X100/301

NATIONAL QUALIFICATIONS 2011

WEDNESDAY, 18 MAY
9.00 AM – 10.30 AM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

SECTION A

ALL questions should be attempted.

1. Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.

A $\begin{pmatrix} 1 \\ 9 \\ -15 \end{pmatrix}$

B $\begin{pmatrix} 1 \\ 11 \\ -13 \end{pmatrix}$

C $\begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$

D $\begin{pmatrix} 5 \\ 11 \\ -15 \end{pmatrix}$

2. A line l has equation $3y + 2x = 6$.

What is the gradient of any line parallel to l?

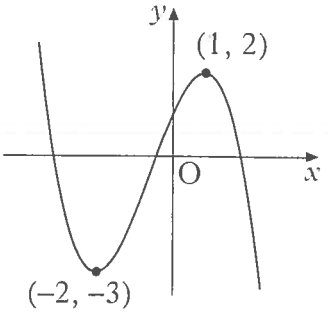
A -2

B $-\frac{2}{3}$

C $\frac{3}{2}$

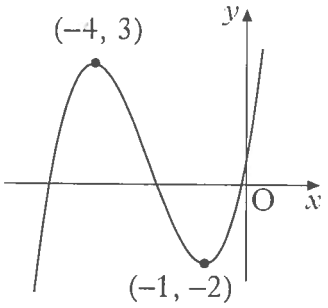
D 2

3. The diagram shows the graph of $y = f(x)$.

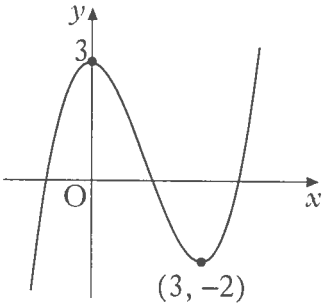


Which of the following shows the graph of $y = f(x + 2) - 1$?

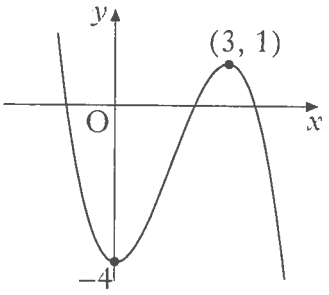
A



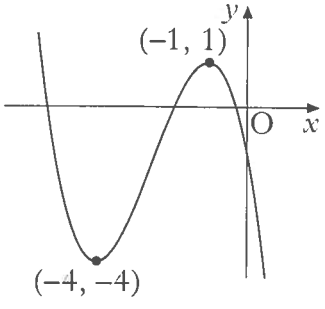
B



C



D



[Turn over

4. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point (2, 4).

What is the gradient of this tangent?

A 2

B 3

C 4

D 10

5. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?

A −9

B −1

C 7

D 23

6. The point P(2, −3) lies on the circle with centre C as shown.

The gradient of CP is −2.

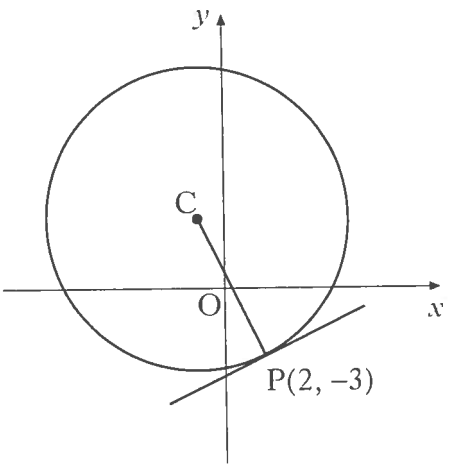
What is the equation of the tangent at P?

A $y + 3 = -2(x - 2)$

B $y - 3 = -2(x + 2)$

C $y + 3 = \frac{1}{2}(x - 2)$

D $y - 3 = \frac{1}{2}(x + 2)$



7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$?

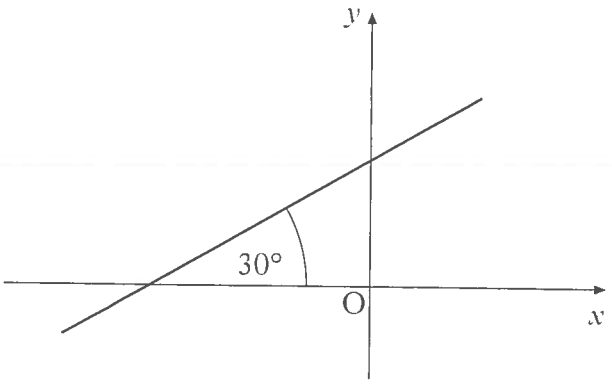
A 0

B 2

C 3

D 4

8. A line makes an angle of 30° with the positive direction of the x-axis as shown.



What is the gradient of the line?

