CfE Higher Maths 2016 Paper 2

1 (a) (i) $M = \left(\frac{-6+10}{2}, \frac{2+6}{2}\right)$

$= \underline{(2, 4)}$

(ii) $m_{PM} = \frac{4-(-4)}{2-0}$

$= \frac{8}{2}$

$= 4$

Equation $y - b = m(x-a)$

$y + 4 = 4(x-0)$

$\underline{\underline{y = 4x - 4}}$

(b) $m_{PR} = \frac{6+4}{10-0}$

$= 1$ $m_{perp} = -1$

Equation $y - b = m(x-a)$

$y - 4 = -1(x-2)$

$y - 4 = -x + 2$

$\underline{\underline{y = -x + 6}}$

(c) midpoint of PR $= \left(\frac{0+10}{2}, \frac{-4+6}{2}\right)$

$= (5, 1)$

When $x = 5$ in $y = -x + 6$

$y = -5 + 6$

$y = 1$ so $(5,1)$ lies on line L.

ie L passes through midpoint of PR.

2 $x^2 - 2x + 3 - p = 0$

For no real roots $b^2 - 4ac < 0$

Here $a = 1$ $b = -2$ $c = (3-p)$

$b^2 - 4ac < 0$

$\Rightarrow$ $4 - 4 \times 1 \times (3-p) < 0$

$4 - 12 + 4p < 0$

$4p < 8$

$p < 2$

3 (a) (i)

-1	2	-9	3	14
		-2	11	-14
	2	-11	14	$\boxed{0}$

remainder 0
so $x+1$ is a factor

(ii) $2x^3 - 9x^2 + 3x + 14 = 0$

$(x+1)(2x^2 - 11x + 14) = 0$

$(x+1)(2x - 7)(x - 2) = 0$

$x = -1$ $x = \frac{7}{2}$ $x = 2$

(b) (i) $A(-1, 0)$ $B(2, 0)$

(ii) area $= \int_{-1}^{2} (2x^3 - 9x^2 + 3x + 14)\,dx$

$= \left[\frac{x^4}{2} - 3x^3 + \frac{3x^2}{2} + 14x \right]_{-1}^{2}$

$= \left(\frac{2^4}{2} - 3(2)^3 + 6 + 28 \right) - \left(\frac{1}{2} + 3 + \frac{3}{2} - 14 \right)$

$= 18 - (-9)$

$= 27$ square units

4 (a) C_1 $(x+5)^2+(y-6)^2=9$

centre $(-5,6)$ radius 3.

C_2 $x^2+y^2-6x-16=0$

$2g=-6$ $2f=0$ $c=-16$

$g=-3$ $f=0$

centre $(3,0)$ radius $=\sqrt{9+0-(-16)}$

$=5.$

(b). distance between centres $=\sqrt{(3-(-5))^2+(0-6)^2}$

$=\sqrt{64+36}$

$=10$

sum of radii $=3+5$

$=8.$

Since sum of radii < distance between centres

the circles do not intersect.

5 (a) $\vec{AB}=\underline{b}-\underline{a}$

$=\begin{pmatrix}-10\\18\\7\end{pmatrix}-\begin{pmatrix}-2\\2\\5\end{pmatrix}$

$=\begin{pmatrix}-8\\16\\2\end{pmatrix}$

$\vec{AC}=\underline{c}-\underline{a}$

$=\begin{pmatrix}-4\\-6\\21\end{pmatrix}-\begin{pmatrix}-2\\2\\5\end{pmatrix}$

$=\begin{pmatrix}-2\\-8\\16\end{pmatrix}$

(b) $\cos B\hat{A}C=\dfrac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|}$

$|\vec{AB}|=\sqrt{64+256+4}$

$=18$

$|\vec{AC}|=\sqrt{4+64+256}$

$=18$

$\vec{AB}\cdot\vec{AC}=16-128+32=-80$

So $\cos B\hat{A}C = \frac{-80}{18 \times 18}$

$\cos B\hat{A}C = -0.2469\ldots$

$B\hat{A}C = 104.3°$ (1d.p)

6 (a) $B(t) = 200e^{0.107t}$

When $t = 0$ $B(0) = 200e^{0}$

$= 200.$

(b) $B(t) = 400$

$\Rightarrow 400 = 200e^{0.107t}$

$e^{0.107t} = 2$

$0.107t = \ln 2$

$t = \frac{\ln 2}{0.107}$

$t = 6.478\ldots$ hrs.

