

$m_{perp} = \frac{1}{3}$

$y - b = m(x - a)$

$y - 3 = \frac{1}{3}(x - 1)$

$3y - 9 = x - 1$

$3y = x + 8$

(b) $m = -3$ $\quad y - b = m(x - a)$

$y + 2 = -3(x - 1)$

$y + 2 = -3x + 3$

$y = -3x + 1$

(c) $3y = x + 8$

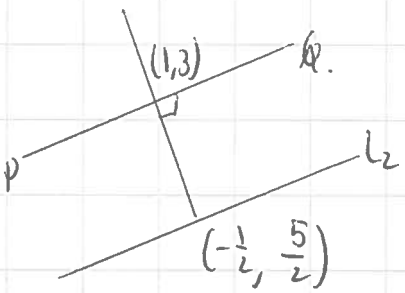
$y = -3x + 1$

$-9x + 3 = x + 8$

$-10x = 5$

$x = -\frac{1}{2}$ $\quad y = \frac{5}{2}$ $\quad \left(-\frac{1}{2}, \frac{5}{2}\right)$

(a)



distance $= \sqrt{(1 + \frac{1}{2})^2 + (3 - \frac{5}{2})^2}$

$= \sqrt{\frac{9}{4} + \frac{1}{4}}$

$= \frac{\sqrt{10}}{2}$.