FORTROSE ACADEMY
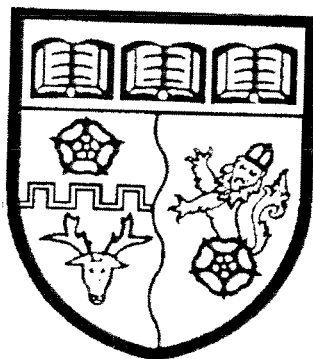
MATHEMATICS DEPARTMENT

Established 1791

Mathematics

Higher End of Course Assessment
Examination 2009/2010

NATIONAL

QUALIFICATIONS

Assessing Unit 3 + revision from Units 1 & 2
Paper 2

Time allowed - 50 minutes

Read carefully

1. **Calculators may be used in this paper.**

2. Full credit will be given only where the solution contains appropriate working.

3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Scalar Product: $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$
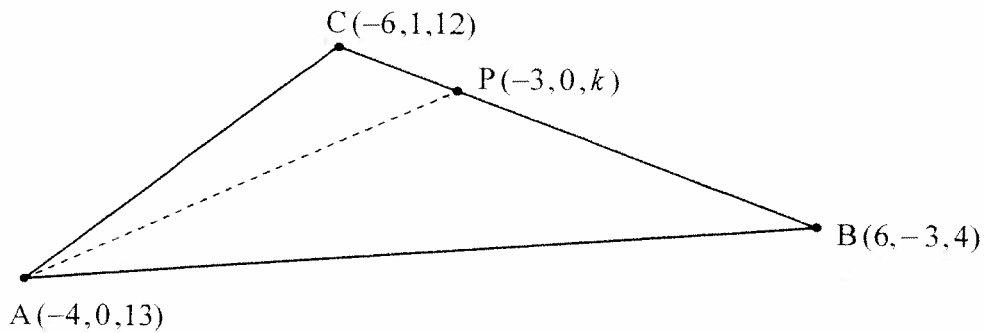
Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

1. In the diagram below A, B and C have coordinates $(-4,0,13)$, $(6,-3,4)$ and $(-6,1,12)$ respectively.

P lies on BC and has coordinates $(-3,0,k)$

C$(-6,1,12)$

P$(-3,0,k)$

B$(6,-3,4)$

A$(-4,0,13)$

    (a)   Find the value of k.   **3**

    (b)   Hence calculate the size of angle APB.   **5**
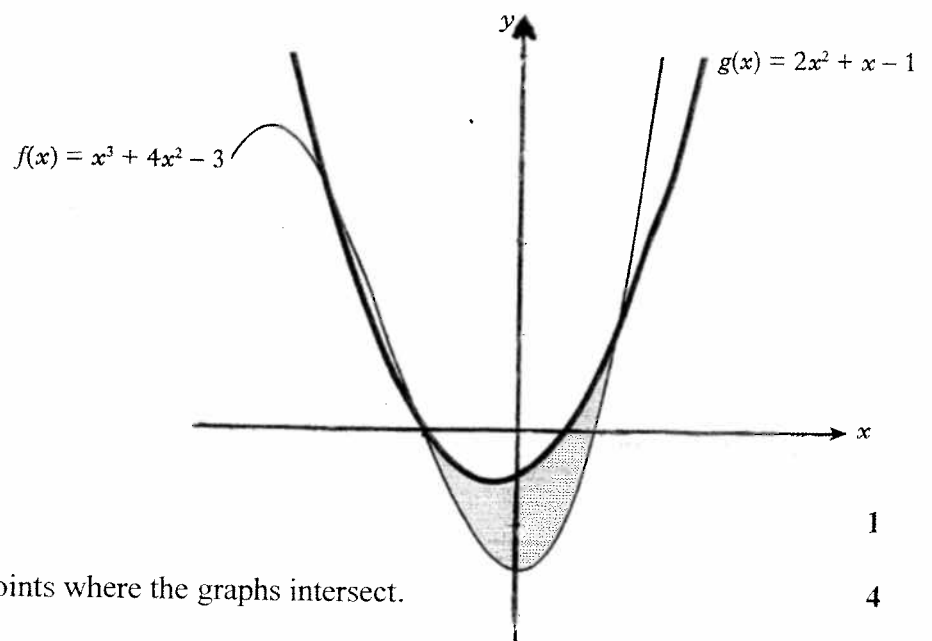
2. A formulae for mass decay is given as $M_t = M_0 e^{-0.02t}$, where t is time elapsed in hours, M_0 is the initial mass in grams and M_t is the mass remaining after t hours.

