Further Calculus Past Papers Unit 3 Outcome 2

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Differentiate $3\cos\left(2x - \frac{\pi}{6}\right)$ with respect to x.

   A. $-3\sin(2x)$

   B. $-3\sin\left(2x - \frac{\pi}{6}\right)$

   C. $-6\sin\left(2x - \frac{\pi}{6}\right)$

   D. $6\sin\left(2x - \frac{\pi}{6}\right)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0.68	0.23	NC	C20	HSN 096

$$\frac{d}{dx}\left(3\cos\left(2x - \frac{\pi}{6}\right)\right) = -3 \times 2\sin\left(2x - \frac{\pi}{6}\right)$$
$$= -6\sin\left(2x - \frac{\pi}{6}\right) \qquad \text{Option } \boxed{C}$$

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

2. Differentiate $\sin 2x + \dfrac{2}{\sqrt{x}}$ with respect to x.   4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
.	4	3.2	2	2					3.2.2	1.3.1	1989 P1 qu.10

- •¹ $2x^{-\frac{1}{2}}$
- •² $\cos 2x$
- •³ $\times 2$
- •⁴ $-x^{-\frac{3}{2}}$

[SQA]   3. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.   3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

$\bullet^1$ pd: differentiate power $\bullet^2$ pd: differentiate 2nd function $\bullet^3$ pd: evaluate $f'(x)$	$\bullet^1$ $\frac{1}{2}(5x - 4)^{-\frac{1}{2}}$ $\bullet^2$ $\times 5$ $\bullet^3$ $f'(4) = \frac{5}{8}$

[SQA]   4. Given $f(x) = \cos^2 x - \sin^2 x$, find $f'(x)$.   3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
		C	A/B	C	A/B	C	A/B	Main	Additional	Source
. 3	3.2	1	2					3.2.2	3.2.1	1999 P1 qu.19

$\bullet^1$ $f(x) = \cos 2x$ $\bullet^2$ $-\sin 2x$ $\bullet^3$ $\times 2$	For $\frac{d}{dx}(\cos^2 x)$ $\bullet^1$ $2\cos x$ $\bullet^2$ $x - \sin x$ For $\frac{d}{dx}(-\sin^2 x)$ $\bullet^3$ $-2\sin x \times \cos x$	OR For $\frac{d}{dx}(-\sin^2 x)$ $\bullet^1$ $-2\sin x$ $\bullet^2$ $\times \cos x$ For $\frac{d}{dx}(\cos^2 x)$ $\bullet^3$ $2\cos x \times -\sin x$

[SQA]   5. Given that $f(x) = 5(7 - 2x)^3$, find the value of $f'(4)$.   4

part marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
		C	A/B	C	A/B	C	A/B	Main	Additional	Source
. 4	3.2		4					3.2.2		1991 P1 qu.13

$\bullet^1$ $(7 - 2x)^2$ $\bullet^2$ $\times 15$ $\bullet^3$ $\times -2$ $\bullet^4$ -30

[SQA]   6. Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x.   4

part marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
		C	A/B	C	A/B	C	A/B		Source
. 4	3.2	1	3					3.2.2	1992 P1 qu.11

$\bullet^1$ $3x^{\frac{1}{2}}$

$\bullet^2$ $(\sin x)^2$ stated or implied by $\bullet^3$

$\bullet^3$ $2\sin x$

$\bullet^4$ $x\cos x$

[SQA]   7. Find the derivative, with respect to x, of $\dfrac{1}{x^3} + \cos 3x$.   4

part marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
		C	A/B	C	A/B	C	A/B		Source
. 4	3.2		4					3.2.2	1994 P1 qu.10

$\bullet^1$ x^{-3} stated or implied by $\bullet^2$

$\bullet^2$ $-3x^{-4}$

$\bullet^3$ $-\sin 3x$

$\bullet^4$ $\times 3$

[SQA]   8. If $f(x) = \cos^2 x - \dfrac{2}{3x^2}$, find $f'(x)$.   4

part marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
		C	A/B	C	A/B	C	A/B		Source
. 4	3.2	2	2					3.2.2 1.3.4	1990 P1 qu.19