A $\frac{1}{\sqrt{3}}$

B $\frac{1}{\sqrt{2}}$

C $\frac{1}{2}$

D $\frac{\sqrt{3}}{2}$

9. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

(1) the roots are real;

(2) the roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

[Turn over

10. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

A $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

C $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

11. Find $\int \left(4x^{\frac{1}{2}} + x^{-3} \right) dx$, where $x > 0$.

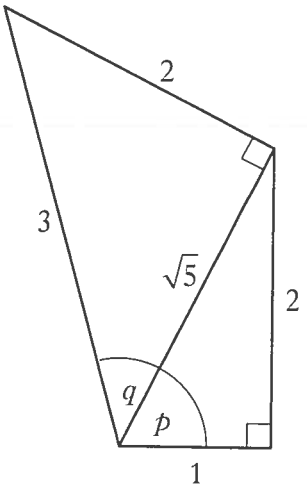
A $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

12. The diagram shows two right-angled triangles with sides and angles as given.



What is the value of $\sin(p+q)$?

A $\frac{2}{\sqrt{5}}+\frac{2}{3}$

B $\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{3}$

C $\frac{2}{3}+\frac{2}{3\sqrt{5}}$

D $\frac{4}{3\sqrt{5}}+\frac{1}{3}$

13. Given that $f(x)=4\sin 3x$, find $f'(0)$.

A 0

B 1

C 12

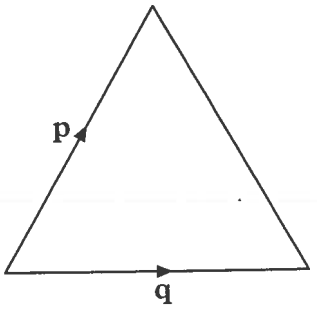
D 36

[Turn over

14. An equilateral triangle of side 3 units is shown.

The vectors **p** and **q** are as represented in the diagram.

What is the value of **p . q**?



A 9

B $\frac{9}{2}$

C $\frac{9}{\sqrt{2}}$

D 0

15. Given that the points S(–4, 5, 1), T(–16, –4, 16) and U(–24, –10, 26) are collinear, calculate the ratio in which T divides SU.

