CfE Higher Maths 2016 Paper 1

(1) $y + 4x = 7$

$y = -4x + 7 \qquad m = -4$

Equation $\quad y - b = m(x - a) \qquad (a, b) = (-2, 3)$

$y - 3 = -4(x - (-2)) \qquad m = -4$

$y - 3 = -4x - 8$

$\underline{\underline{y = -4x - 5}}$

(2) $y = 12x^3 + 8\sqrt{x}$

$y = 12x^3 + 8x^{\frac{1}{2}}$

$\frac{dy}{dx} = 36x^2 + 4x^{-\frac{1}{2}}$

$= 36x^2 + \frac{4}{\sqrt{x}}$

(3) (a) $u_{n+1} = \frac{1}{3}u_n + 10$

$u_4 = \frac{1}{3}u_3 + 10$

$= \frac{1}{3} \times 6 + 10$

$= \underline{\underline{12}}.$

(b) Limit exists since $-1 < \frac{1}{3} < 1$

(c) $L = \frac{b}{1-a}$

$= \frac{10}{1-\frac{1}{3}}$

$= 10 \div \frac{2}{3}$

$= 10 \times \frac{3}{2}$

$= \underline{\underline{15}}$

(4) midpoint $\left(\frac{-7+1}{2} \quad \frac{3+5}{2}\right)$

$= (-3, 4)$

radius $= \sqrt{(1-(-3))^2 + (5-4)^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

circle $\underline{(x+3)^2 + (y-4)^2 = 17}$

(5) $\int 8\cos(4x+1)\,dx$

$= \frac{1}{4} \times 8\sin(4x+1) + c$

$= 2\sin(4x+1) + c$

(6) (a) If $y = f(x)$ then $x = f^{-1}(y)$

$y = 3x+5$

$y-5 = 3x$

$x = \frac{y-5}{3}$

$f^{-1}(y) = \frac{y-5}{3}$

$\underline{f^{-1}(x) = \frac{x-5}{3}}$

(b) $g(2) = 7$

$\Rightarrow g^{-1}(7) = 2.$

(7) (a) $\vec{FH} = \vec{FG} + \vec{GH}$

$= -2\underline{i} - 6\underline{j} + 3\underline{k} + 3\underline{i} + 9\underline{j} - 7\underline{k}$

$= \underline{i} + 3\underline{j} - 4\underline{k}$

(b) $\vec{FE} = \vec{FH} + \vec{HE}$

$= \vec{FH} - \vec{EH}$

$= \underline{i} + 3\underline{j} - 4\underline{k} - (2\underline{i} + 3\underline{j} + \underline{k})$

$= -\underline{i} - 5\underline{k}$

8 $y = 3x - 5 \quad \dots \text{①}$

$x^2 + y^2 + 2x - 4y - 5 = 0 \quad \dots \text{②}$

Substitute ① in ②

$x^2 + (3x-5)^2 + 2x - 4(3x-5) - 5 = 0$

$x^2 + 9x^2 - 30x + 25 + 2x - 12x + 20 - 5 = 0$

$10x^2 - 40x + 40 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$ equal roots so line is a tangent.

When $x = 2$ in ① $y = 6 - 5$
$= 1$ point $(2, 1)$.

9 $y = x^3 + 3x^2 - 24x$

$\frac{dy}{dx} = 3x^2 + 6x - 24$

$= 3(x^2 + 2x - 8)$

$= 3(x+4)(x-2)$

For stationary points $\frac{dy}{dx} = 0$

$3(x+4)(x-2) = 0$

$x = -4$ or $x = 2$.

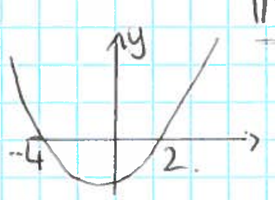
(ii) Nature

x	$\overset{(-5)}{\rightarrow}$	-4	$\overset{(0)}{\rightarrow}$	2	$\overset{(3)}{\rightarrow}$
$\frac{dy}{dx} = 3(x+4)(x-2)$	21	0	-24	0	21
	/	—	\	—	/

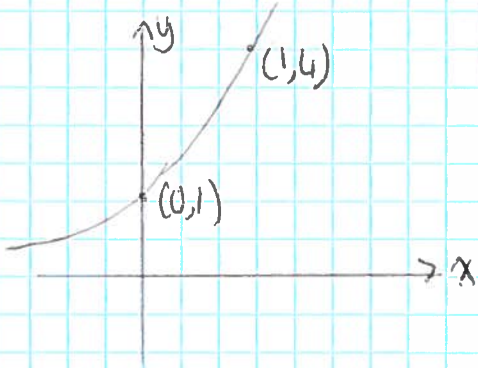
Increasing for $x < -4$ and $x > 2$.