$\bullet^1$ $-\frac{2}{3}x^{-2}$

$\bullet^2$ $2\cos x$

$\bullet^3$ $x(-\sin x)$

$\bullet^4$ $\frac{4}{3}x^{-3}$

[SQA]   9. Differentiate $4\sqrt{x} + 3\cos 2x$ with respect to x.   4

part marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
										Source
. 4	3.2	2	2					3.2.2	1.3.4	1993 P1 qu.9

- $\bullet^1$ $4x^{\frac{1}{2}}$
- $\bullet^2$ $2x^{-\frac{1}{2}}$
- $\bullet^3$ $-\sin 2x$
- $\bullet^4$ $\times 2$

[SQA]   10. Find $\dfrac{dy}{dx}$ given that $y = \sqrt{1 + \cos x}$.   3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
										Source
. 3	3.2		3					3.2.2	3.2.1	1996 P1 qu.13

- $\bullet^1$ $(1 + \cos x)^{\frac{1}{2}}$ stated or implied by $\bullet^2$
- $\bullet^2$ $\frac{1}{2}(1 + \cos x)^{-\frac{1}{2}}$
- $\bullet^3$ $\times - \sin x$

[SQA]   11. Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'(\frac{\pi}{6})$.   3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
										Source
. 3	3.2		3					3.2.1	3.2.2	1998 P1 qu.16

		Alternative
$\bullet^1$ $2(\sin x + 1)$		$\bullet^1$ expand and differentiate $2\sin x + 1$
$\bullet^2$ $\times \cos x$		$\bullet^2$ differentiate $\sin^2 x$
$\bullet^3$ $\dfrac{3\sqrt{3}}{2}$		$\bullet^3$ $\dfrac{3\sqrt{3}}{2}$

[SQA] 12. Find the equation of the tangent to the curve $y = 2\sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$. 4

Part	Marks	Level	Calc.	Content		Answer	U3 OC2
	4	C	CN	C5, C20		$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	2002 P2 Q6

- •^{1} pd: find derivative
- •^{2} ss: know derivative at $x = \ldots$ represents grad.
- •^{3} pd: find corresponding y-coordinate
- •^{4} ic: state equation of tangent

- •^{1} $\frac{dy}{dx} = 2\cos(x - \frac{\pi}{6})$
- •^{2} $m = \sqrt{3}$
- •^{3} $y_{x=\frac{\pi}{3}} = 1$
- •^{4} $y - 1 = \sqrt{3}(x - \frac{\pi}{3})$

[SQA] 13. Find $\int \sqrt{1 + 3x}\, dx$ and hence find the exact value of $\int_0^1 \sqrt{1 + 3x}\, dx$. 4

part marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
		C	A/B	C	A/B	C	A/B		Source
. 4	3.2		4					3.2.3	1993 P1 qu.16

- •^{1} $(1 + 3x)^{\frac{1}{2}}$
- •^{2} $\frac{1}{\frac{3}{2}}(1 + 3x)^{\frac{3}{2}}$
- •^{3} $\div 3$
- •^{4} $\frac{14}{9}$

[SQA] 14. Differentiate $\sin^3 x$ with respect to x.

Hence find $\int \sin^2 x \cos x\, dx$. 4

part marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
		C	A/B	C	A/B	C	A/B		Source
. 4	3.2	1	3					3.2.2 2.2.3	1994 P1 qu.17

- •^{1} using $(\sin x)^3$ stated or implied by •^{2}
- •^{2} $3\sin^2 x$
- •^{3} $\times \cos x$
- •^{4} $\frac{1}{3}\sin^3 x$

[SQA] 15. Find $\int \dfrac{1}{(7-3x)^2} \, dx$.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\dfrac{1}{3(7-3x)} + c$	2000 P2 Q10

$\bullet^1$ pd: integrate function $\bullet^2$ pd: deal with function of function	$\bullet^1 \ \frac{1}{-1}(7-3x)^{-1}$ $\bullet^2 \ \times \frac{1}{-3}$

[SQA] 16. Evaluate $\displaystyle\int_{-3}^{0} \left(2x+3\right)^2 dx$.