How long will it take for an initial mass of 40 grams to decay down to 28 grams?
Give your answer correct to the nearest minute.   **5**

3. The graph shows the functions $f(x) = x^3 + 4x^2 - 3$ and $g(x) = 2x^2 + x - 1$

$g(x) = 2x^2 + x - 1$

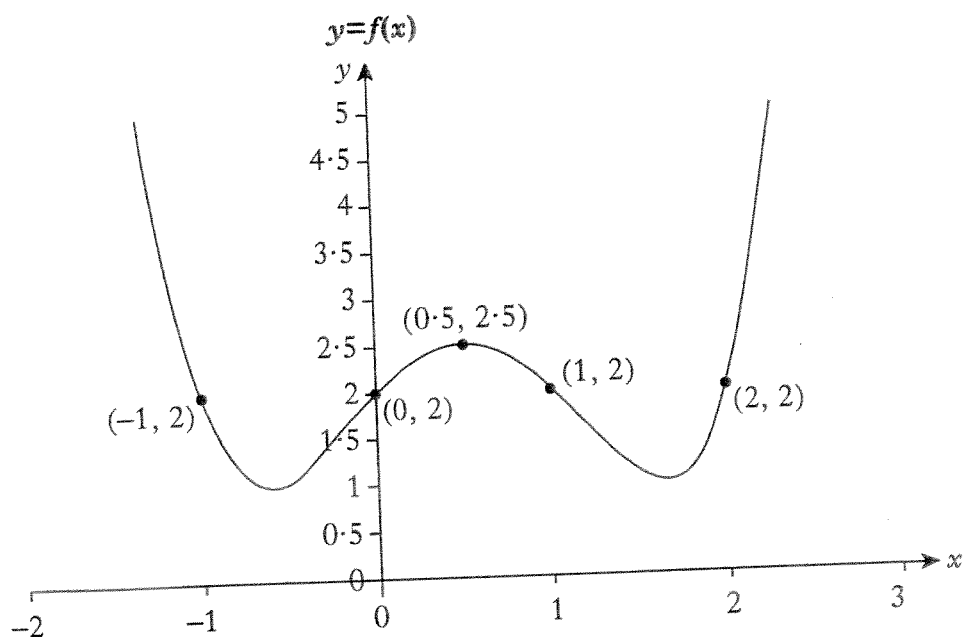$f(x) = x^3 + 4x^2 - 3$

    (a)   Show $f(-2) = g(-2)$.   **1**

    (b)   Hence find all three points where the graphs intersect.   **4**

    (c)   Calculate the grey area trapped between these curves.   **4**

4. If $\dfrac{dy}{dx} = \sqrt{4x+1}$, find an expression for y in terms of x given that $y = 9 \cdot 5$ when $x = 2$. 5

5. The diagram shows part of the graph $y = f(x)$.

(a) State the coordinates of **five** points on the graph for
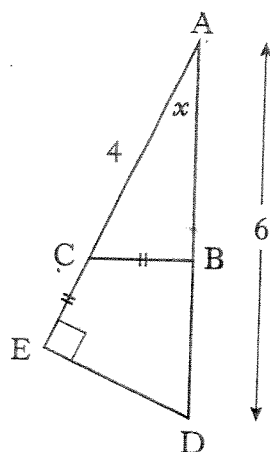
    (i) $y = 2f(x)$ **and** 2
    (ii) $y = f(2x)$ 1

(b) Sketch the graph of $y = 2 - f(x)$ 3

6.

(a) Express *6cos x – 4sinx* in the form *k sin(x - a)*, where $k > 0$ and $0 \le a \le 2\pi$ 4

(b) A right angled triangle ABC has been constructed with the right angle at B and AC = 4cm. AC is extended to E so that CE = BC.

It is found that AD = 6cm and triangle AED is right angled at E.
Show that *6cosx – 4sinx = 4* where *x* is the size of angle CAB.   2

(c) Solve this equation to find all solutions in the region $0 \le x \le 2\pi$ 3

(d) Find the value of *x* in the context of the diagram.   1

[END OF QUESTION PAPER]