

**Higher Maths – End of Course Assessment
Paper 2**

2009/2010 (Answers + Marking Scheme)

	Give 1 mark for each •	Illustration(s) for awarding each mark										
1 (a)	ans: $k = 10$ (3 marks)	<ul style="list-style-type: none"> •¹ finds change in x-coords/y-coords •² realises $\vec{CP} = \frac{1}{4}\vec{CB}$ •³ establishes z-coordinate of P <ul style="list-style-type: none"> •¹ $CB - 6$ to $6 = 12$; $CP - 6$ to $-3 = 3$ [x] CB 1 to $-3 = 4$; CP 1 to $0 = 1$ •² evidence •³ $12 - 4 = 8$ so change in z-coord is 2; $k = 10$ 										
(b)	ans: 139.5° (5 marks)	<ul style="list-style-type: none"> •¹ know how to find angles •² finds \vec{PA} and \vec{PB} •³ finds scalar product •⁴ finds magnitudes of vectors •⁵ substitutes in formula and finds angle <ul style="list-style-type: none"> •¹ evidence of $\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a} \mathbf{b} }$ •² $\vec{PA} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}; \vec{PB} = \begin{pmatrix} 9 \\ -3 \\ -6 \end{pmatrix}$ •³ $\vec{PA} \cdot \vec{PB} = -9 + 0 - 18 = -27$ •⁴ $\vec{PA} = \sqrt{10}; \vec{PB} = \sqrt{126}$ •⁵ $\cos\theta = \frac{-27}{\sqrt{10}\sqrt{126}}; \theta = 139.5^\circ$ 										
2	ans: 17 hours 50 minutes (5 marks)	<ul style="list-style-type: none"> •¹ substitutes values in formula and simplifies •² takes natural logs of both sides •³ releases power and removes $\log_e e$ •⁴ evaluates for t •⁵ changes hours to hours and minutes <ul style="list-style-type: none"> •¹ $28 = 40e^{-0.02t}; e^{-0.02t} = 0.7$ •² $\log_e e^{-0.02t} = \log_e 0.7$ •³ $-0.02t \log_e e = \log_e 0.7; 0.02t = \log_e 0.7$ •⁴ $t = \frac{\log_e 0.7}{-0.02} = 17.8337\dots$ •⁵ 17 hours 50 minutes 										
3(a)	ans proof (1 mark)	<ul style="list-style-type: none"> •¹ proves they are equal <ul style="list-style-type: none"> •¹ $f(-2) = 5 = g(-2)$ 										
(b)	ans (-2.5), (-1,0), (1,2) (4marks)	<ul style="list-style-type: none"> •¹ form equation •² uses known root to synthetically divide <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="border: 1px solid black; padding: 2px;">-2</td> <td style="border: 1px solid black; padding: 2px;">1</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td style="border: 1px solid black; padding: 2px;">-1</td> <td style="border: 1px solid black; padding: 2px;">-2</td> </tr> <tr> <td></td> <td style="border: 1px solid black; padding: 2px;">-2</td> <td style="border: 1px solid black; padding: 2px;">0</td> <td style="border: 1px solid black; padding: 2px;">2</td> <td></td> </tr> </table> <ul style="list-style-type: none"> •³ solves for x •⁴ determines points of intersection <ul style="list-style-type: none"> •³ $x = -2, -1, 1$ •⁴ (-2.5), (-1,0) and (1,2) 	-2	1	2	-1	-2		-2	0	2	
-2	1	2	-1	-2								
	-2	0	2									

(c)	<p>ans $2\frac{2}{3}$ units² (4marks)</p> <ul style="list-style-type: none"> •¹ sets up correct integral •² integrates correctly •³ substitutes limits in correctly •⁴ evaluates correctly 	<ul style="list-style-type: none"> •¹ $\int_{-1}^1 (2x^2 + x - 1) - (x^3 + 4x^2 - 3) dx$ •² $[-\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + 2x]_{-1}^1$ •³ •⁴ $2\frac{2}{3}$
4 ans:	<p>$y = \frac{1}{6}(4x+1)^{\frac{3}{2}} + 5$ (5 marks)</p>	<ul style="list-style-type: none"> •¹ knows to integrate and prepares •² starts to integrate •³ completes integration and adds C •⁴ knows to substitute to find C •⁵ evaluates for C
(b)	<p>5(a) ans (2 marks) (1 mark)</p> <p>(i) (-1,4) (0,4) (0.5, 5) (1,4) (2,4) (ii) (-0.5, 2) (0,2) (0.25,2.5) (0.5,2) (1,2)</p> <ul style="list-style-type: none"> •¹ double all y - coords •² all points correct •¹ halved all x-coords <p>ans - Graph</p>	<ul style="list-style-type: none"> •¹ $y = \int (4x+1)^{\frac{1}{2}} dx$ •² $y = \frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}} \dots\dots$ •³ $\dots\dots \times \frac{1}{4} + C; y = \frac{1}{6}(4x+1)^{\frac{3}{2}} + C$ •⁴ $9 \cdot 5 = \frac{1}{6}(4(2)+1)^{\frac{3}{2}} + C;$ •⁵ $C = 9 \cdot 5 - \frac{27}{6} = 5$ <ul style="list-style-type: none"> •¹ reflected in x-axis •² vertical shift •³ annotation

6(a) ans: $y = \sqrt{52}\sin(x - 4.125)^\circ$ (4 marks)

- ¹ uses correct expansion
- ² equates coefficients
- ³ calculates value of k
- ⁴ finds value for a

- ¹ $k \sin x \cos a - k \cos x \sin a$
- ² $k \cos a = -4, k \sin a = -6$
- ³ $k = \sqrt{52}$
- ⁴ $a = 4.125$

(b) ans: proof (2 marks)

- ¹ marshalls facts
- ² completes proof

- ¹ $AE = 6\cos x, BC = 4\sin x, CE = BC$
 $CE = AE - 4$
- ² $6\cos x - 4 = 4\sin x$
 $6\cos x - 4\sin x = 4$

(c) ans: 0.396, 4.713 (3 marks)

- ¹ method
- ² finds related angle
- ³ solves in range

- ¹ $\sqrt{52}\sin(x - 4.125)^\circ = 4$
- ² 0.588
- ³ $x = 0.396, 4.713$

(d) ans 0.396 (1 mark)

- ¹ chooses the one $< \frac{\pi}{2}$

- ¹ 0.396

Total: 43 marks

Total for both papers – 80 marks

