X100/302

NATIONAL QUALIFICATIONS 2010

FRIDAY, 21 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LI X100/302 6/31310

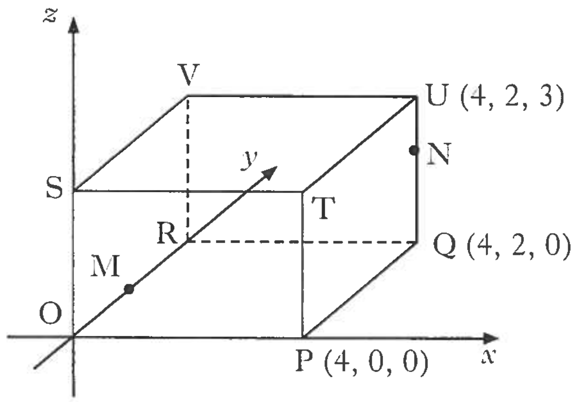
ALL questions should be attempted.

1. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

(*c*) Calculate the size of angle MVN. **5**

2. (*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a. **4**

(*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$.

(ii) Determine the values of x, in the interval $0 \leq x < 360$, at which these maximum and minimum values occur. **3**

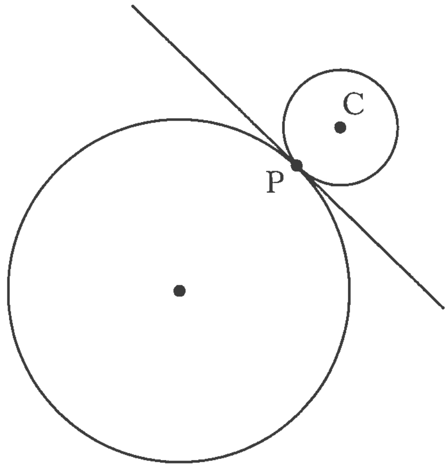
[Turn over

Marks

3\. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



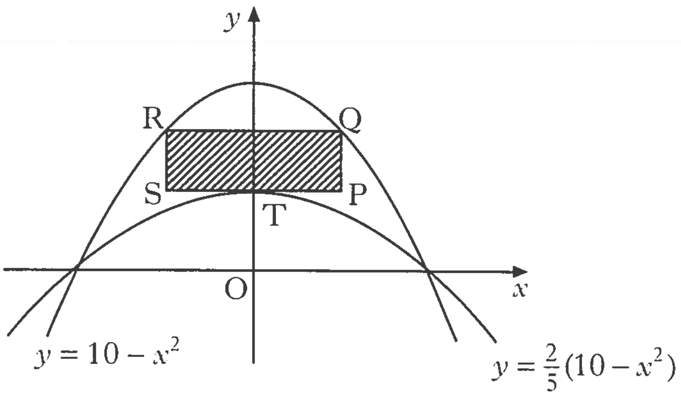
The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

4\. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$. **5**

5. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(*a*) (i) If $TP = x$ units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$ **3**

(*b*) Find the maximum area of this rectangle. **6**

[Turn over for Questions 6 and 7 on *Page six*

6. (*a*) A curve has equation $y = (2x - 9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

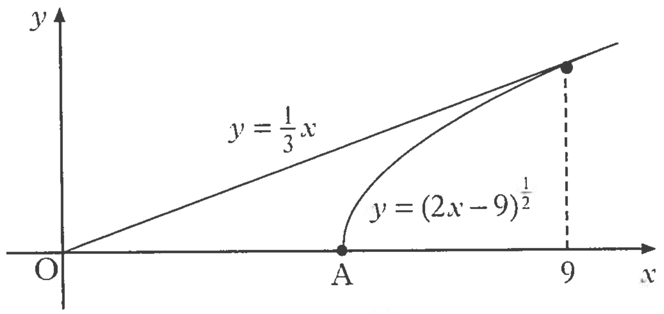


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

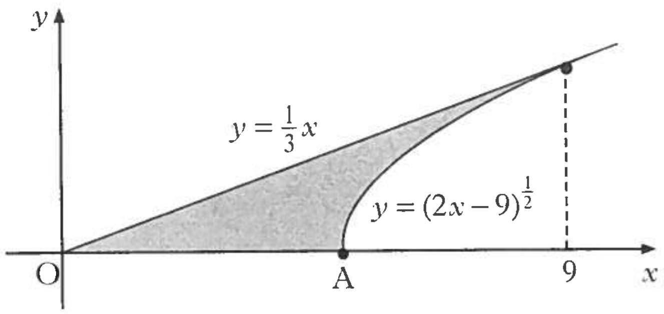


Diagram 2

7. (*a*) Given that $\log_4 x = P$, show that $\log_{16} x = \frac{1}{2}P$. **3**

(*b*) Solve $\log_3 x + \log_9 x = 12$. **3**

[*END OF QUESTION PAPER*]

Higher Maths Paper 2 2010.

