H

National
Qualifications
2016

**Mathematics
Paper 1
(Non-Calculator)**

THURSDAY, 12 MAY
9:00 AM – 10:10 AM

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer should not be taken as an indication of how much to write. It is not necessary to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space **you must clearly identify the question number** you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA

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A/PB

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

| $f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + c$
$\cos ax$	$\dfrac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks – 60

1. Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$.

    2

2. Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\dfrac{dy}{dx}$.

    3

3. A sequence is defined by the recurrence relation $u_{n+1} = \dfrac{1}{3}u_n + 10$ with $u_3 = 6$.

    (a) Find the value of u_4.

    1

    (b) Explain why this sequence approaches a limit as $n \to \infty$.

    1

    (c) Calculate this limit.

    2

4. A and B are the points $(-7, 3)$ and $(1, 5)$.

    AB is a diameter of a circle.
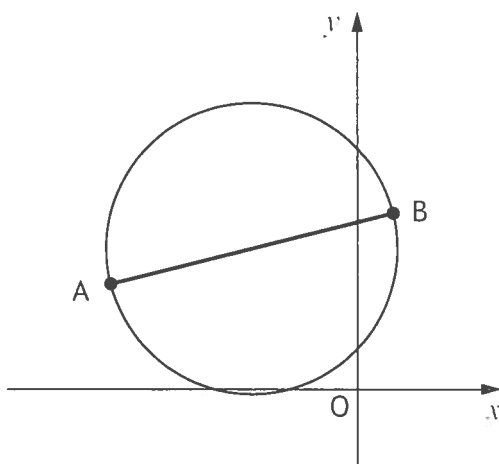
    Find the equation of this circle.

    3

[Turn over

5. Find $\int 8\cos(4x+1)\,dx$.　　　　2

6. Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
   The inverse functions f^{-1} and g^{-1} both exist.

   (a) Given $f(x) = 3x + 5$, find $f^{-1}(x)$.　　　3

   (b) If $g(2) = 7$, write down the value of $g^{-1}(7)$.　　　1

7. Three vectors can be expressed as follows:

   $$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$$

   $$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$$

   $$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$

   (a) Find $\overrightarrow{FH}$.　　　2

   (b) Hence, or otherwise, find $\overrightarrow{FE}$.　　　2

8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact.　　　5

9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$.

4

(b) Hence determine the range of values of x for which the function f is strictly increasing.

2

10. The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.
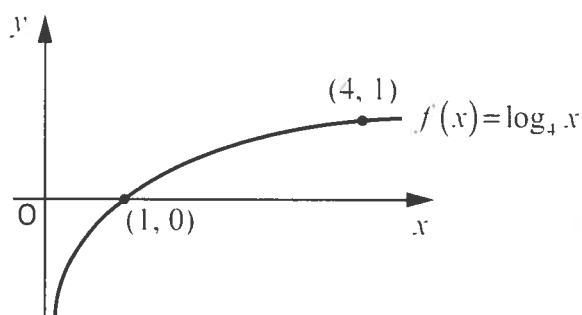
The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function.

2

11. (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.

Point B divides AC in the ratio $1 : 2$.

Find the coordinates of B.

2

(b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.

Determine the value of k.

3

[Turn over

12. The functions f and g are defined on $\mathbb{R}$, the set of real numbers by

$f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.
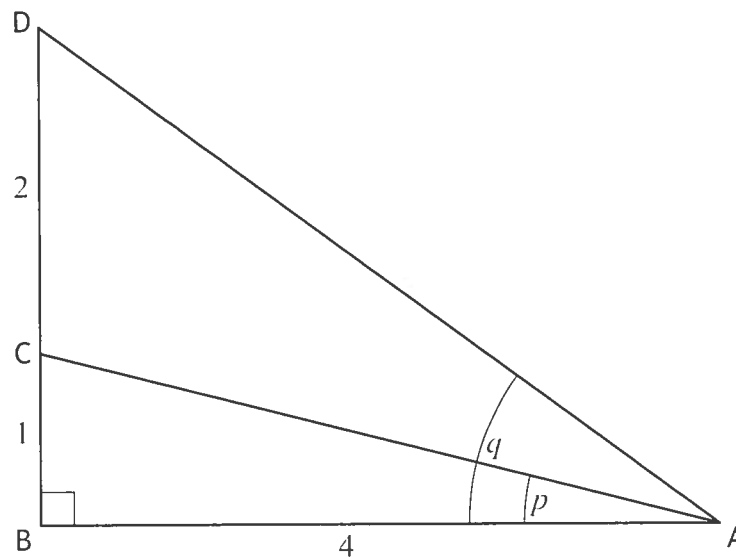
(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$.   **2**

(b) Express $h(x)$ in the form $p(x+q)^2 + r$.   **3**

13. Triangle ABD is right-angled at B with angles BAC = p and BAD = q and lengths as shown in the diagram below.

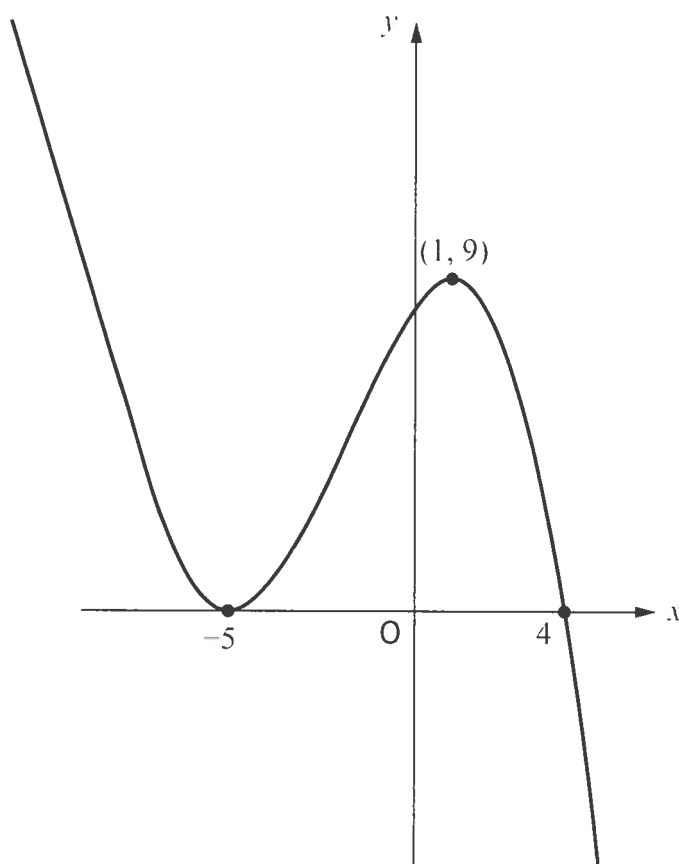Show that the exact value of $\cos(q - p)$ is $\dfrac{19\sqrt{17}}{85}$.   **5**

14. (a) Evaluate $\log_5 25$.

1

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$.

5

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.

(a) Find the values of a, b and k.

3

(b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.

1

[END OF QUESTION PAPER]