Variation (1)

Direct Variation

1. Re-write each of the following statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) C varies directly as t. (b) V varies directly as the square of r.

   (c) The time (T) of swing of a simple pendulum varies as the square root of its length (l).

   (d) The volume (V) of a spherical object varies as the cube of its radius (r).

2. If $y \propto x$ and $y = 4$ when $x = 2$, find the constant of variation k.

   Hence find y when $x = 5$.

3. If y varies as the square of x and if $y = 3$ when $x = 2$, find the value of y when $x = 12$.

4. The value of a diamond (v) varies as the square of its weight (w).

   Given that a diamond weighing 2 carats is worth £1200, find the value of a diamond of similar quality which weighs 5 carats.

5. The time of vibration (T) of a pendulum varies as the square root of its length (l).

   Given that the length of a pendulum with a vibration time of 2 seconds is 100 cm, find the time of vibration of a pendulum measuring 64 cm.

6. The height (H) reached by a small fountain of water varies as the square of the water's velocity (v) leaving the nozzle.

   A velocity of 2 metres per second causes a height of 14 cm.

   What height is reached by a water velocity of 4 metres per second?

Inverse Variation

1. Re-write each of the following statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) N varies inversely as p. (b) V varies inversely as the square root of h.

   (c) The resistence (e) of a wire varies inversely as the square of its diameter (d).

2. If y varies inversely as the cube of x and $y = 16$ when $x = 2$, find y when $x = 4$.

3. The maximum speed (v) at which a railway engine can travel, on the flat, varies inversely as the square of the number of loaded wagons (n) attached to it.

   With four wagons attached, the train has a maximum speed of 40 km/h.

   What is the maximum speed of the train when 8 wagons are attached?

4. The time (T) of one oscillation of a vibrating magnetometer varies inversely as the square root of the horizontal intensity (h) of the earth's magnetic field.

   In Iceland, $T = 0 \cdot 75$ seconds when $h = 0 \cdot 12$ gauss.

   Find the time of one oscillation in Hollywood where $h = 0 \cdot 25$ gauss.

Variation (2) - Joint Variation

1. Re-write each of the following variation statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) E varies directly as t and inversely as p^2.

   (b) Q varies directly as the square of h and t^3.

   (c) M varies as e and g^2 and inversely as $\sqrt{d}$.

   (d) T varies as m, p and g and inversely as s^2.

   (e) The volume (V) of a cone varies jointly as the height (h) of the cone and the square of the radius (r) of the base.

   (f) The time of oscillation (T) of a pendulum varies directly as the square root of the length (l) and inversely as the square of the gravitational constant (g).

2. Given that $x \propto yz$ and $x = 24$ when $y = 3$ and $z = 2$, find the value of x when $y = 2$ and $z = 6$.

3. If x varies directly as y and inversely as z, and if $x = 8$ when $y = 4$ and $z = 3$, find the value of x when $y = 5$ and $z = 2$.

4. If a varies directly as b and inversely as the square root of c, and if $a = 9$ when $b = 3$ and $c = 4$, find the equation connecting a, b and c and hence find a when $b = 5$ and $c = 9$.

5. A number e varies directly as f and inversely as the square of g.

   (a) If $e = 10$ when $f = 5$ and $g = 4$, express e in terms of f and g.

   (b) Hence find the value of e when $f = 8$ and $g = 2$.

6. The air resistance (R) to an outboard engine running at certain speeds varies jointly as the area (a) of the surface exposed and as the square of the speed of the engine (s).
   When the engine is travelling at a speed of 50 m.p.h. with a surface area of 40 square centimetres exposed, the resistance to the engine is 3000 lb. wt.

   

   Find the resistance to the engine if the surface exposed is 25 square centimetres and the speed is 60 m.p.h.

7. The time (T) required to plough a field varies directly as its area (A) and inversely as the number of men (n) employed.
   If 10 men plough a 15 hectare field in 3 days, how many men would be required to plough a field of 12 hectares in 4 days?

   

8. The horse power (P) of a windmill varies directly as the total sail area (A) and as the cube of the velocity of the wind (v).
   If the sail area is 150 square metres and the velocity of the wind is 20 km/h, the horse power is 35.

   

   Calculate the horse power generated by a windmill with a sail area of 200 square metres when the velocity of the wind is 30 km/h.

9. The volume (V) of a right circular cone varies jointly as its height (h) and as the square of its base radius (r). The volume of a cone which is 7 cm high with a base radius of 3 cm is 66 cm^3.

   (a) Show that the constant of variation (k) is equal to $\frac{22}{21}$.

   (b) Hence find the volume of a cone which is twice as high standing on a base with a radius half as large as the previous one.

Variation (1)

Direct Variation

1. (a) $C \propto t$, $C = kt$ (b) $V \propto r^2$, $V = kr^2$ (c) $T \propto \sqrt{r}$, $T = k\sqrt{r}$ (d) $V \propto r^3$, $V = kr^3$

2. 2, 10

3. 108

4. £7500

5. 1.6 seconds

6. 56 cm

Inverse Variation

1. (a) $N \propto \frac{1}{p}$, $N = \frac{k}{p}$ (b) $V \propto \frac{1}{\sqrt{h}}$, $V = \frac{k}{\sqrt{h}}$ (c) $e \propto \frac{1}{d^2}$, $e = \frac{k}{d^2}$

2. 2

3. 10 km/h

4. 0·52 seconds

Variation (2) - Joint Variation

1. (a) $E \propto \frac{I}{p^2}$, $E = \frac{k}{p^2}$ (b) $Q \propto h^2t^3$, $Q = kh^2t^3$ (c) $M \propto \frac{eg^2}{\sqrt{d}}$, $M = \frac{keg^2}{\sqrt{d}}$ (d) $T \propto \frac{mpg}{s^2}$, $V = \frac{kmpg}{s^2}$

   (e) $V \propto hr^2$, $V = khr^2$ (f) $T \propto \frac{\sqrt{l}}{g^2}$, $T = \frac{k\sqrt{l}}{g^2}$

2. 48

3. 15

4. 10

5. (a) $e = \frac{32f}{g^2}$ (b) 64

6. 2700 lb.wt.

7. 6 men

8. 157.5

9. (a) proof
   (b) 33 cm^3