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
.	1	2.2	4						2.2.4	2.2.5	1996 P1 qu.5

$\bullet^1 \ \frac{1}{3}(2x+3)^3$		$\bullet^1 \ \frac{4}{3}x^3$	
$\bullet^2 \ +2$	OR	$\bullet^2 \ 6x^2 + 9x$	
$\bullet^3 \ \frac{1}{6}(3)^3 - \frac{1}{6}(-6+3)^3$		$\bullet^3 \ |0| - \left[\frac{4}{3}(-3)^3 + 6(-3)^2 + 9(-3)\right]$	
$\bullet^4 \ 9$		$\bullet^4 \ 9$	
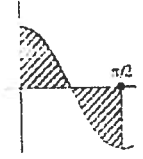
[SQA] 17.

(a) Evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos 2x \, dx$.

3

(b) Draw a sketch and explain your answer.

2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
(a)	3	3.2		3					3.2.4		
(b)	2	1.2	1	1					1.2.3	2.2.6	1992 P1 qu.14

$\bullet^1 \ \frac{1}{2}$	
$\bullet^2 \ \sin 2x$	
$\bullet^3 \ 0$	
$\bullet^4 \ $ diagram	
$\bullet^5 \ $ −ve and −ve cancel out	

[SQA]   18.

(a) Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$.   1

(b) Hence find $\int (\cos x + \sin x)^2 \, dx$.   3

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
			C	A/B	C	A/B	C	A/B		
(a)	1	2.3	1						2.3.3	Source
(b)	3	3.2		3					3.2.4	1993 P1 qu.19

- •¹ $\sin^2 x + \cos^2 x + 2\sin x \cos x$ and complete
- •² $x + c$
- •³ $-\cos 2x$
- •⁴ $\times \frac{1}{2}$

[SQA]   19. Find $\int \left(6x^2 - x + \cos x \right) dx$.   4

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
			C	A/B	C	A/B	C	A/B		
.	4	3.2	4						3.2.4	Source
										1995 P1 qu.3

- •¹ $2x^3$
- •² $-\frac{1}{2}x^2$
- •³ $\sin x$
- •⁴ $+c$

[SQA] 20.

(a) By writing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. 4

(b) Hence find $\int \sin^3 x\,dx$. 4

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
			C	A/B	C	A/B	C	A/B		Source
(a)	4	2.3	2	2					2.3.2, 2.3.3	**1995 Paper 2**
(b)	4	3.2		4					3.2.4	**Qu.9**

(a)   •¹ $\sin 2x\cos x + \cos 2x\sin x$

    •² $2\sin x\cos x\cos x + \ldots\ldots$

    •³ $\ldots\ldots + \left(1 - 2\sin^2 x\right)\sin x$

    •⁴ $2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

(b)   •⁵ $\int \frac{1}{4}(3\sin x - \sin 3x)\,dx$

    •⁶ $-3\cos x$

    •⁷ $+\cos 3x$

    •⁸ $+3$

[SQA] 21. (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. 2

(b) Hence write down $\int \dfrac{x^2}{(8 - x^3)^{\frac{1}{2}}}\,dx$. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	2	A/B	CN	C21	$-\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$	2002 P1 Q10
(b)	1	A/B	CN	C24	$-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$	

•¹ pd: process differentiation	•¹ $\frac{1}{2}(8 - x^3)^{-\frac{1}{2}}$
•² pd: use the chain rule	•² $\ldots \times -3x^2$
•³ ic: interpret answer from (a)	•³ $-\frac{2}{3}f(x)$ **or** $-\frac{2}{3}(8 - x^3)^{\frac{1}{2}}$

[SQA] 22. The curve $y = f(x)$ passes through the point $(\frac{\pi}{12}, 1)$ and $f'(x) = \cos 2x$.

