

# Functions/Graphs Past Papers Unit 1 Outcome 2

## Written Questions

[SQA] 1.  $f(x) = 3 - x$  and  $g(x) = \frac{3}{x}, x \neq 0$ .

(a) Find  $p(x)$  where  $p(x) = f(g(x))$ .

2

(b) If  $q(x) = \frac{3}{3-x}, x \neq 3$ , find  $p(q(x))$  in its simplest form.

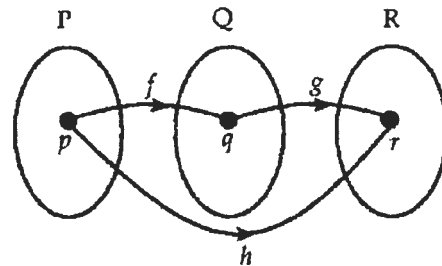
3

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(b)	2	C	CN	A4	$x$	
(b)	1	A/B	CN	A4		

<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: interpret composite func.</li> <li>•<sup>2</sup> pd: process</li> <li>•<sup>3</sup> ic: interpret composite func.</li> <li>•<sup>4</sup> pd: process</li> <li>•<sup>5</sup> pd: process</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f\left(\frac{3}{x}\right)</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>3 - \frac{3}{x}</math></li> <li>•<sup>3</sup> <math>p\left(\frac{3}{3-x}\right)</math> stated or implied by •<sup>4</sup></li> <li>•<sup>4</sup> <math>3 - \frac{3}{\frac{3}{3-x}}</math></li> <li>•<sup>5</sup> <math>x</math></li> </ul>
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[SQA] 2. The diagram illustrates three functions  $f, g$  and  $h$ . The functions are defined by  $f(x) = 2x + 5$  and  $g(x) = x^2 - 3$ .

The function  $h$  is such that whenever  $f(p) = q$  and  $g(q) = r$  then  $h(p) = r$ .



2  
2

- (a) If  $q = 7$ , find the values of  $p$  and  $r$ .
- (b) Find a formula for  $h(x)$ , in terms of  $x$ .

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.6		Source 1991 P1 qu.19
(b)	2	1.2		2					1.2.6		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>p = 1</math></li> <li>•<sup>2</sup> <math>r = 46</math></li> <li>•<sup>3</sup> <math>h(x) = g(f(x))</math></li> <li>•<sup>4</sup> <math>h(x) = (2x + 5)^2 - 3</math></li> </ul>
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- [SQA] 3. On a suitable set of real numbers, functions  $f$  and  $g$  are defined by  $f(x) = \frac{1}{x+2}$  and  $g(x) = \frac{1}{x} - 2$ .

Find  $f(g(x))$  in its simplest form.

3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
3	1.2	3						1.2.6		Source 1992 P1 qu.6

•<sup>1</sup>  $f\left(\frac{1}{x}-2\right)$

•<sup>2</sup>  $\frac{1}{\frac{1}{x}-2+2}$

•<sup>3</sup>  $x$

- [SQA] 4.  $f(x) = 2x - 1$ ,  $g(x) = 3 - 2x$  and  $h(x) = \frac{1}{4}(5 - x)$ .

(a) Find a formula for  $k(x)$  where  $k(x) = f(g(x))$ .

2

(b) Find a formula for  $h(k(x))$ .

2

(c) What is the connection between the functions  $h$  and  $k$ ?

1

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
(a) 2	1.2	2						1.2.6		Source 1993 P1 qu.13
(b) 2	1.2	2						1.2.6		
(c) 1	0.1		1					0.1		

•<sup>1</sup>  $f(3-2x)$

•<sup>2</sup>  $5-4x$

•<sup>3</sup>  $h(5-4x)$

•<sup>4</sup>  $x$

•<sup>5</sup> inverse of each other

[SQA] 5. A function  $f$  is defined on the set of real numbers by  $f(x) = \frac{x}{1-x}$ ,  $x \neq 1$ .

Find, in its simplest form, an expression for  $f(f(x))$ .

3

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
3	1.2	1	2					1.2.6		Source 1994 P1 qu.19

• <sup>1</sup>	$f\left(\frac{x}{1-x}\right)$
• <sup>2</sup>	$\frac{\frac{x}{1-x}}{1-\frac{x}{1-x}}$
• <sup>3</sup>	$\frac{x}{1-2x}$

[SQA] 6. The functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = \frac{1}{x^2-4}$  and  $g(x) = 2x + 1$ .

(a) Find an expression for  $h(x)$  where  $h(x) = g(f(x))$ . Give your answer as a single fraction.

3

(b) State a suitable domain for  $h$ .

