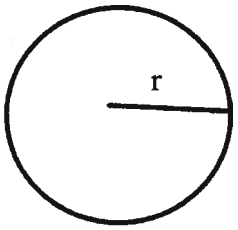
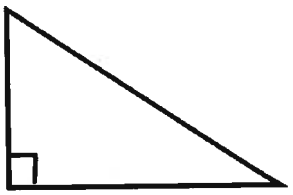
11. Calculating the Volume of a Standard Solid

(Textbook – Chapter 10)

Reminder



$A = \pi r^2$



$A = \frac{1}{2} \times base \times height$

Volume of Prisms

A Prism is a solid shape with a uniform 'cross section', meaning that if the shape was cut anywhere, the 'cut bit' would look the same each time.

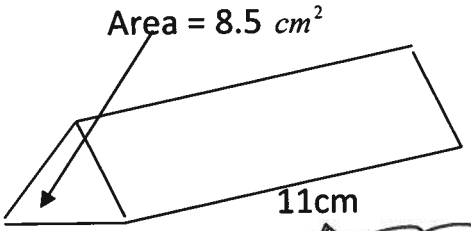
V = (area of the cross section) x height

V = Ah.

* learn.

Examples

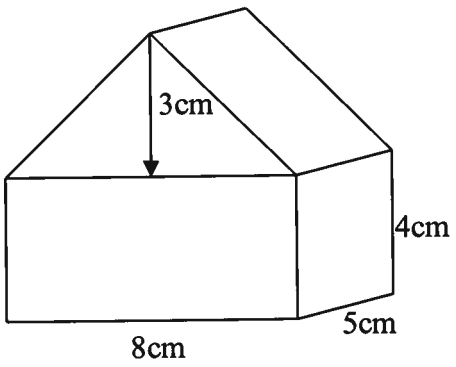
1. Calculate the volume of this shape



V = Ah ← formula
V = 8.5 x 11 ← working
V = 93.5 cm^3 ← answer with units

height ie distance between two ends

2. Calculate the volume of this solid.



Area of end

Rectangle.
A = lb
= 8 x 4
= 32 cm^2

Triangle
A = $\frac{1}{2}$bh
= $\frac{1}{2}$ x 8 x 3
= 12 cm^2.

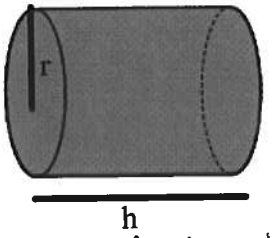
total = 32 + 12
= 44 cm^2.

V = Ah
= 44 x 5
= 220 cm^3.

Cylinders

$$V = \pi r^2 h$$

Learn.



↑ height is distance between two ends

Example

Calculate the volume (to 2 s. f.) of cylinders with

(a) radius = 3.2cm
height = 6.2cm

(b) diameter = 3m
height = 1.5m

(a) $V = \pi r^2 h$

$V = \pi \times 3.2^2 \times 6.2$

$V = 199.453...$

$V = 200\ cm^3$ (2 s.f)

(b) $r = 3 \div 2$

$= 1.5m$

$V = \pi r^2 h$

$V = \pi \times 1.5^2 \times 1.5$

$V = 10.602...$

$V = 11m^3$ (2 s.f)

Remember
- formula
- working
- answer, rounded with units.

Volume of a Sphere

$$V = \frac{4}{3}\pi r^3$$

Examples

1. Calculate the volume of a sphere with

(a) radius = 8cm

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \pi \times 8^3$

$V = 2144.66...$

$V = 2100 cm^3$ (2 s.f)

(b) diameter = 3m

$r = 1.5m$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \pi \times 1.5^3$

$V = 14.137...$

$V = 14 m^3$ (2 s.f)

2. A metal cube with sides 8cm is melted down to make metal balls of radius 0.4cm.
How many balls can be made from the block?

Volume of cube.

