X100/302

NATIONAL QUALIFICATIONS 2011

WEDNESDAY, 18 MAY
10.50 AM – 12.00 NOON

MATHEMATICS HIGHER
Paper 2

Read Carefully

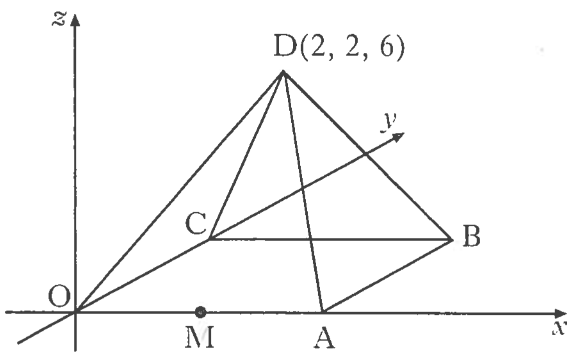
1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

ALL questions should be attempted.

1. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(*a*) State the coordinates of B. **1**

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(*c*) Find the size of angle BDM. **5**

2. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

[Turn over

3. (*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2. **1**

(*b*) A second sequence is given by 4, 5, 7, 11, ...
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q. **3**

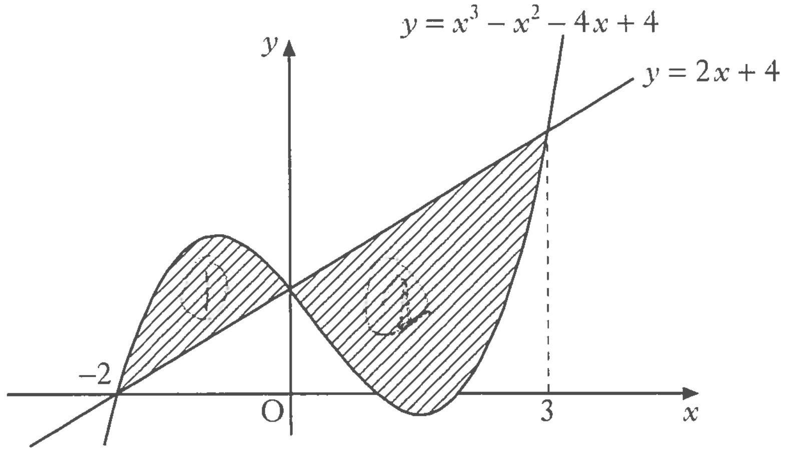
(*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit? **3**

4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points (–2, 0), (0, 4) and (3, 10).



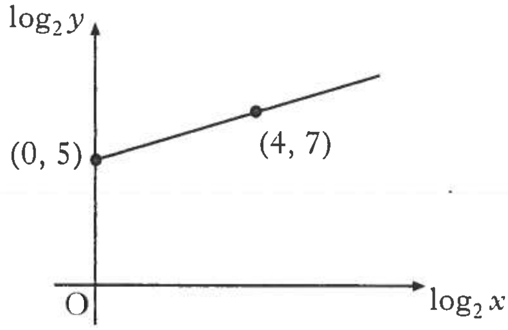
Calculate the total shaded area. **10**

Marks

5. Variables x and y are related by the equation $y = kx^n$.

The graph of $\log_2 y$ against $\log_2 x$ is a straight line through the points (0, 5) and (4, 7), as shown in the diagram.

Find the values of k and n. **5**



6. (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\, dx = 3.$$ **7**

7. Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

The circles have no points of contact.

What is the range of values of p? **9**

[*END OF QUESTION PAPER*]

Higher Maths Paper 2 2011

① (a) $B(4,4,0)$

(b) $\overrightarrow{DB} = \underline{b} - \underline{d}$

$= \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$

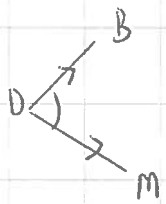
$= \begin{pmatrix}2\\2\\-6\end{pmatrix}$

$\overrightarrow{DM} = \underline{m} - \underline{d}$

$= \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix}$

$= \begin{pmatrix}0\\-2\\-6\end{pmatrix}$

(c).