1

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	3	2	1					1.2.6		Source
(b)	1		1					1.2.1		1995 P1 qu.11

• <sup>1</sup>	$g\left(\frac{1}{x^2-4}\right)$	• <sup>3</sup>	$\frac{x^2-2}{x^2-4}$
• <sup>2</sup>	$2\left(\frac{1}{x^2-4}\right)+1$	• <sup>4</sup>	"any domain which excludes 2"

- [SQA] 7. Functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = 2x$  and  $g(x) = \sin x + \cos x$ .

Find  $f(g(x))$  and  $g(f(x))$ .

4

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
4	1.2	4						1.2.6		Source 1997 P1 qu.3

- <sup>1</sup>  $f(\sin x + \cos x)$
- <sup>2</sup>  $2(\sin x + \cos x)$
- <sup>3</sup>  $g(2x)$
- <sup>4</sup>  $\sin 2x + \cos 2x$

- [SQA] 8. Functions  $f$  and  $g$  are defined by  $f(x) = 2x + 3$  and  $g(x) = \frac{x^2 + 25}{x^2 - 25}$  where  $x \in \mathbb{R}$ ,  $x \neq \pm 5$ .

The function  $h$  is given by the formula  $h(x) = g(f(x))$ .

For which real values of  $x$  is the function  $h$  **undefined**?

4

part marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
		C	A/B	C	A/B	C	A/B	Main	Additional	
4	1.2					2	2	1.2.6	1.2.1	Source 1989 P1 qu.19

- <sup>1</sup>  $g(2x+3)$
- <sup>2</sup>  $\frac{(2x+3)^2+25}{(2x+3)^2-25}$
- <sup>3</sup>  $(2x+3)^2 - 25 = 0$
- <sup>4</sup>  $x = 1, -4$

[SQA] 9. The functions  $f$  and  $g$  are defined on a suitable domain by  $f(x) = x^2 - 1$  and  $g(x) = x^2 + 2$ .

(a) Find an expression for  $f(g(x))$ .

2

(b) Factorise  $f(g(x))$ .

2

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2					2		1.2.6		Source 1998 P1 qu.6
(b)	2	1.2					1	1	0.1		

						OR
• <sup>1</sup>	$f(x^2 + 2)$	• <sup>3</sup>	$((x^2 + 2) + 1)((x^2 + 2) - 1)$	• <sup>3</sup>	$x^4 + 4x^2 + 3$	
• <sup>2</sup>	$(x^2 + 2)^2 - 1$	• <sup>4</sup>	$(x^2 + 3)(x^2 + 1)$	• <sup>4</sup>	$(x^2 + 3)(x^2 + 1)$	

- [SQA] 10. (a)  $f(x) = 2x + 1$ ,  $g(x) = x^2 + k$ , where  $k$  is a constant.
- (i) Find  $g(f(x))$ . (2)
- (ii) Find  $f(g(x))$ . (2)
- (b) (i) Show that the equation  $g(f(x)) - f(g(x)) = 0$  simplifies to  $2x^2 + 4x - k = 0$ . (2)
- (ii) Determine the nature of the roots of this equation when  $k = 6$ . (2)
- (iii) Find the value of  $k$  for which  $2x^2 + 4x - k = 0$  has equal roots. (3)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	1.2	4						1.2.6		Source 1996 Paper 2 Qu.4
(b)	7	2.1	7						2.1.6, 2.1.7, 0.1		

<p>(a) •<sup>1</sup> <math>g(2x+1)</math></p> <p>•<sup>2</sup> <math>(2x+1)^2 + k</math></p> <p>•<sup>3</sup> <math>f(x^2+k)</math></p> <p>•<sup>4</sup> <math>2(x^2+k)+1</math></p>	<p>(b) •<sup>5</sup> <math>4x^2 + 4x + k + 1</math> AND <math>2x^2 + 2k + 1</math></p> <p>•<sup>6</sup> <math>4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0</math></p> <p>so <math>2x^2 + 4x - k = 0</math></p> <p>•<sup>7</sup> <math>b^2 - 4ac = 16 - 4 \times 2 \times (-k) = 64</math></p> <p>•<sup>8</sup> so roots real &amp; distinct</p> <p>•<sup>9</sup> <math>b^2 - 4ac = 16 - 4 \times 2 \times (-k)</math></p> <p>•<sup>10</sup> <math>b^2 - 4ac = 0</math> for equal roots</p> <p>•<sup>11</sup> <math>k = -2</math></p>
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[SQA] 11. Functions  $f$  and  $g$  are defined on the set of real numbers by  $f(x) = x - 1$  and  $g(x) = x^2$ .

(a) Find formulae for

(i)  $f(g(x))$

(ii)  $g(f(x))$ .

4

(b) The function  $h$  is defined by  $h(x) = f(g(x)) + g(f(x))$ .

Show that  $h(x) = 2x^2 - 2x$  and sketch the graph of  $h$ .