$V = l^3$

$V = 8 \times 8 \times 8$

$V = 512 cm^3$

Volume of ball

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3} \times \pi \times 0.4^3$

$V = 0.26808... cm^3$

↑ keep number on calculator to maintain accuracy

Number of balls $= 512 \div 0.26808...$

$= 1909.859...$

$= 1909$ whole balls.

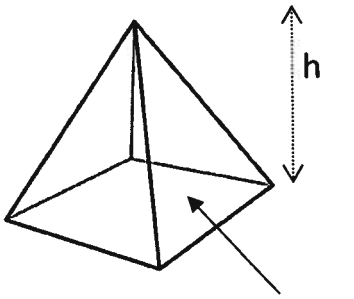
don't round up because there is not enough to make 1910 balls!!

- p77 Ex 10A Q1,2,4,5,8

Volume of a Pyramid

$$V = \frac{1}{3} A h$$

A = area of the base
h = height



Example

Calculate the volume of a pyramid with a square base of side 8 cm and height of 12cm.

$V = \frac{1}{3} Ah$

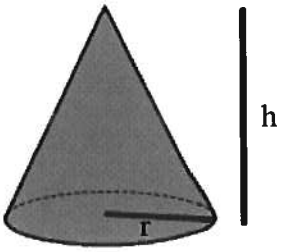
Area of base $A = lb$
$= 8 \times 8$
$= 64 cm^2$.

So $V = \frac{1}{3} Ah$
$V = \frac{1}{3} \times 64 \times 12$
$V = 256 cm^3$.

- p79 Ex 10B Q1 & 4

Volume of a Cone

$$V = \frac{1}{3}\pi r^2 h$$



r = radius of the base
h = height

Examples

Calculate (to 2 s. f.) the volume of a cone with:

(a) radius = 6mm
height = 8mm

(b) diameter = 7cm
height = 4cm

(a) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3} \times \pi \times 6^2 \times 8$

$V = 301.592\ldots$

$V = 300\ mm^3$ (2 s.f)

(b) $r = 7 \div 2$

$= 3.5cm$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3.5^2 \times 4$

$= 51.3126\ldots$

$= 51cm^3$ (2 s.f)

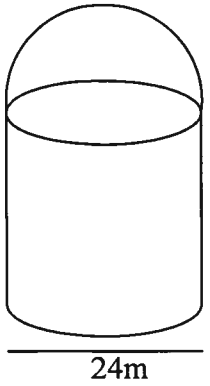
- p81 Ex 10C Q1, 4, 5

Composite Shapes

These need to be split up appropriately and worked out separately.
Add up these bits to find total volume.

Examples

Calculate the volume (to 2 s. f.) of
(a) An astronomical observatory (a cylinder and a hemisphere)



Cylinder

$V = \pi r^2 h$

$r = 24 \div 2 = 12\text{cm}$

$= \pi \times 12^2 \times 40$

$= 5760\pi$

↖ leave in terms to avoid error here.

Hemisphere ← half a sphere.

$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 12^3$

$= 1152\pi$

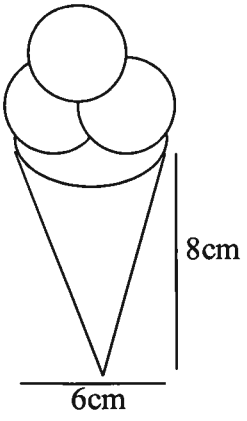
total volume $= 5760\pi + 1152\pi$

$= 6912\pi$

$= 21714.688...$

$= 22000\ m^3$ (2 s.f)

(b) This ice cream cone (each scoop is a sphere of radius 2cm) to 3 s. f.



