Higher Maths Homework (17)

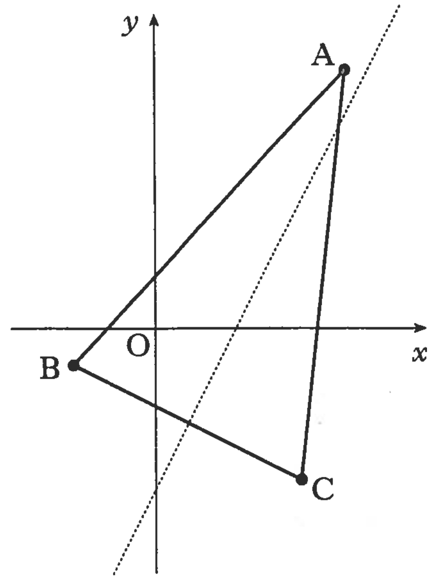
1. The vertices of triangle ABC are A(7, 9), B(−3, −1) and C(5, −5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.

(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. **4**

(*b*) Find the equation of the median from C. **3**

(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. **3**



2. The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

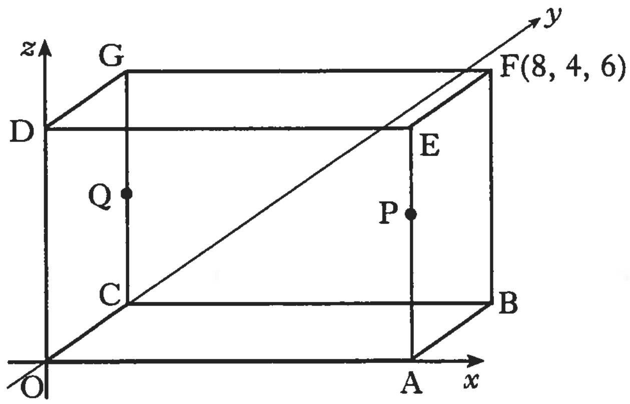
P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

(*c*) Find the size of angle QPA. **5**

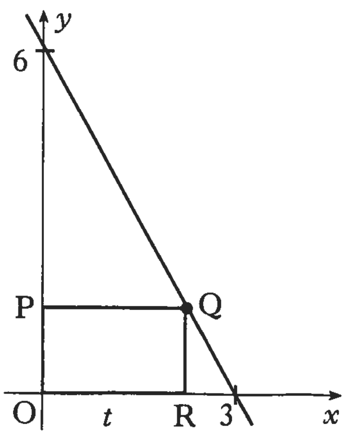


3. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

(*a*) Show that QR = $6 - 2t$. **3**

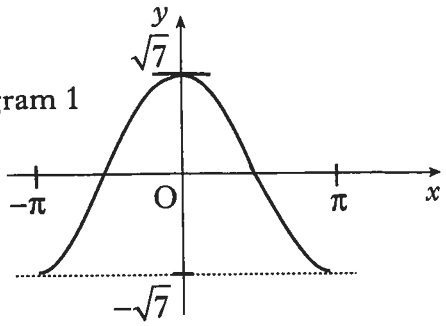
(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**



(4) (a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

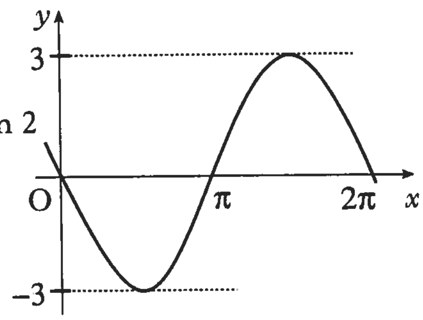
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2 **2**



(b) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

(5) (a) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(b) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(c) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. **5**

(6) Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \leq x < 360$. **5**