3

(c) Find the area enclosed between this graph and the  $x$ -axis.

4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	1.2	4						1.2.6		Source 1999 Paper 2 Qu. 6
(b)	3	1.2	3					1.2.9	0.1		
(c)	4	2.2	4					2.2.6			

(a)	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(x^2)</math> stated or implied by •<sup>2</sup></li> <li>•<sup>2</sup> <math>x^2 - 1</math></li> <li>•<sup>3</sup> <math>g(x-1)</math> stated or implied by •<sup>4</sup></li> <li>•<sup>4</sup> <math>(x-1)^2</math></li> </ul>	(c)	<ul style="list-style-type: none"> <li>•<sup>8</sup> <math>\int_0^1 (2x^2 - 2x) dx</math></li> <li>•<sup>9</sup> <math>[\frac{2}{3}x^3 - x^2]</math></li> <li>•<sup>10</sup> <math>-\frac{1}{3}</math></li> <li>•<sup>11</sup> dealing with - ve</li> </ul>
(b)	<ul style="list-style-type: none"> <li>•<sup>5</sup> <math>(x-1)^2 + x^2 - 1</math> and complete proof</li> <li>•<sup>6</sup> sketch as shown</li> </ul>		
	<ul style="list-style-type: none"> <li>•<sup>7</sup> minimum at <math>(\frac{1}{2}, -\frac{1}{2})</math> calculated or on sketch</li> </ul>		

[SQA] 12. Functions  $f(x) = \sin x$ ,  $g(x) = \cos x$  and  $h(x) = x + \frac{\pi}{4}$  are defined on a suitable set of real numbers.

(a) Find expressions for:

(i)  $f(h(x))$ ;

(ii)  $g(h(x))$ .

2

(b) (i) Show that  $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$ .

(ii) Find a similar expression for  $g(h(x))$  and hence solve the equation  $f(h(x)) - g(h(x)) = 1$  for  $0 \leq x \leq 2\pi$ .

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	2	C	NC	A4	(i) $\sin(x + \frac{\pi}{4})$ , $\cos(x + \frac{\pi}{4})$	2001 P1 Q7
(b)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: interpret composite functions</li> <li>•<sup>2</sup> ic: interpret composite functions</li> <li>•<sup>3</sup> ss: expand <math>\sin(x + \frac{\pi}{4})</math></li> <li>•<sup>4</sup> ic: interpret</li> <li>•<sup>5</sup> ic: substitute</li> <li>•<sup>6</sup> pd: start solving process</li> <li>•<sup>7</sup> pd: process</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin(x + \frac{\pi}{4})</math></li> <li>•<sup>2</sup> <math>\cos(x + \frac{\pi}{4})</math></li> <li>•<sup>3</sup> <math>\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}</math> and <b>complete</b></li> <li>•<sup>4</sup> <math>g(h(x)) = \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x</math></li> <li>•<sup>5</sup> <math>(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x) - (\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x)</math></li> <li>•<sup>6</sup> <math>\frac{2}{\sqrt{2}} \sin x</math></li> <li>•<sup>7</sup> <math>x = \frac{\pi}{4}, \frac{3\pi}{4}</math> accept only radians</li> </ul>
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[SQA] 13. Functions  $f$  and  $g$  are defined on suitable domains by  $f(x) = \sin(x^\circ)$  and  $g(x) = 2x$ .

(a) Find expressions for:

(i)  $f(g(x))$ ;

(ii)  $g(f(x))$ .

2

(b) Solve  $2f(g(x)) = g(f(x))$  for  $0 \leq x \leq 360$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	2	C	CN	A4	(i) $\sin(2x^\circ)$ , (ii) $2\sin(x^\circ)$	2002 P1 Q3
(b)	5	C	CN	T10	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	