Cone

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 3^2 \times 8$

$= 24\pi$

Sphere

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 2^3$

$= \frac{32}{3}\pi$

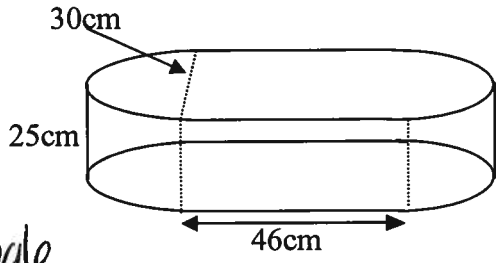
total volume $= 24\pi + 3 \times \frac{32}{3}\pi$

$= 56\pi$

$= 175.929...$

$= 176\ cm^3$ (3 s.f)

(c) A garden trough is shaped as a prism with height 25cm. It's cross section is a rectangle and 2 semicircles. Calculate it's volume in cm^3 (2 s. f.)



Area of end

= area of circle + area of rectangle

$= \pi r^2 + lb$

$= \pi \times 15^2 + 46 \times 30$

$= 225\pi + 76$

$V = Ah$

$= (225\pi + 76) \times 25$

$= 19571.4586...$

$= 20\,000\ cm^3$ (2 s.f)

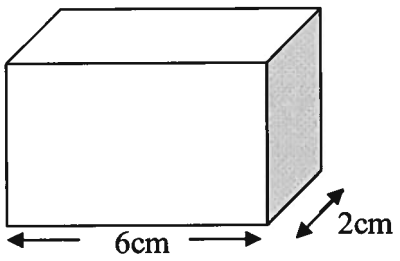
- p82 Ex 10D Q1 - 6

Finding the Dimensions of Shapes Given the Volume

We need to be able to work back from knowing the volume of a shape to find particular lengths of the shape

Examples

1.



The volume of this cuboid is 36 cm^3
What is its height ?

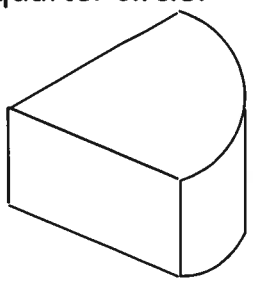
$V = lbh$ ← Formula

$36 = 6 \times 2 \times h$ ← put in what yar know

$36 = 12h$

$h = 3cm$ (solve)

2. A new design of trough has a cross section which is a quarter circle.
It has a height of 20cm.
It holds 30 000cm^3.
Find the radius of the cross section.



Quarter of a cylinder

so $V = \frac{1}{4}\pi r^2 h$

$30\,000 = \frac{1}{4}\pi r^2 h$

$30\,000 = \frac{1}{4} \times \pi \times r^2 \times 20$

$30\,000 = 5\pi \times r^2$

$r^2 = 30000 \div 5\pi$

$r^2 = \frac{6000}{\pi}$

$r = \sqrt{\frac{6000}{\pi}} = 43.701\ldots$

$= 43.7cm$ (3 s.f)

3. Calculate the radius of a sphere with a volume of $30m^3$.

Sphere $V = \frac{4}{3}\pi r^3$

$30 = \frac{4}{3}\pi r^3$

$(\times 3)$ $90 = 4\pi r^3$

$r^3 = \frac{90}{4\pi}$

$r = \sqrt[3]{\frac{90}{4\pi}}$

$= 1.9275...$

$= 1.93\,m$ (3s.f)

4. Calculate the height of a pyramid with a volume of $40cm^3$ and base area of $11\ cm^2$.

$V = \frac{1}{3}Ah$

$40 = \frac{1}{3} \times 11 \times h$

$120 = 11h$

$h = \frac{120}{11}$

$h = 10.9090...$

$h = 10.9\ cm$ (3s.f)

5. Calculate the height of a cone with a volume of $87cm^3$ and a radius of 4 cm.

$V = \frac{1}{3}\pi r^2 h$

$87 = \frac{1}{3} \times \pi \times 4^2 \times h$

$(\times 3) \quad 261 = 16\pi h$

$h = \frac{261}{16\pi}$

$h = 5.192...$

$h = 5.19\ cm\ (3sf)$

- p77 Ex 10A Q3, 6
- p79 Ex 10B Q2, 3
- p81 Ex 10C Q2, 3