FORTROSE ACADEMY
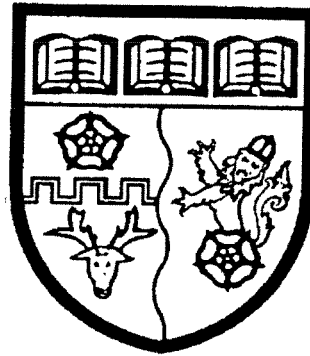
MATHEMATICS DEPARTMENT

Established 1791

Mathematics

Higher End of Course Assessment
Examination 2011/2012

Assessing Unit 3 + revision from Units 1 & 2
Paper 2

Time allowed - 50 minutes

41 marks

Read carefully

1. **Calculators may be used in this paper.**

2. Full credit will be given only where the solution contains appropriate working.

3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Scalar Product:

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

or

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$
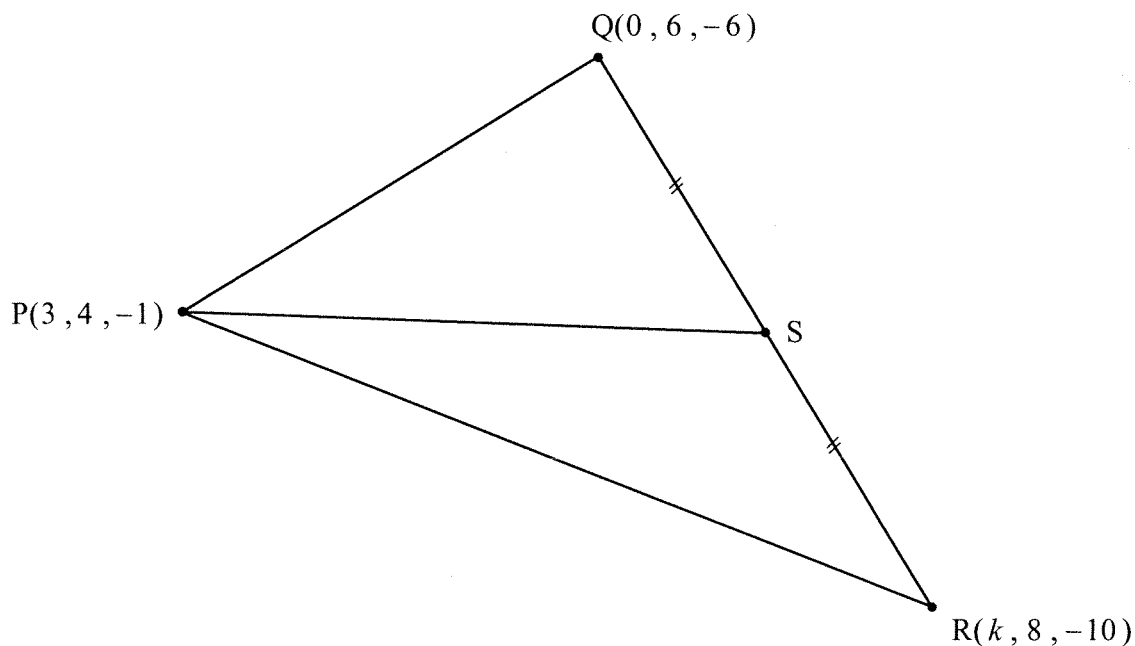
Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

1. In the diagram P, Q, and R have coordinates P($3 , 4 , -1$), Q($0 , 6 , -6$) and R($k , 8 , -10$) respectively.

Q($0 , 6 , -6$)

P($3 , 4 , -1$)

S

R($k , 8 , -10$)

(a) Given that angle PQR is a right-angle, find the value of k. **4**

(b) Calculate the size of angle RPS where S is the mid-point of QR. **6**

2. A Baryon particle decays according to the formula $M_t = M_o\, e^{-0.0009t}$, where M_o is the intitial mass of the substance and M_t is the mass remaining after t seconds.
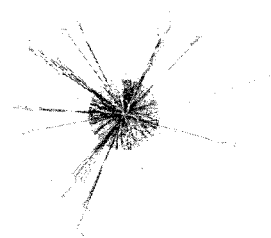
Calculate , **to the nearest ten seconds**, how long a sample would take to lose 30% of its original mass. **5**

3. (a) Express $3\cos x° + \sqrt{7}\sin x°$ in the form $k\sin(x+a)°$, where k and a are constants and $k > 0$. **5**

    (b) Hence state the **minimum** value of f given that $f(x) = \dfrac{20}{3\cos x° + \sqrt{7}\sin x°}$. **1**

4. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 8$, where a is a constant.