Find $f(x)$. 3

part marks	Unit	non-calc		calc		calc neut		Content Reference :	3.2
		C	A/B	C	A/B	C	A/B	Main Additional	
3	3.2		3					3.2.4	Source
									1997 P1 qu.15

- $\bullet^1$ $\frac{1}{2}\sin 2x$
- $\bullet^2$ $1 = \frac{1}{2}\sin\frac{\pi}{6} + c$
- $\bullet^3$ $c = \frac{3}{4}$

[SQA] 23. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$ express y in terms of x. 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	2000 P1 Q8

$\bullet^1$ ss: know to integrate $\bullet^2$ pd: integrate $\bullet^3$ ic: interpret $(\frac{\pi}{9}, 1)$ $\bullet^4$ pd: process	$\bullet^1$ $y = \int \sin(3x)\,dx$ stated or implied by $\bullet^2$ $\bullet^2$ $-\frac{1}{3}\cos(3x)$ $\bullet^3$ $1 = -\frac{1}{3}\cos(\frac{3\pi}{9}) + c$ or equiv. $\bullet^4$ $c = \frac{7}{6}$

[SQA] 24. A curve for which $\dfrac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.

Find y in terms of x. 4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	CN	C18, C23	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	2001 P2 Q10

$\bullet^1$ pd: integrate trig function $\bullet^2$ pd: integrate composite function $\bullet^3$ ss: use given point to find "c" $\bullet^4$ pd: evaluate "c"	$\bullet^1$ $\int 3\sin(2x)\,dx$ stated or implied by $\bullet^2$ $\bullet^2$ $-\frac{3}{2}\cos(2x)$ $\bullet^3$ $\sqrt{3} = -\frac{3}{2}\cos(2 \times \frac{5}{12}\pi) + c$ $\bullet^4$ $c = \frac{1}{4}\sqrt{3} \ (\approx 0{\cdot}4)$

[SQA] 25. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$.

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C18, C22	$V = -\frac{4}{3}(4 - t)^{\frac{3}{2}} + \frac{32}{3}$	2002 P2 Q8

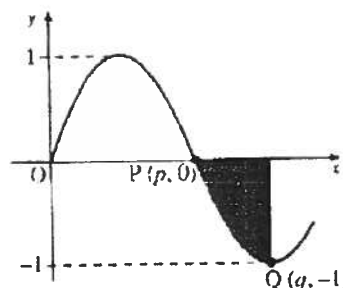|	
•¹ ss: know to integrate acceleration •² pd: integrate •³ ic: use initial conditions with const. of int. •⁴ pd: process solution	•¹ $V = \int(2(4 - t)^{\frac{1}{2}})\,dt$ *stated or implied by* •² •² $2 \times \frac{1}{-\frac{3}{2}}(4 - t)^{\frac{3}{2}}$ •³ $0 = 2 \times \frac{1}{-\frac{3}{2}}(4 - 0)^{\frac{3}{2}} + c$ •⁴ $c = 10\frac{2}{3}$

[SQA] 26. A sketch of part of the graph of $y = \sin 2x$ is shown in the diagram.

The points P and Q have coordinates $(p, 0)$ and $(q, -1)$.

(a) Write down the values of p and q.

(b) Find the area of the shaded region.

1

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
(a)	1	1.2	1						1.2.3		
(b)	4	3.2	1	3					2.2.6	3.2.1	1998 P1 qu.15

•¹ $\quad p = \frac{\pi}{2}$ and $q = \frac{3\pi}{4}$	•² $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (\sin 2x)\,dx$ •³ $-\frac{1}{2}\cos 2x$ •⁴ $-\frac{1}{2}$ •⁵ deal with −ve correctly giving $\frac{1}{2}$

[SQA] 27. An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape. (6)

| part | marks | Unit | non-calc | | calc | | calc neut | | Content Reference : | 3.2 |
			C	A/B	C	A/B	C	A/B	Main Additional	
										Source
-	6	3.2	2	4					3.2.4, 2.2.6	1996 Paper 2 Qu.5

(-) $\bullet^1$ evidence of two integrals

$\bullet^2$ $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \ dx \ and \ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \ dx$

$\bullet^3$ $\frac{1}{2}\sin 2x$

$\bullet^4$ $\frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin\frac{\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{4}$

