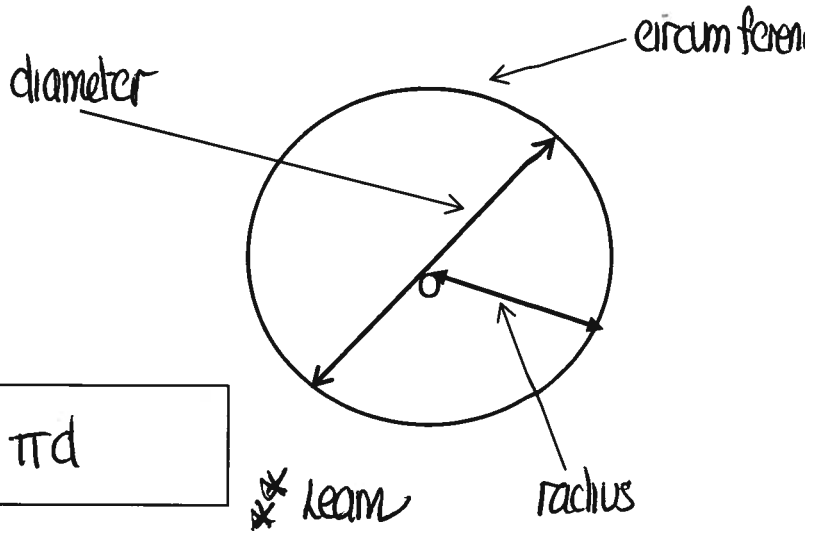
9. Calculating the Length of an Arc or the Area of a Sector of a Circle

(Textbook – Chapter 9)

You should already know:

PARTS OF A CIRCLE



CIRCUMFERENCE

$$C = \pi d$$

** learn

Examples

Calculate the circumference (to 2 s.f.) of a circle with

(a) diameter = 6cm

$C = \pi d$
$= \pi \times 6$
$= 18.849...$
$= 19\text{cm}$ (2s.f)

(b) radius = 6cm

$d = 2 \times 6$
$= 12\text{cm}$

$C = \pi d$
$= \pi \times 12$
$= 37.699....$
$= 38\text{cm}$ (2s.f)

AREA

$$A = \pi r^2$$

** learn.

Examples

Find the area (to 2s.f.) of the circles with

(a) radius = 3cm

$A = \pi r^2$
$A = \pi \times 3^2$
$A = 28.274....$
$A = 28\text{cm}^2$ (2s.f)

(b) diameter = 3cm

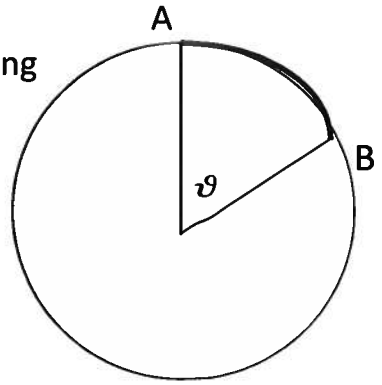
$r = 1.5\text{cm}$
$A = \pi r^2$
$A = \pi \times 1.5^2$
$A = 7.0685...$
$A = 7.1\text{cm}^2$ (2s.f)

Arc Length

An arc of a circle is a 'piece' of the circumference

Working out the length of an arc is found by simply finding the matching fraction of the whole circumference (πd)

$$Minor\ Arc\ AB = \frac{\vartheta}{360}\pi d$$



Examples

Calculate the length (to 3 s.f.) of

(a) Minor arc AB

$= \frac{110}{360} \times \pi d$ ← circumference

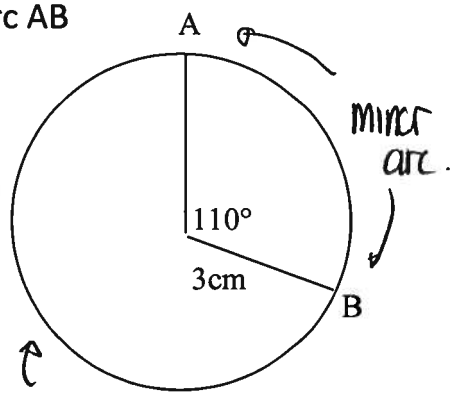
angle fraction

$= \frac{110}{360} \times \pi \times 6$

$= 5.7595...$

$= 5.76\ cm$ (3 s.f)

(b) Major arc AB



(b) $360 - 110°$

$= 250°$

Major arc $AB = \frac{250}{360} \times \pi \times 6$

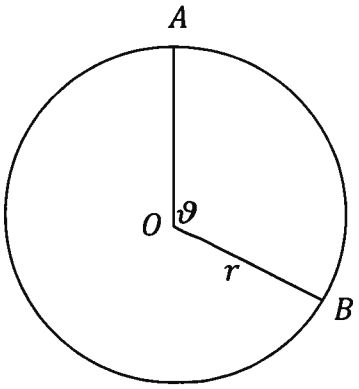
$= 13.0899...$

$= 13.1cm$ (3 s.f)