(a) Given that $U_0 = 16$, show that, in terms of a, $U_2 = 8(2a^2 + a + 1)$. **2**

(b) Hence find a, where $a > 0$, given that $U_2 = 11$. **3**

5. The diagram shows how two circles can have three common tangents.
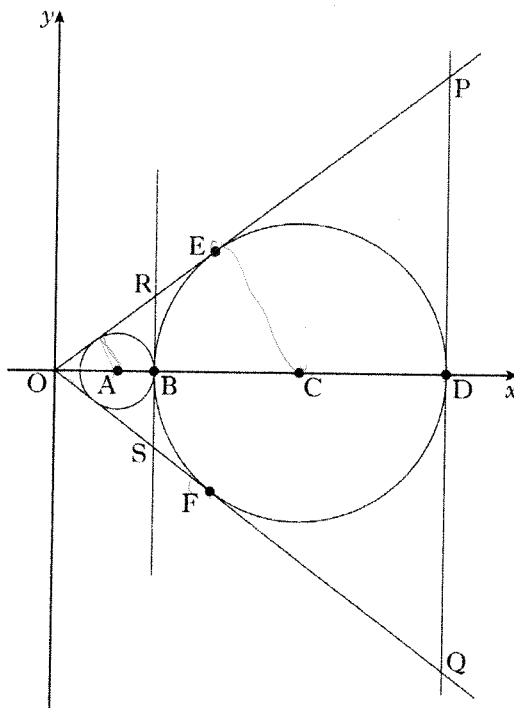
The circle centre C cuts the x-axis at B and D and has equation $x^2 + y^2 - 10x + 16 = 0$.

(a) (i) Find the centre and radius of this circle.
(ii) State the coordinates of B and D. **3**

(b) Two of the common tangents pass through the origin and so have equations of the form $y = mx$ where m is a constant.

Find the values of m for which $y = mx$ is a tangent to the circle centre C. **4**

(c) The triangle OPQ is an enlargement of triangle ORS. By considering the enlargement factor, find the equation of the circle centre A. **3**

6. The graph shows the two curves $y = \sin x^0$ and $y = 1 - 2\cos 2x^0$ intersecting in the interval $0 \leq x < 360$.
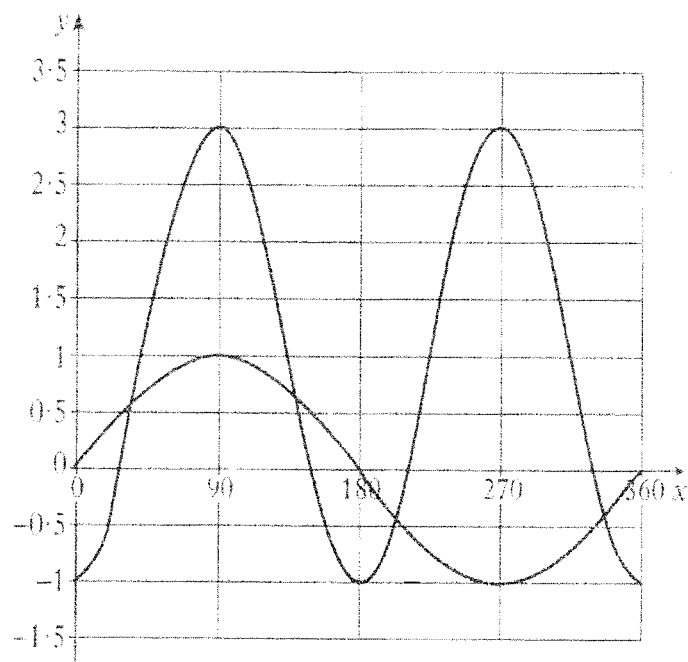
Calculate to one decimal place, the values of x at which this occurs.   **5**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]