FORTROSE ACADEMY

MATHEMATICS DEPARTMENT

Established 1791
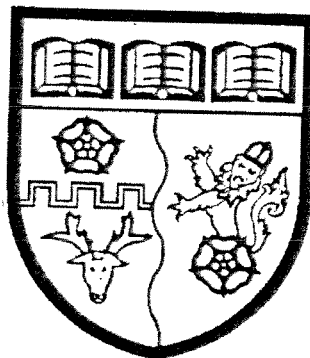
Mathematics	NATIONAL
Higher End of Course Assessment	QUALIFICATIONS
Examination 2009/2010	

Assessing Unit 3 + revision from Units 1 & 2

Paper 1

Time allowed - 50 minutes

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A - Questions 1 - 10 (20 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (17 marks)

1. Full credit will be given only where the solution contains appropriate working.

2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Scalar Product:

$a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

In this section the correct answer to each question is given by one of the alternatives **A. B. C** *or* **D.**
Indicate the correct answer on the answer grid provided.
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1.   The function $f(x) = 2\sin x° + \cos x°$ has a **minimum** value of

      A -2

      B 0

      C -3

      D $-\sqrt{5}$

2.   Which of the following is a correct assumption from the statement $\log_b a = c$?

      A $a^c = b$

      B $c^a = b$

      C $b^c = a$

      D $c^b = a$

3.   What is the value of $\displaystyle\int_0^\pi \sin x \; dx$?

      A -2

      B $+1$

      C $+2$

      D 0

4.   P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

The length of PQ is

      A 1

      B $\sqrt{17}$

      C $\sqrt{21}$

      D $\sqrt{13}$

5. Given that $\cos P = \dfrac{1}{\sqrt{6}}$, where $0 < p < \dfrac{\pi}{2}$, the value of $\cos 2P$ is

A $\dfrac{1}{\sqrt{3}}$

B $\dfrac{2}{\sqrt{6}}$

C $\dfrac{\sqrt{5}}{\sqrt{6}}$

D $-\dfrac{2}{3}$

6. An equation is such that $\log x + \log(x+1) = \log 6$, where $x > 0$.

The value of x is

A   2

B   1

C   3

D   6

7. The gradient of the tangent to the curve $y = \sin x^{\circ}$ at the point where $x = \dfrac{\pi}{3}$ is

A $\dfrac{\sqrt{3}}{2}$

B $\dfrac{1}{2}$

C $-\dfrac{1}{2}$

D   0

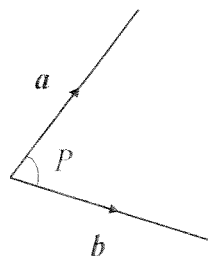**8.** Vectors a and b are such that $|a| = |b| = 2$ with P being the angle between the vectors.

If $a \cdot b = 0\cdot8$, the value of $\cos P$ is

A $3\cdot2$

B $0\cdot4$

C $0\cdot2$

D $0\cdot05$

9. The gradient of the tangent to the circle $x^2 + y^2 + 6x - 4y - 4 = 0$ at the point $(1,3)$ on the circumference is

A $\frac{1}{4}$

B -4

C $-\frac{1}{4}$

D $\frac{2}{5}$

10. The recurrence relation $u_{n+1} = ku_n + 10$ has a limit of 25. What is the value of k?

A -0.4

B 0.4

C 0.6

D -0.6

[END OF SECTION A]

SECTION B

ALL questions should be attempted

11. 9. A function is defined on a suitable domain as $f(x) = \dfrac{-16}{(2x-1)^2}$.

(a) Show clearly that the derivative of this function can be written in the form
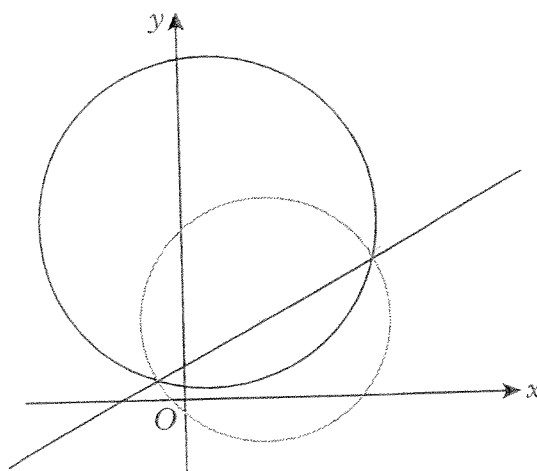
$$f'(x) = \frac{k}{(2x-1)^n}$$

and write down the values of k and n.

4

(b) Hence find x when $f'(x) = 1$ and $x > 0$.

3

12. 10. A circle, $x^2 + y^2 - 2x - 10y + 6 = 0$, is cut by a line $x - 3y + 4 = 0$

(a) Find the two points of intersection.

5

(b) These two points are the end points of a diameter of another circle.
Find the equation of this circle.

3

(c) Decide whether the point (0 , 4) lies within, without or on this second circle.

2

[END OF SECTION B]