$=$ 6hours. 29 mins.

7 length $= 3x$ width $= 2y$

$A = lb$

$108 = 6xy$

$y = \frac{18}{x}$

total length of fencing $= 9x + 8y$

$L(x) = 9x + 8\left(\frac{18}{x}\right)$

$L(x) = 9x + \frac{144}{x}$

(b) $L'(x) = 9 - 144x^{-2}$

$= 9 - \frac{144}{x^2}.$

For minimum length $L'(x) = 0$

$$9 - \frac{144}{x^2} = 0$$

$$9x^2 - 144 = 0$$

$$9x^2 = 144$$

$$x^2 = \frac{144}{9}$$

$$x = \frac{12}{3}$$ $(x > 0)$ so take +ve square root

$$x = 4.$$

<u>Nature</u>

x	$\overset{1}{\rightarrow}$	4	$\overset{10}{\rightarrow}$
$L'(x) = 9 - \frac{144}{x^2}$	-135	0	7.56
	$\searrow$	$-$	$\nearrow$

so $x = 4$ gives a minimum length.

8 (a) $5\cos x - 2\sin x = k\cos(x + a)$

$= k(\cos x \cos a - \sin x \sin a)$

$= k\cos a \cos x - k\sin a \sin x$

Compare $k\cos a = 5$

$k\sin a = 2.$

Square and add $k^2 = 25 + 4$

$k = \sqrt{29}$

Divide. $\tan a = \frac{2}{5}$

$a = 0.381$ (3 s f)

$$5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.381)$$

(b) Solve $10 + 5\cos x - 2\sin x = 12$

$$5\cos x - 2\sin x = 2.$$

$$\sqrt{29}\cos(x + 0.381) = 2$$

$$\cos(x + 0.381) = \frac{2}{\sqrt{29}}$$

ra 1.19

S	A ✓
T	C ✓

$$x + 0.381 = 1.19,\ 5.09$$

$$\underline{\underline{x = 0.81,\ 4.71}}$$

(9) $f(x) = \int \frac{2x+1}{\sqrt{x}}\,dx$

$$= \int \left(\frac{2x}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}\right) dx$$

$$= \int \left(2x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$$

$$= \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

$$f(9) = 40 \Rightarrow \frac{4}{3}\sqrt{9}^{3} + 2\sqrt{9} + c = 40$$

$$36 + 6 + c = 40$$

$$c = -2$$

$$\underline{\underline{f(x) = \frac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 2}}$$

10 (a)

$$y = (x^2+7)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+7)^{-\frac{1}{2}} \times 2x$$

$$= \frac{x}{(x^2+7)^{\frac{1}{2}}}$$

(b)

$$\int \frac{4x}{\sqrt{x^2+7}}\, dx$$

$$= 4\int \frac{x}{(x^2+7)^{\frac{1}{2}}}\, dx$$

$$= 4(x^2+7)^{\frac{1}{2}} + C.$$

11 (a) Show that $\sin 2x \tan x = 1 - \cos 2x$

LHS $= \sin 2x \tan x$

$$= 2\sin x \cos x \cdot \frac{\sin x}{\cos x}$$

for $\frac{\pi}{2} < x < \frac{3\pi}{2}$ $\tan x$ is defined ie $\cos x \neq 0$.

$$= 2\sin^2 x$$

$$= 1 - (1 - 2\sin^2 x)$$

$$= 1 - \cos 2x$$

$$= \text{RHS}.$$

(b) $f(x) = \sin 2x \tan x$

$$= 1 - \cos 2x$$

so $f'(x) = 2\sin 2x.$