<ul style="list-style-type: none"> <li>•<sup>1</sup> ic: interpret <math>f(g(x))</math></li> <li>•<sup>2</sup> ic: interpret <math>g(f(x))</math></li> <li>•<sup>3</sup> ss: equate for intersection</li> <li>•<sup>4</sup> ss: substitute for <math>\sin 2x</math></li> <li>•<sup>5</sup> pd: extract a common factor</li> <li>•<sup>6</sup> pd: solve a 'common factor' equation</li> <li>•<sup>7</sup> pd: solve a 'linear' equation</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\sin(2x^\circ)</math></li> <li>•<sup>2</sup> <math>2\sin(x^\circ)</math></li> <li>•<sup>3</sup> <math>2\sin(2x^\circ) = 2\sin(x^\circ)</math></li> <li>•<sup>4</sup> appearance of <math>2\sin(x^\circ)\cos(x^\circ)</math></li> <li>•<sup>5</sup> <math>2\sin(x^\circ)(2\cos(x^\circ) - 1)</math></li> <li>•<sup>6</sup> <math>\sin(x^\circ) = 0</math> and <math>0, 180, 360</math></li> <li>•<sup>7</sup> <math>\cos(x^\circ) = \frac{1}{2}</math> and <math>60, 300</math></li> </ul> <p>or</p> <ul style="list-style-type: none"> <li>•<sup>6</sup> <math>\sin(x^\circ) = 0</math> and <math>\cos(x^\circ) = \frac{1}{2}</math></li> <li>•<sup>7</sup> <math>0, 60, 180, 300, 360</math></li> </ul>
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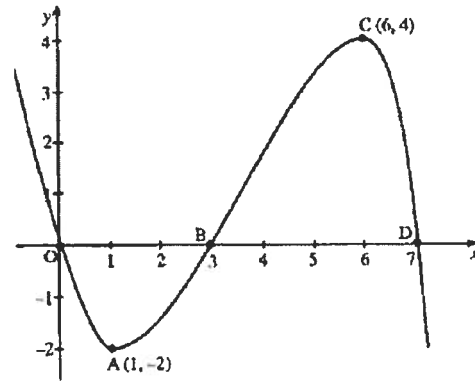
[SQA] 14. Part of the graph of  $y = f(x)$  is shown in the diagram.

On separate diagrams sketch the graphs of

(a)  $y = f(x+1)$

(b)  $y = -2f(x)$ .

Indicate on each graph the images of O, A, B, C and D.



1

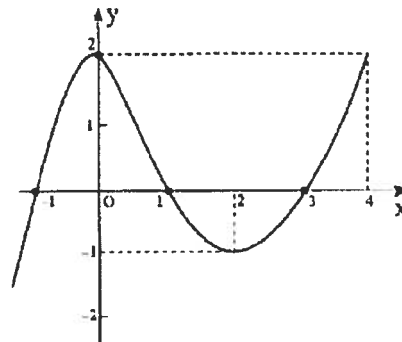
part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.4		Source
(b)	3	1.2	1	2					1.2.4		1999 P1 qu.10

<ul style="list-style-type: none"> <li>•<sup>1</sup> translation of <math>\begin{pmatrix} -1 \\ 0 \end{pmatrix}</math></li> <li>•<sup>2</sup> positions of images of A, B, C, D, O clear from the sketch</li> </ul>		<ul style="list-style-type: none"> <li>•<sup>3</sup> reflect in x-axis</li> <li>•<sup>4</sup> double y-coordinates</li> <li>•<sup>5</sup> positions of images of A, B, C, D, O clear from the sketch</li> </ul>	
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[SQA] 15. The diagram shows the graph of  $y = f(x)$ .

Sketch the graph of  $y = 2 - f(x)$ .



3

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	3	1.2	1	2					1.2.4		Source
											1993 P1 qu.8

<ul style="list-style-type: none"> <li>•<sup>1</sup> reflection in Ox</li> <li>•<sup>2</sup> translation <math>\begin{pmatrix} 0 \\ 2 \end{pmatrix}</math></li> <li>•<sup>3</sup> two trans. in correct order, annotate diagram</li> </ul>	
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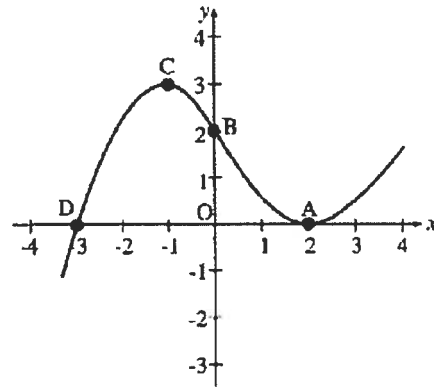
[SQA] 16. Part of the graph of  $y = f(x)$  is shown in the diagram.

On separate diagrams sketch the graphs of

(i)  $y = f(x - 1)$

(ii)  $y = -f(x) - 2$

indicating on each graph the images of A, B, C and D.



5

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(i)	2	1.2					2		1.2.4		Source
(ii)	3	1.2					3		1.2.4		1996 P1 qu.8

<ul style="list-style-type: none"> <li>•<sup>1</sup> translation of <math>\begin{pmatrix} 1 \\ 0 \end{pmatrix}</math></li> <li>•<sup>2</sup> <math>A(3, 0) B(1, 2) C(0, 3) D(-2, 0)</math></li> <li>•<sup>3</sup> reflect in <math>x</math> - axis</li> <li>•<sup>4</sup> translation of <math>\begin{pmatrix} 0 \\ -2 \end{pmatrix}</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>5</sup> <math>A(2, -2) B(0, -4) C(-1, -5) D(-3, -2)</math></li> </ul>		
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[END OF WRITTEN QUESTIONS]