1. Working With Surds

(Textbook - Chapter 1)

Sets of Numbers

In mathematics we use different sets of numbers.

The set of **natural numbers** N = {1 , 2 , 3 , 4 , 5 , ...}

The set of **whole numbers** W = {0 , 1 , 2 , 3 , 4 , ...}

The set of **integers** Z = {..... , -3 , -2 , -1 , 0 , 1 , 2 , 3 .....}

The set of **rational numbers** $\mathbb{Q}$ = all numbers that can be written as fractions.

eg $\frac{2}{7}$, $\frac{-3}{99}$, 4.

The set of real numbers R → all numbers that you know (includes all of the above, π, $\sqrt{3}$ etc)

SURDS

A surd is a square root of a number which can't be found exactly e.g $\sqrt{2}$, $\sqrt{3}$

Examples

Which of the following are surds?

1. $\sqrt{49}$
   $= 7$ no

2. $\sqrt{\frac{9}{4}} = \frac{3}{2}$ no

3. $\sqrt{18}$ yes.
   $= 4.2426\ldots$ (not exact)

4. $\sqrt{0.25}$
   $= 0.5$ no

5. $\sqrt[3]{27}$
   $= 3$ no

Simplifying Surds

e.g. $\sqrt{12}$

Write 12 as the product of two numbers where one of the numbers is a square number

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

SQUARE NUMBERS

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225

Examples

1. $\sqrt{48}$

$= \sqrt{16 \times 3}$

$= 4\sqrt{3}$

2. $\sqrt{56}$

$= \sqrt{4 \times 14}$

$= 2\sqrt{14}$

3. $\sqrt{500}$

$= \sqrt{100 \times 5}$

$= 10\sqrt{5}$

4. $2\sqrt{72}$

$= 2\sqrt{36 \times 2}$

$= 2 \times 6\sqrt{2}$

$= 12\sqrt{2}$

- p5 Ex 1A Q 1 & 3

Adding and Subtracting Surds

Adding and subtracting surds is similar to algebra. Only like surds can be added or subtracted.

1. $3\sqrt{3}+5\sqrt{3}-\sqrt{3}$

$= 7\sqrt{3}$

2. $4\sqrt{7}+\sqrt{5}-3\sqrt{7}+4\sqrt{5}$

$= \sqrt{7}+5\sqrt{5}$

Sometimes the surds need to be simplified first.

3. $\sqrt{18}+4\sqrt{2}$

$= \sqrt{9\times2}+4\sqrt{2}$

$= 3\sqrt{2}+4\sqrt{2}$

$= 7\sqrt{2}$

4. $\sqrt{119}+\sqrt{45}-3\sqrt{18}$

$= \sqrt{119}+\sqrt{9\times5}-3\sqrt{9\times2}$

$= \sqrt{119}+3\sqrt{5}-9\sqrt{2}$

- p5 Ex 1A Q 2 & 4

Multiplying and Dividing Surds

Rules

1) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

2) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Remember to simplify answers if possible.

Examples

1. $\sqrt{5} \times \sqrt{6}$

$= \sqrt{30}$

2. $\frac{\sqrt{20}}{\sqrt{4}}$

$= \sqrt{5}$

3. $3\sqrt{2} \times \sqrt{2}$

$= 3\sqrt{4}$

$= 3 \times 2$

$= 6$

4. $\sqrt{6} \times \sqrt{3}$

$= \sqrt{18}$

$= \sqrt{9 \times 2}$

$= 3\sqrt{2}$

5. $(\sqrt{5})^2$

$= 5$

6. $\frac{2\sqrt{3} \times \sqrt{4} \times 3\sqrt{2}}{\sqrt{8}}$

$= \frac{6\sqrt{24}}{\sqrt{8}}$ divide

$= 6\sqrt{3}$

- p6 Ex 1B

Using Surds

Examples

1. Find the area of this rectangle.

$A = lb$

$= \sqrt{6} \times 2\sqrt{3}$

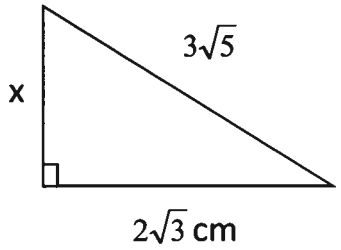
$= 2\sqrt{18}$

$= 2\sqrt{9 \times 2}$

$= 6\sqrt{2}$

since $\sqrt{9} = 3$ and $2 \times 3 = 6$

2. Calculate x leaving your answer as a surd in simplest form.



$x^2 = (3\sqrt{5})^2 - (2\sqrt{3})^2$ ← square $\sqrt{3}$

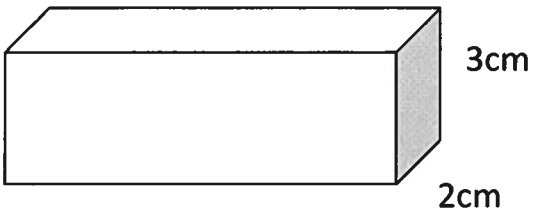
$x^2 = 9 \times 5 - 4 \times 3$ ← square 2

$x^2 = 45 - 12$

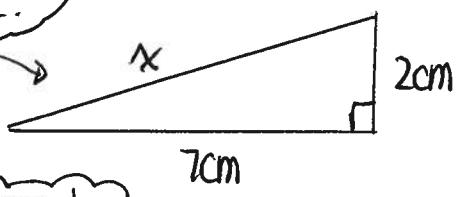
$x^2 = 33$

$x = \sqrt{33}$ cm.

3. Find the exact length of the space diagonal of this cuboid.



diagonal on base.



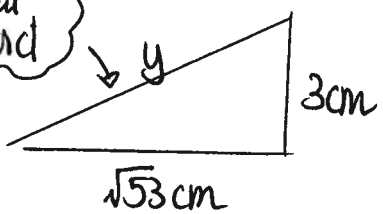
$x^2 = 7^2 + 2^2$

$x^2 = 49 + 4$

$x^2 = 53$

$x = \sqrt{53}$.

space diagonal through cuboid



$y^2 = (\sqrt{53})^2 + 3^2$

$y^2 = 53 + 9$

$y^2 = 62$

$y = \sqrt{62}$

Space diagonal $= \sqrt{62}$cm

- p8 Ex 1C

Rationalizing The Denominator

Rationalizing the denominator means making the bottom of the fraction into a whole number.
If we have a single surd on the bottom of a fraction we multiply the top and bottom of the fraction by that surd.

Examples

1. $\frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{4}}$

$= \frac{\sqrt{2}}{2}$

2. $\frac{3}{\sqrt{5}}$

$= \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{3\sqrt{5}}{\sqrt{25}}$

$= \frac{3\sqrt{5}}{5}$

3. $\frac{4}{3\sqrt{7}}$

$= \frac{4}{3\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{4\sqrt{7}}{3\sqrt{49}}$

$= \frac{4\sqrt{7}}{3\times 7}$

$= \frac{4\sqrt{7}}{21}$

4. $\frac{1}{\sqrt{12}}$

$= \frac{1}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$

$= \frac{\sqrt{12}}{12}$

$= \frac{\sqrt{4\times 3}}{12}$

$= \frac{2\sqrt{3}}{12}$

$= \frac{\sqrt{3}}{6}$

- p10 Ex 1D