$|\overrightarrow{DB}| = \sqrt{4+4+36}$

$= \sqrt{44}$

$|\overrightarrow{DM}| = \sqrt{0+4+36}$

$= \sqrt{40}$

ca $\overrightarrow{DB} . \overrightarrow{DM} = \begin{pmatrix}2\\2\\-6\end{pmatrix} . \begin{pmatrix}0\\-2\\-6\end{pmatrix}$

$= -4+36$

$= 32$

$\cos\theta = \dfrac{\overrightarrow{DB} . \overrightarrow{DM}}{|\overrightarrow{DB}||\overrightarrow{DM}|}$

$= \dfrac{32}{\sqrt{44}\sqrt{40}}$

$= 0.7628$

$\underline{\underline{\theta = 40.3°}}$

② (a) $g(f(x)) = g(x^3 - 1)$

$= 3(x^3-1)+1$

$= 3x^3 - 2.$

(b) $g(f(x)) + x(h(x))$

$= 3x^3 - 2 + x(4x - 5)$

$= 3x^3 - 2 + 4x^2 - 5x$

$= 3x^3 + 4x^2 - 5x - 2$ as required.

(c) (i)

$$\begin{array}{c|cccc} 1 & 3 & 4 & -5 & -2 \\ & & 3 & 7 & +2 \\ \hline & 3 & 7 & +2 & \boxed{0} \end{array}$$

remainder $= 0$

so $(x-1)$ is a factor.

(ii) $(x-1)(3x^2+7x+2)$

$(x-1)(3x+1)(x+2)$

(d) $g(f(x)) + x(h(x)) = 0$

$3x^3 + 4x^2 - 5x - 2 = 0$

$(x-1)(3x+1)(x+2) = 0$

$x = 1, \quad x = -\frac{1}{3} \quad x = -2.$

3 (a) $u_{n+1} = -\frac{1}{2}u_n \qquad u_0 = -16.$

$u_1 = -\frac{1}{2}u_0 \qquad u_2 = -\frac{1}{2}u_1$

$= 8 \qquad = -4.$

(b) ~~[illegible]~~ $v_1 = 4$

$v_2 = pv_1 + q \Rightarrow 5 = 4p + q$

$v_3 = pv_2 + q \Rightarrow 7 = 5p + q$

Subtract $2 = p$

$\Rightarrow q = -3$ $\qquad$ $\underline{\underline{p = 2,\ q = 3}}$

(c) For limit $-1 < a < 1$ so sequence in (a) has a limit

$L = \dfrac{b}{1-a}$

$= \dfrac{0}{1+\frac{1}{2}}$

$= 0.$

(ii) No limit since $-1 < a < 1$ for limit to exist and here $a = 2$.

(4) LH area $= \displaystyle\int_{-2}^{0} (x^3 - x^2 - 6x + 4) - (2x + 4)\, dx$

$= \displaystyle\int_{-2}^{0} (x^3 - x^2 - 6x\ )\, dx$

$= \left[\dfrac{x^4}{4} - \dfrac{x^3}{3} - 3x^2 \right]_{-2}^{0}$

$= 0 - \left(\dfrac{16}{4} + \dfrac{8}{3} - 12 \right)$

$= \dfrac{16}{3}$

RH area $= \displaystyle\int_{0}^{3} ((2x-4) - (x^3 - x^2 - 6x + 4))\, dx$

$$= \left[-\frac{x^4}{4} + \frac{x^3}{3} + 3x^2 \right]_0^3$$

$$= \left(-\frac{81}{4} + \frac{27}{3} + 27 \right) - 0$$

$$= \frac{63}{4}$$

total area $= \frac{16}{3} + \frac{63}{4}$

$= \frac{253}{12}$ square units

(5) straight line $y = mx + c$.

$m = \frac{7-5}{4-0} = \frac{1}{2}$.

Here. $\log_2 y = \frac{1}{2}\log_2 x + 5$

$$\log_2 y - \log_2 x^{\frac{1}{2}} = 5$$

$$\log_2 \frac{y}{x^{\frac{1}{2}}} = 5$$

$$\frac{y}{x^{\frac{1}{2}}} = 25$$

$$y = 25x^{\frac{1}{2}}.$$

$k = 25 \quad n = \frac{1}{2}$.

(6) (a) $3\sin x - 5\cos x = R\sin(x+a)$

$= R\sin x\cos a + R\cos x\sin a$.

⑦ C_1 $\quad (x+1)^2 + (y-1)^2 = 121$

centre $(-1, 1)$ $\qquad$ radius 11

C_2 $\quad x^2 + y^2 - 4x + 6y + p = 0$

centre $(2, -3)$ $\qquad$ radius $= \sqrt{4+9-p}$

$= \sqrt{13-p}.$ $\qquad$ $p < 13$ for radius to exist

distance between centres $= \sqrt{(-1-2)^2 + (1+3)^2}$

$= \sqrt{9+16}$

$= 5.$

So ~~max~~ radius of C_2 must be less than $11-5$

$=6.$

ie $\quad \sqrt{13-p} < 6$

$13-p < 36$

$-p < 23$

$p > -23.$

So $\quad -23 < p < 13.$