

2. Simplifying Expressions Using the Laws of Indices

(Textbook – Chapter 2)

Revision

1. $4 \times 4 \times 4 = 4^3$

2. $p \times p \times p \times y \times y \times y \times y \times 14 = 14p^3y^4$

3. $3p^4 = 3 \times p \times p \times p \times p$

4. $(2p)^3 = 2p \times 2p \times 2p$

5. If $a = 2$, $b = 3$ evaluate

(i) $a^2 + b^2$
 $= 2^2 + 3^2$
 $= 4 + 9$
 $= 13$

(ii) $(a+b)^2$
 $= (2+3)^2$
 $= 5^2$
 $= 25$

(iii) $3a^2$
 $= 3 \times 2^2$
 $= 3 \times 4$
 $= 12$

(iv) $(3a)^2$
 $= (3 \times 2)^2$
 $= 6^2$
 $= 36$

Rule 1

$a^0 = 1$

(anything to the power 0 is 1)

e.g. 5^0
 $= 1$

x^0
 $= 1$

$(4a^4bc^7df)^0$
 $= 1$

• p14 Ex 2A

Negative Indices

Rule 2	$a^{-m} = \frac{1}{a^m}$
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Examples

1. Evaluate

(a) 3^{-2}
 $= \frac{1}{3^2}$
 $= \frac{1}{9}$

(b) 2^{-5}
 $= \frac{1}{2^5}$
 $= \frac{1}{32}$

2. Write with negative indices.

(a) $\frac{1}{x^3}$
 $= x^{-3}$

(b) $\frac{3}{x^3}$
 $= 3x^{-3}$

(c) $\frac{1}{3x^3}$
 $= \frac{1}{3}x^{-3}$

(d) $\frac{5}{6y^4}$
 $= \frac{5}{6}y^{-4}$

↑
 numbers stay
 in the same place

3. Write with positive indices.

(a) p^{-2}
 $= \frac{1}{p^2}$

(b) $2x^{-1}$
 $= \frac{2}{x}$

(c) $\frac{p^{-5}}{5}$
 $= \frac{1}{5p^5}$

(d) $\frac{3c^{-2}}{7}$
 $= \frac{3}{7c^2}$

• p15 Ex 2B

Multiplying and Dividing in Index Form

Rule 3 $a^m \times a^n = a^{m+n}$

When multiplying you add the indices

Examples

1. $5^6 \times 5^7$
 $= 5^{13}$

2. $x^7 \times x^5$
 $= x^{12}$

3. $5b^3 \times 2b$
 $= 10b^4$

↑
 multiply numbers first

4. $a^4 \times a^{-5}$
 $= a^{-1}$
 $= \frac{1}{a}$

always leave answers with positive powers

Rule 4 $a^m \div a^n = a^{m-n}$

When dividing you subtract the indices

Examples

(i) $6^5 \div 6^3$
 $= 6^2$

(ii) $y^8 \div y^3$
 $= y^5$

(iii) $p^4 \div p^{-3}$
 $= p^7$

(iv) $20a^6 \div 4a^2$
 $= 5a^4$

↑
 divide numbers first

(v) $\frac{b^5 \times 4b^3}{2b^2}$
 $= \frac{4b^8}{2b^2}$
 $= 2b^6$

• p17 Ex 2C

Raising a Power to a Further Power

Rule 5 $(a^m)^n = a^{mn}$

To find the power of a power multiply the indices

Examples

(i) $(y^6)^3$
 $= y^{18}$

(ii) $(2xy^2)^2$
 $= 2^2 \times y^4$
 $= 4x^2y^4$

(iii) $(9^4)^{-2}$
 $= 9^{-8}$
 $= \frac{1}{9^8}$

(iii) Solve for x $(2^x)^3 = 2^9$
 $2^{3x} = 2^9$
 so $3x = 9$
 $x = 3$

Fractional Indices**Rule 6**

$$a^n = \sqrt[n]{a^m}$$

The numerator (top) is the power, the denominator (bottom) is the root.

Examples

1. Write in root form.

(a) $x^{\frac{3}{4}}$

$$= \sqrt[4]{x^3}$$

(b) $y^{\frac{2}{5}}$

$$= \sqrt[5]{y^2}$$

(c) $a^{\frac{5}{4}}$

$$= \sqrt[4]{a^5}$$

2. Write in index form.

(a) $\sqrt[3]{a}$

$$= a^{\frac{1}{3}}$$

(b) $\sqrt[3]{a^2}$

$$= a^{\frac{2}{3}}$$

(c) $\sqrt{u^5}$

$$= u^{\frac{5}{2}}$$

3. Evaluate

(a) $16^{\frac{1}{2}}$

$$= \sqrt{16}$$

$$= 4$$

(b) $16^{\frac{1}{4}}$

$$= \sqrt[4]{16}$$

$$= 2$$

(c) $16^{\frac{3}{4}}$

$$= \sqrt[4]{16^3}$$

$$= 2^3$$

$$= 8$$

4. Simplify

(a) $x^{\frac{1}{2}} \times x^2$

$= x^{\frac{5}{2}}$

$\frac{1}{2} + 2$
 $= 2\frac{1}{2}$
 $= \frac{5}{2}$

write as top heavy fraction

(b) $p^{\frac{3}{4}} \times p^{\frac{-1}{4}}$

$= p^{\frac{2}{4}}$

$= p^{\frac{1}{2}}$

(c) $p^{\frac{3}{4}} \div p$

$= p^{-\frac{1}{4}}$

(d) $(p^{\frac{3}{4}})^2$

$= p^{\frac{3}{2}}$

(e) $\frac{x^3 \times x^{-1}}{x^2}$

$= \frac{x^2}{x^2}$

$= 1$

(g) $\frac{y^{\frac{2}{3}} \times y^{\frac{1}{3}}}{y^2}$

$= y^{\frac{1}{3}}$

$= \frac{y^2}{y^{\frac{5}{3}}}$

$= \frac{1}{y^{\frac{1}{3}}}$

Scientific Notation (Standard Form)

In scientific notation all numbers can be written in the form

$$a \times 10^b \quad \text{where } 1 \leq a \leq 10$$

Examples

1. Write these numbers in standard form.

(a) 34000
 $= 3.4 \times 10^4$

↑
 decimal point goes after the first number

(b) 0.000007
 $= 7 \times 10^{-6}$

negative since small number

(c) 60.9 million
 $= 6$

(d) 0.001004
 $= 1.004 \times 10^{-5}$

2. Write the following numbers in full.

(a) 3.8×10^5
 $= 380\,000$

(b) 2.08×10^{-4}
 $= 0.000208$

• p23 Ex 2G

Calculations Using Scientific Notation

Examples

Calculate expressing your answer in scientific notation.

1. $(3.4 \times 10^9) \times (5 \times 10^{12})$
 $= 3.4 \times 5 \times 10^9 \times 10^{12}$
 $= 17 \times 10^{21}$
 $= 1.7 \times 10^{22}$

2. $(1.4 \times 10^{15}) \div (2 \times 10^{11})$
 $= 14 \div 2 \times 10^{15} \div 10^{11}$
 $= 0.7 \times 10^4$
 $= 7 \times 10^3$

↑
 first number must be between 1 and 10

• p24 Ex 2H