

# Cfe Higher Homework (13)

①  $A$  and  $B$  are acute angles such that  $\tan A = \frac{3}{4}$  and  $\tan B = \frac{5}{12}$ .

Find the exact value of

- |                      |   |
|----------------------|---|
| (a) $\sin 2A$        | 2 |
| (b) $\cos 2A$        | 1 |
| (c) $\sin(2A + B)$ . | 2 |

② Find the equation of the tangent to the curve  $y = 3x^3 + 2$  at the point where  $x = 1$ . 4

③ (a) The function  $f$  is defined by  $f(x) = x^3 - 2x^2 - 5x + 6$ .

The function  $g$  is defined by  $g(x) = x - 1$ .

Show that  $f(g(x)) = x^3 - 5x^2 + 2x + 8$ . 4

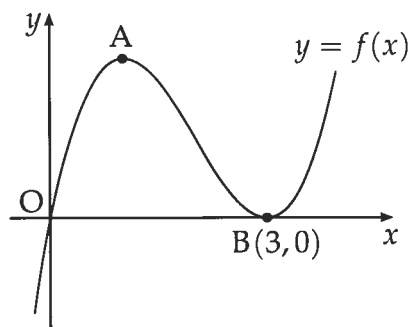
(b) Factorise fully  $f(g(x))$ . 3

(c) The function  $k$  is such that  $k(x) = \frac{1}{f(g(x))}$ .

For what values of  $x$  is the function  $k$  not defined? 3

④ A sketch of the graph of  $y = f(x)$  where  $f(x) = x^3 - 6x^2 + 9x$  is shown below.

The graph has a maximum at  $A$  and a minimum at  $B(3, 0)$ .



(a) Find the coordinates of the turning point at  $A$ . 4

(b) Hence sketch the graph of  $y = g(x)$  where  $g(x) = f(x + 2) + 4$ .

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. 2

(c) Write down the range of values of  $k$  for which  $g(x) = k$  has 3 real roots. 1

5 Solve the equation  $\cos 2x^\circ + \cos x^\circ = 0$ ,  $0 \leq x < 360$ .

5

6 (a) For a particular radioactive substance the mass  $m$  (in grams) at time  $t$  (in years) is given by

$$m = m_0 e^{-0.02t}$$

where  $m_0$  is the original mass.

If the original mass is 500 grams, find the mass after 10 years.

(2)

(b) The half-life of any material is the time taken for half of the mass to decay.

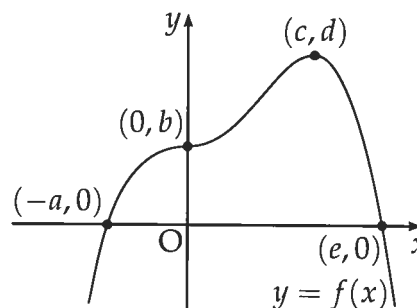
Find the half-life of this substance.

(3)

7 The graph of a function  $f$  intersects the  $x$ -axis at  $(-a, 0)$  and  $(e, 0)$  as shown.

There is a point of inflexion at  $(0, b)$  and a maximum turning point at  $(c, d)$ .

Sketch the graph of the derived function  $f'$ .



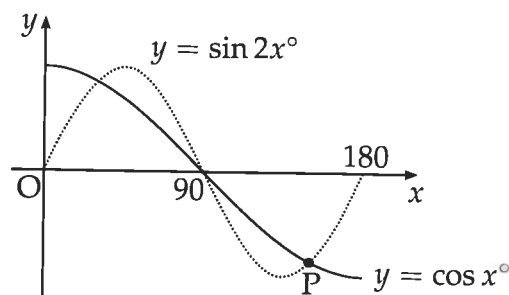
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8 (a) Solve the equation  $\sin 2x^\circ - \cos x^\circ = 0$  in the interval  $0 \leq x \leq 180$ .

4

(b) The diagram shows parts of two trigonometric graphs,  $y = \sin 2x^\circ$  and  $y = \cos x^\circ$ .

Use your solutions in (a) to write down the coordinates of the point P.



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