A 2 : 3

B 3 : 2

C 2 : 5

D 3 : 5

16. Find $\int \frac{1}{3x^4}\,dx$, where $x \neq 0$.

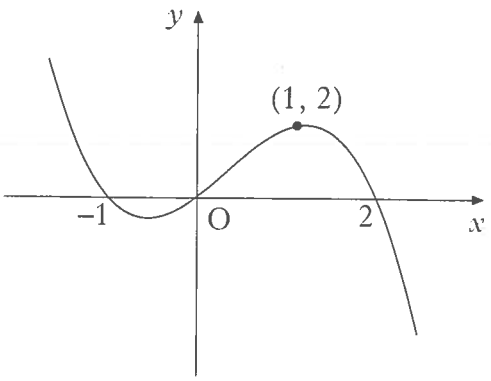
A $-\frac{1}{9x^3}+c$

B $-\frac{1}{x^3}+c$

C $\frac{1}{x^3}+c$

D $\frac{1}{12x^3}+c$

17. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A $y = -x(x + 1)(x - 2)$

B $y = -x(x - 1)(x + 2)$

C $y = x(x + 1)(x - 2)$

D $y = x(x - 1)(x + 2)$

18. If $f(x) = (x - 3)(x + 5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A $-5 < x < 3$

B $-3 < x < 5$

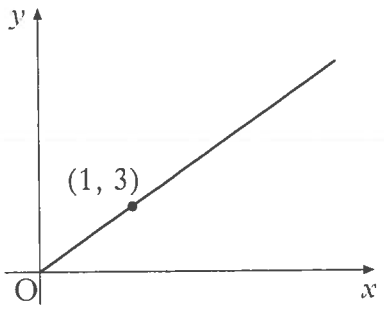
C $x < -5, x > 3$

D $x < -3, x > 5$

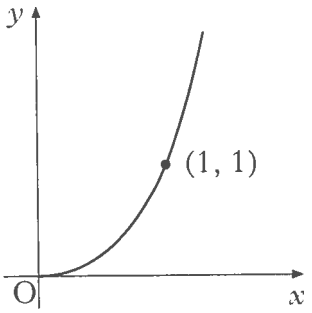
[Turn over

19. Which of the following diagrams represents the graph with equation $\log_3 y = x$?

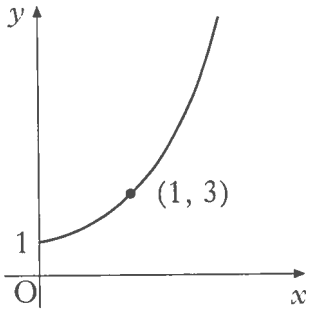
A



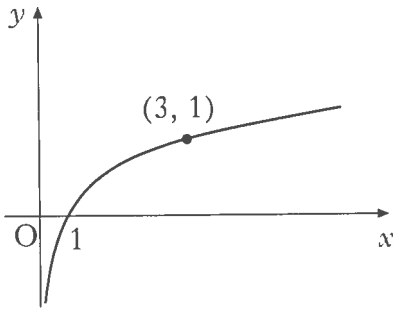
B



C



D



20. On a suitable domain, D, a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

A $x \geq 0$ and $-1 \leq g(x) \leq 1$

B $x \geq 0$ and $0 \leq g(x) \leq 1$

C $x \geq 2$ and $-1 \leq g(x) \leq 1$

D $x \geq 2$ and $0 \leq g(x) \leq 1$

[END OF SECTION A]

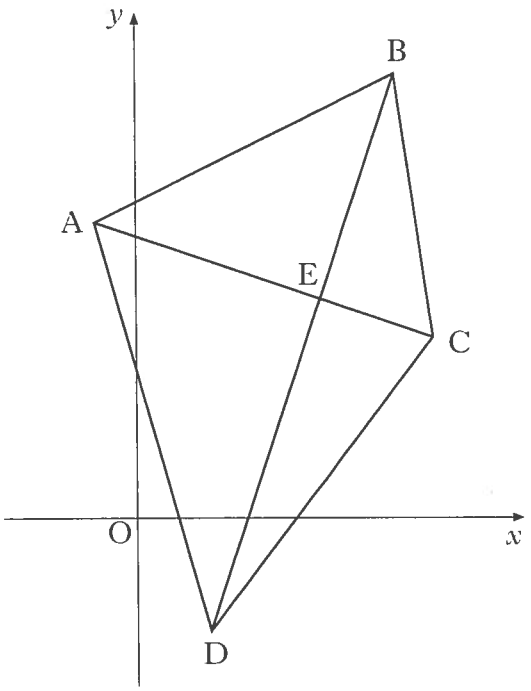
[Turn over for SECTION B

Marks

SECTION B

ALL questions should be attempted.

21. A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.



(*a*) Find the equation of diagonal BD. **2**

(*b*) The equation of diagonal AC is $x + 3y = 23$.
Find the coordinates of E, the point of intersection of the diagonals. **3**

(*c*) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. **5**

Marks

22. A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. **2**

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$. **3**

23. (*a*) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(*b*) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

[END OF SECTION B]

[END OF QUESTION PAPER]

Higher 2011 Paper 1

① $2\underline{p} - \underline{q} - \frac{1}{2}\underline{r}$

$= \begin{pmatrix} 4 \\ 10 \\ -14 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ 9 \\ -13 \end{pmatrix}$ (C)

② $3y + 2x = 6$.

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$. $m = -\frac{2}{3}$ (B)

③ $y = f(x+2) - 1$.

$(1, 2) \to (-1, 2) \to (-1, 1)$

$(-2, 3) \to (-4, 3) \to (-4, 4)$

(D)

④ $y = x^3 - 2x$

$\frac{dy}{dx} = 3x^2 - 2$.

When $x = 2$ $\frac{dy}{dx} = 3 \times 2^2 - 2$

$= 12 - 2$

$= 10$. (D)

⑤ $x^2 - 8x + 7$

$= (x-4)^2 + 7 - (-4)^2$

$= (x-4)^2 + 7 - 16$

$= (x-4)^2 - 9$ $q = -9$ (A)

⑥ $y - b = m(x-a)$ $\quad m = \frac{1}{2}$

$y + 3 = \frac{1}{2}(x-2)$

Ⓒ.

⑦

1	1	−1	1	3
		1	0	1
	1	0	1	4

Ⓓ.