1 (a) $M(0,1,0)$ $N(4,2,2)$

(b) $\overrightarrow{VM} = \underline{m} - \underline{v}$

$= \begin{pmatrix}0\\1\\0\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix}$

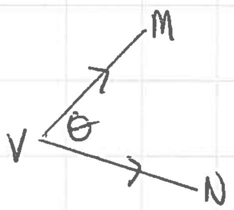
$= \begin{pmatrix}0\\-1\\-3\end{pmatrix}$

$\overrightarrow{VN} = \underline{n} - \underline{v}$

$= \begin{pmatrix}4\\2\\2\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix}$

$= \begin{pmatrix}4\\0\\-1\end{pmatrix}$

(c) $M\hat{V}N$



$|\overrightarrow{VM}| = \sqrt{0+1+9}$

$= \sqrt{10}$

$|\overrightarrow{VN}| = \sqrt{16+1}$

$= \sqrt{17}$

$\overrightarrow{VM} \cdot \overrightarrow{VN} = \begin{pmatrix}0\\-1\\-3\end{pmatrix} \cdot \begin{pmatrix}4\\0\\-1\end{pmatrix}$

$= 3$

$\cos\theta = \dfrac{\overrightarrow{VM} \cdot \overrightarrow{VN}}{|\overrightarrow{VM}||\overrightarrow{VN}|}$

$= \dfrac{3}{\sqrt{10}\sqrt{17}}$

$= 0.230$

$\underline{\underline{\theta = 76.7^\circ}}$ (3 s.f)

② (a) $12\cos x - 5\sin x = k\cos(x+a)$

$= k\cos x\cos a - k\sin x\sin a.$

$k\cos a = 12$

$k\sin a = 5$

Square and add $k^2 = 144 + 25$

$\underline{k = 13}$

Divide $\tan a = \frac{5}{12}$

$\underline{a = 22.6°}$ (3 s.f)

So $12\cos x - 5\sin x = 13\cos(x + 22.6°)$

(b) (i) max 13

Min −13.

(ii) max when $x + 22.6 = 0$ or 360

$x = 337.4°$

min when $x + 22.6 = 180$

$x = 157.4°.$

③ $x^2+y^2+16x+4y-19=0$

$y=3-x$

$$x^2+(3-x)^2+16x+4(3-x)-19=0$$

$$x^2+9-6x+x^2+16x+12-4x-19=0$$

$$\cancel{2x^2+4} \quad 2x^2+4x+2=0$$

$$x^2+2x+1=0$$

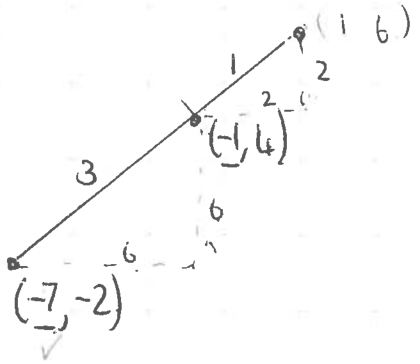
$$(x+1)^2=0$$

$$x=-1$$ one solution → line is a tangent.

$P(-1,4)$

$$r=\sqrt{7^2+2^2-(-19)}$$
$$=\sqrt{49+4+19}$$
$$=\sqrt{72}$$
$$=6\sqrt{2}$$

r smaller circle $=2\sqrt{2}$



$C(1,6)$

equation $=(x-1)^2+(y-6)^2=8$

(4) $2\cos 2x - 5\cos x - 4 = 0$

$$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$$

$$4\cos^2 x - 2 - 5\cos x - 4 = 0$$

$$4\cos^2 x - 5\cos x - 6 = 0$$

$$(4\cos x + 3)(\cos x - 2) = 0$$

$\cos x = -\frac{3}{4}$ $\qquad$ $\cos x = 2$ not possible.

r.a 0.723.

S ✓ | A

T ✓ | C

$x = \pi - 0.723$ or $\pi + 0.723$

$x = 2.42$ or 3.86.

(5) (i) $TP = x$ so Q has co-ords $(x, 10 - x^2)$

T when $x = 0$ $y = \frac{2}{5}(10)$

$= 4.$

point T $(0, 4)$

P $(x, 4)$

so $PQ = 10 - x^2 - 4$

$= 6 - x^2.$

(ii) Area $= lb$

$= 2x(6 - x^2)$

$A = 12x - 2x^3$ as required

(b) $\frac{dA}{dx} = 12 - 6x^2$

For stationary points $\frac{dA}{dx} = 0$

$$\Rightarrow \quad 12 - 6x^2 = 0$$
$$x^2 = 2$$
$$x = \sqrt{2}.$$

Nature

x		$\rightarrow$	$\sqrt{2}$	$\rightarrow$
$\frac{dA}{dx} = 12 - 6x^2$	$+$		0	$-$
	/		—	\

so $x = \sqrt{2}$ gives a maximum.

$$\text{Area} = 12x - 2x^3$$
$$= 12\sqrt{2} - 2(\sqrt{2})^3$$
$$= 12\sqrt{2} - 4\sqrt{2}$$
$$= \underline{\underline{8\sqrt{2}}}$$

⑥ (a) $y = (2x-9)^{\frac{1}{2}}$.

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-\frac{1}{2}} \times 2.$

$= \frac{1}{\sqrt{2x-9}}$

When $x = 9$ $\quad \frac{dy}{dx} = \frac{1}{\sqrt{9}}$ $\quad y = 3.$

$= \frac{1}{3}$

$(y-b) = m(x-a)$

$(y-3) = \frac{1}{3}(x-9)$

$3y - 9 = x - 9$

$3y = x$

$y = \frac{1}{3}x$.

(b) $y = (2x-9)^{\frac{1}{2}}$.

Cuts x-axis $\quad y = 0$

$2x - 9 = 0$

$x = 4.5$ $\quad A(4.5, 0)$

⑦ (a) $\log_4 x = p$

$x = 4^p$

$x = (\sqrt{16})^p$

$x = 16^{\frac{1}{2}p}$

$\Rightarrow \log_{16} x = \frac{1}{2}p$

(b) $\log_3 x + \log_9 x = 12$

$\log_3 x + \frac{1}{2}\log_3 x = 12.$

$\log_3 x + \log_3 x^{\frac{1}{2}} = 12$

$\log_3 x^{\frac{3}{2}} = 12$

$x^{\frac{3}{2}} = 3^{12}$

$x = 3^{\frac{24}{3}}$

$x = 3^8$