or $\frac{dy}{dx} > 0$

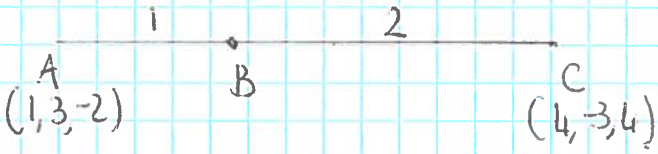
$\Rightarrow (x+4)(x-2) > 0$



10 $f(x) = \log_4 x$ $f^{-1}(x) = 4^x$



11 (a)



$$\frac{AB}{BC} = \frac{1}{2}$$

$$2\vec{AB} = \vec{BC}$$

$$2(\underline{b} - \underline{a}) = \underline{c} - \underline{b}$$

$$2\underline{b} - 2\underline{a} = \underline{c} - \underline{b}$$

$$3\underline{b} = \underline{c} + 2\underline{a}$$

$$\underline{b} = \frac{1}{3}\left(\begin{pmatrix}4\\-3\\4\end{pmatrix} + \begin{pmatrix}2\\6\\-4\end{pmatrix}\right)$$

$$\underline{b} = \frac{1}{3}\begin{pmatrix}6\\3\\0\end{pmatrix}$$

$$= \begin{pmatrix}2\\1\\0\end{pmatrix}$$

$\underline{\underline{B(2,1,0)}}$

(b) $$\vec{AC} = \underline{c} - \underline{a}$$

$$= \begin{pmatrix}4\\-3\\4\end{pmatrix} - \begin{pmatrix}1\\3\\-2\end{pmatrix}$$

$$= \begin{pmatrix}3\\-6\\6\end{pmatrix}$$

$$|\vec{AC}| = \sqrt{9+36+36}$$

$$= \sqrt{81} = 9$$

$\underline{\underline{k = \frac{1}{9}}}$

12 (a)

$$\begin{aligned} h(x) &= f(g(x)) \\ &= f(3-x) \\ &= 2(3-x)^2 - 4(3-x) + 5 \\ &= 2(9-6x+x^2) - 12 + 4x + 5 \\ &= 18 - 12x + 2x^2 - 12 + 4x + 5 \\ &= 2x^2 - 8x + 11 \end{aligned}$$

as required

(b)

$$\begin{aligned} h(x) &= 2x^2 - 8x + 11 \\ &= 2(x^2 - 4x) + 11 \\ &= 2\left((x-2)^2 - (-2)^2\right) + 11 \\ &= 2(x-2)^2 - 8 + 11 \\ &= 2(x-2)^2 + 3 \end{aligned}$$

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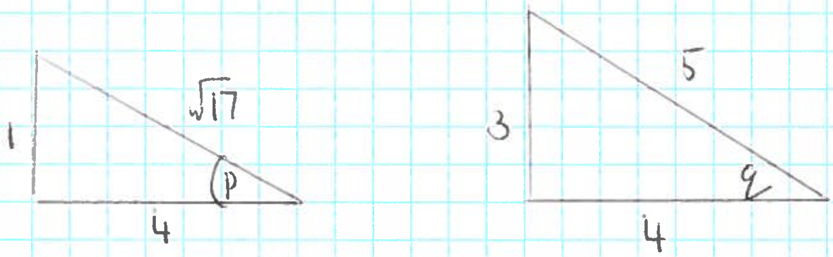

$$\begin{aligned} \cos(q-p) &= \cos q \cos p + \sin q \sin p \\ &= \frac{4}{5} \times \frac{4}{\sqrt{17}} + \frac{3}{5} \times \frac{1}{\sqrt{17}} \\ &= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}} \\ &= \frac{19}{5\sqrt{17}} \\ &= \frac{19\sqrt{17}}{5 \times 17} \\ &= \frac{19\sqrt{17}}{85} \end{aligned}$$

as required

14 (a) $\log_5 25$

$= \log_5 5^2$

$= 2\log_5 5$

$= \underline{\underline{2}}$.

(b) $\log_4 x + \log_4 (x-6) = \log_5 25$

$\log_4 x(x-6) = 2$

$x(x-6) = 4^2$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ or $x = \cancel{-2}$ not possible. so $\underline{\underline{x = 8}}$

15 (a) $\underline{\underline{a = 4 \qquad b = -5}}$

$f(x) = k(x-4)(x+5)^2$.

Substitute $(1, 9)$

$9 = k(-3)6^2$

$9 = -108k$

$\underline{\underline{k = -\frac{1}{12}}}$.

b) Graph shifts down. $\underline{\underline{d > 9}}$.