⑧ $m = \tan\theta$

$= \tan 30$

$= \frac{1}{\sqrt{3}}$

Ⓐ.

⑨ Ⓑ

⑩ $2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

S	A ✓
T	C ✓

r.a $\frac{\pi}{6}$

$x = \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$

$x = \frac{\pi}{6}, \quad \frac{11\pi}{6}$.

Ⓓ.

⑪ $\int \left(4x^{\frac{1}{2}} + x^{-3}\right) dx$

$= \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-2}}{-2} + C.$

$= \frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2x^2} + C.$

Ⓓ.

(12) $\sin(p+q)$

$= \sin p \cos q + \cos p \sin q$

$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \frac{1}{\sqrt{5}} \cdot \frac{2}{3}$

$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$

(D)

(13) $f(x) = 4\sin 3x$

$f'(x) = 12\cos 3x$

$f'(0) = 12\cos 0$

$= 12.$

(C)

(14) $\underline{p} \cdot \underline{q} = |\underline{p}||\underline{q}| \cos 60$

$= 3 \times 3 \times \frac{1}{2}$

$= \frac{9}{2}$

(B)

(15) $\vec{ST} = \underline{t} - \underline{s}$

$= \begin{pmatrix} -16 \\ -4 \\ 16 \end{pmatrix} - \begin{pmatrix} -4 \\ 5 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -12 \\ -9 \\ 15 \end{pmatrix}$

$= 3\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$

$\vec{TU} = \underline{u} - \underline{t}$

$= \begin{pmatrix} -24 \\ -10 \\ 26 \end{pmatrix} - \begin{pmatrix} -16 \\ -4 \\ 16 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ -6 \\ 10 \end{pmatrix}$

$= 2\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$

$\frac{ST}{TU} = \frac{3}{2}$

(B)

(16) $\int \frac{1}{3x^4}\,dx$

$= \int \frac{1}{3}x^{-4}\,dx$

$= \frac{1}{3}\cdot\frac{x^{-3}}{-3} + C$

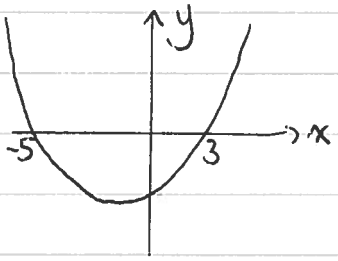
$= -\frac{1}{9x^3} + C$ (A)

(17) $y = kx(x+1)(x-2)$ ↳ so $-x^3$

$y = -x(x+1)(x-2)$

(A).

(18) Sketch $y = (x-3)(x+5)$ $+x^2$ U



$x < -5$ or $x > 3$. (C)

(19) ~~y=max~~ $\log_3 y = x$

$y = 3^x$ (C)

(20) $x - 2 \geq 0$

$x \geq 2$

$0 \leq \sin^2 x \leq 1$ (D).

(21) (a) $m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-3-12}{2-7}$

$= \frac{-15}{-5}$

$= 3$

Equation $y - b = m(x - a)$

$y - 12 = 3(x - 7)$

$y - 12 = 3x - 21$

$y = 3x - 9$

(b) Solve $x + 3y = 23$ --- ①

$y = 3x - 9$ --- ②

Substitute ② in ① $x + 3(3x - 9) = 23$

$x + 9x - 27 = 23$

$10x = 50$

$x = 5.$

$\Rightarrow y = 15 - 9$

$= 6$ $E(5, 6)$

(c) (i) $m_{AB} = \frac{12-8}{7-(-1)}$

$= \frac{4}{8}$

$= \frac{1}{2}.$

$m_{perp} = -2.$

$M_{AB} = \left(\frac{-1+7}{2}, \frac{8+12}{2}\right)$

$= (3, 10)$

$y - b = m(x - a)$

$y - 10 = -2(x - 3)$

$y - 10 = -2x + 6$

$y = -2x + 16$

(ii) $E(5, 6)$ Put $x = 5$ into $y = -2x + 16$

$= -2 \times 5 + 16$

$= -10 + 16$

$y = 6$

So point E lies on the line.