$\bullet^5$ $\frac{1}{2}\sin\pi - \frac{1}{2}\sin\frac{\pi}{2} = -\frac{1}{2}$

$\bullet^6$ $1 - \frac{\sqrt{3}}{4}$

[SQA]   28.   Linktown Church is considering designs for a logo for their
Parish magazine. The 'C' is part of a circle and the centre of the
circle is the mid-point of the vertical arm of the 'L'. Since the 'L'
is clearly smaller than the 'C', the designer wishes to ensure that
the total length of the arms of the 'L' is as long as possible.
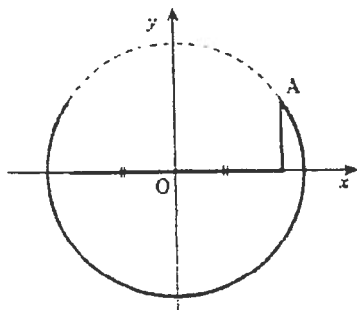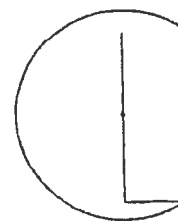
The designer decides to call the point where
the 'L' and 'C' meet A and chooses to draw
co-ordinate axes so that A is in the first
quadrant. With axes as shown, the equation of

the circle is $x^2 + y^2 = 20$.

(a)   If A has co-ordinates (x,y), show that the total length T of the arms of

the 'L' is given by $T = 2x + \sqrt{20 - x^2}$.   **(1)**

(b)   Show that for a stationary value of T, x satisfies the equation

$$x = 2\sqrt{20 - x^2}.$$ **(5)**

(c)   By squaring both sides, solve this equation.
Hence find the greatest length of the arms of the 'L'.   **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	1	0.1						1	0.1		Source
(b)	5	3.2					1	4	3.2.2		1995 Paper 2
(c)	3	1.3					1	2	1.3.15		Qu.11

(a)   •^{1} $T = x + x + y$ and $y^2 = 20 - x^2$

(b)   •^{2} appearance of $\frac{dT}{dx} = 2 + \ldots$

•^{3} $\frac{1}{2}\left(20 - x^2\right)^{-\frac{1}{2}}$

•^{4} $\times -2x$

•^{5} $\frac{dT}{dx} = 0$ stated or implied

•^{6} completing proof

(c)   •^{7} $x^2 = 4\left(20 - x^2\right)$

•^{8} $x = 4$ (accept $x = \pm 4$)

•^{9} justifying $x = 4$ gives $T_{max} = 10$

[SQA] 29. An oil production platform, $9\sqrt{3}$ km offshore, is to be connected by a pipeline to a refinery on shore, 100 km down the coast from the platform as shown in the diagram.

Oil Production Platform

$9\sqrt{3}$ km x km

y km   Refinery

$\longleftarrow$ - - - - - - - - - - 100 km - - - - - - - - $\longrightarrow$

The length of underwater pipeline is x km and the length of pipeline on land is y km. It costs £2 million to lay each kilometre of pipeline underwater and £1 million to lay each kilometre of pipeline on land.

(a) Show that the total cost of this pipeline is £$C(x)$ million where

$$C(x) = 2x + 100 - \left(x^2 - 243\right)^{\frac{1}{2}}.$$

(3)

(b) Show that $x = 18$ gives a minimum cost for this pipeline.
     Find this minimum cost and the corresponding total length of the pipeline.

(7)

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.2
			C	A/B	C	A/B	C	A/B		
(a)	3	0.1	1	2					0.1	Source
(b)	7	1.3	1	6					1.3.15, 3.2.2	1993 Paper 2 Qu.11

(a) •¹ $C = 2x + y$

     •² $\sqrt{x^2 - \left(9\sqrt{3}\right)^2}$

     •³ for completing proof

(b) •¹ knowing to differentiate

     •⁵ $\frac{1}{2}\left(x^2 - 243\right)^{-\frac{1}{2}}$

     •⁶ $\times 2x$

     •⁷ $C'(18) = 0$

     •⁸ justification of minimum e.g. nature table

     •⁹ $C = 127$

     •¹⁰ $x + y = 109$

	18⁻	18	18⁺
$C'(x)$	–	0	+
	\	—	/
		minimum	

[END OF WRITTEN QUESTIONS]