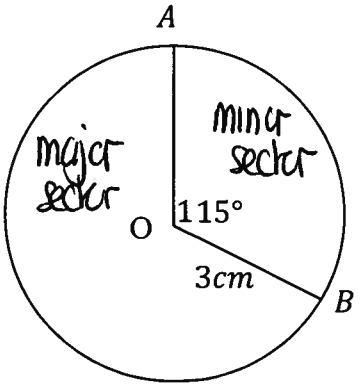
Sector Area

Sectors are 'pieces' of a circle (like slices of pizza or cake)
Area of a sector of a circle is simply the appropriate fraction of the whole area (πr^2)

$$Area\ of\ Minor\ Sector\ AOB = \frac{\vartheta}{360}\pi r^2$$



Example
Find the area (to 2 s.f.) of
(a) minor sector AOB (b) major sector AOB



(a) minor sector AOB

$= \frac{115}{360} \times \pi r^2$

$\frac{115}{360}$ ↑ angle

πr^2 ↑ area

$= \frac{115}{360} \times \pi \times 3^2$

$= 9.03207...$

$= 9.0 cm^2$ (2 s.f.)

(b) major sector AOB

$360 - 115 = 245^\circ$

$= \frac{245}{360} \times \pi r^2$

$= \frac{245}{360} \times \pi \times 3^2$

$= 19.262...$

$= 19 cm^2$ (2 s.f.)

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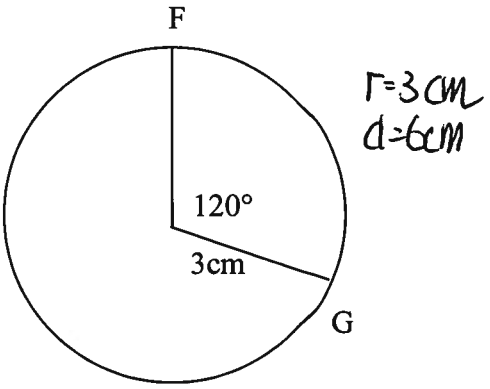
Expressing Arc Lengths and Sector Areas in Terms of π

Examples

1. Without the use of a calculator and leaving your answer in terms of π, calculate the length of the major arc FG.



$360 - 120 = 240°$

major arc FG

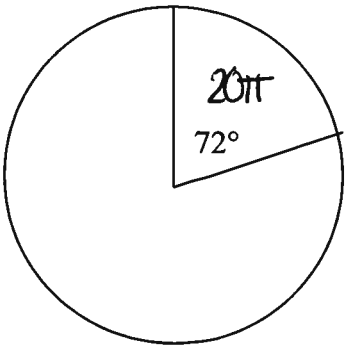
$= \frac{240}{360} \times \pi d$

$= \frac{240}{360} \times \pi \times 6$

$= \frac{2}{3} \times \pi \times 6^2$

$= 4\pi$ cm.

2. The area of this sector is 20π cm^2. Find the length of the radius of this circle.



area of minor sector $= \frac{72}{360} \times \pi r^2$

$20\pi = \frac{72}{360} \times \pi \times r^2$

$20\pi = \frac{1}{5} \times \pi \times r^2$

$100\pi = \pi r^2$

$100 = r^2$

$r = 10$ cm

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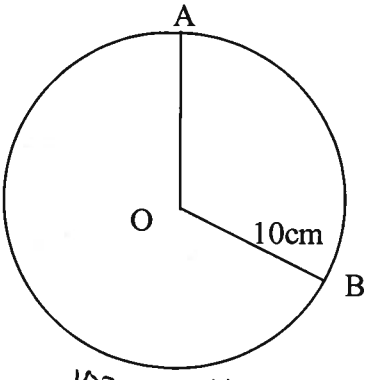
Working Backwards: Finding Sector Angle From Given Arcs and Sectors

We can establish that

$$Fraction\ of\ Circle = \frac{angle\ at\ centre}{360} = \frac{length\ of\ arc}{\pi d} = \frac{area\ of\ sector}{\pi r^2}$$

Examples

1. If the length of the minor arc AB is 18cm, calculate:
   a) the angle at the centre
   b) the area of minor sector OAB



(a) angle $= \frac{18}{\pi d} \times 360$

length of arc ÷ fraction × 360.

$= \frac{18}{\pi \times 20} \times 360$

$= 103.132...$

$= 103°$ (nearest degree)

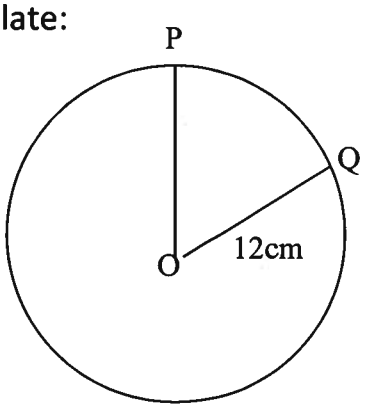
(b) area $= \frac{103}{360} \times \pi r^2$

$= \frac{103}{360} \times \pi \times 10^2$

$= 89.884...$

$= 89.9\ cm^2$ (1d.p)

2. The area of the minor sector POQ is $52cm^2$. Calculate:
   a) the angle at the centre
   b) length of minor arc PQ



(a) angle $= \frac{52}{\pi r^2} \times 360$

area ÷ fraction

$= \frac{52}{\pi \times 12^2} \times 360$

$= 41.382...$

$= 41.4°$ (1d.p)

(b) arc PQ $= \frac{41.4}{360} \times \pi d$

$= \frac{41.4}{360} \times \pi \times 24$

$= 8.6707...$

$= 8.7cm$ (1d.p)

• p74 Ex 9C