(22) (a) $f(x) = (x - 2)(x^2 + 1)$

(i) cuts x-axis $f(x) = 0$

$(x - 2)(x^2 + 1) = 0$

$x = 2$ $(2, 0)$

(ii) cuts y-axis $x = 0$

$y = (-2)(1)$

$= -2$ $(0, -2)$

(b) $f(x) = x^3 + x - 2x^2 - 2$

$f'(x) = 3x^2 + 1 - 4x$

$= 3x^2 - 4x + 1$

For stationary points $f'(x) = 0$

$3x^2 - 4x + 1 = 0$

$(3x - 1)(x - 1) = 0$

$x = \frac{1}{3}$ or $x = 1$

y-co-ords $\quad x = \frac{1}{3} \quad y = \left(\frac{1}{3} - 2\right)\left(\frac{1}{4} + 1\right)$

$= \left(-\frac{5}{3}\right)\left(\frac{10}{4}\right)$

$= -\frac{50}{27}$ $\quad \left(\frac{1}{3}, \frac{-50}{27}\right)$

$x = 1 \quad y = (-1)(2)$

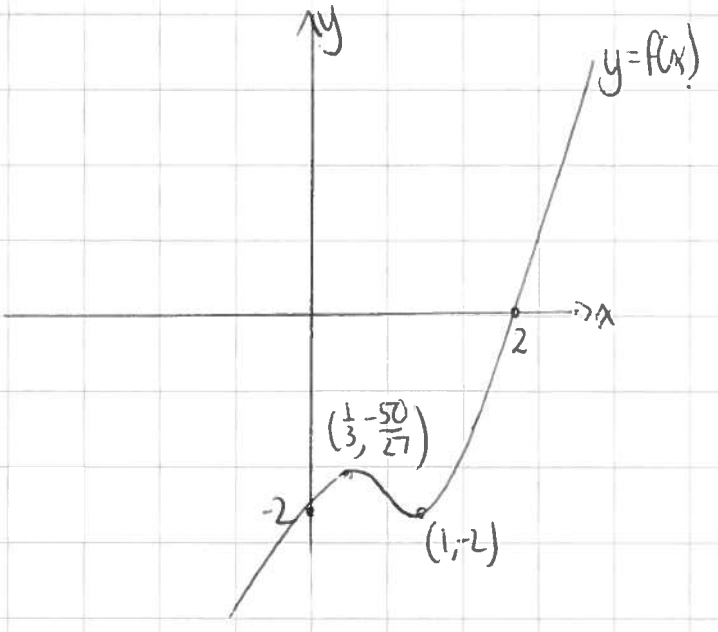
$= -2$ $\quad (1, -2)$

Nature	x	$0 \rightarrow$	$\frac{1}{3}$	$\frac{2}{3} \rightarrow$	1	$\rightarrow 2$
$f'(x) = (3x-1)(x-1)$		1	0	$-\frac{1}{3}$	0	5
		↗	→	↘	→	↗

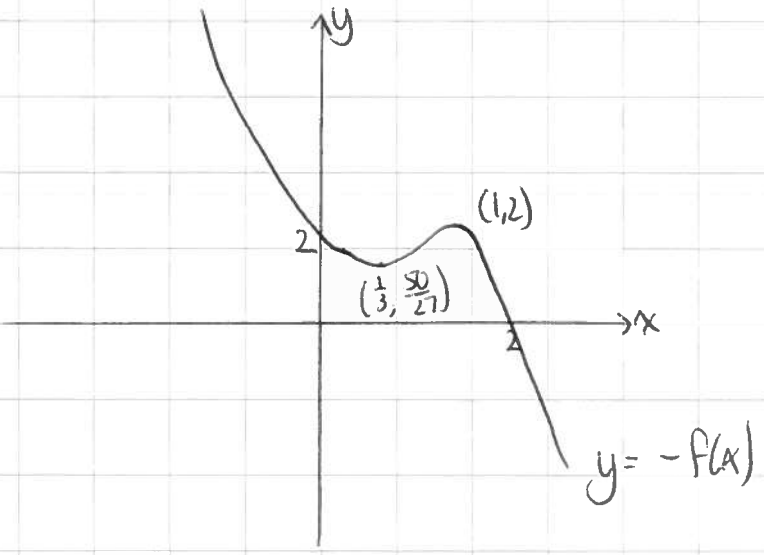
so $\left(\frac{1}{3}, \frac{-50}{27}\right)$ is a maximum TP

$(1, -2)$ is a minimum TP.

(c)



(ii)



(23) (a) $\cos 2x - 3\cos x + 2 = 0$

$2\cos^2 x - 1 - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = 1$

$x = 60°, 300°$ $\quad$ $x = 0°$

$x = 0, 60, 300.$

(b) $2x = 0, 60, 300$

$x = 0, 30, 150$

period 180 so $x = 0, 30, 150, 180, 210, 330.$