NATIONAL
QUALIFICATIONS

Mathematics
Higher Prelim Examination 2010/2011
Paper 2

Assessing Unit 3 + circle + revision from Units 1 & 2

Time allowed - 50 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

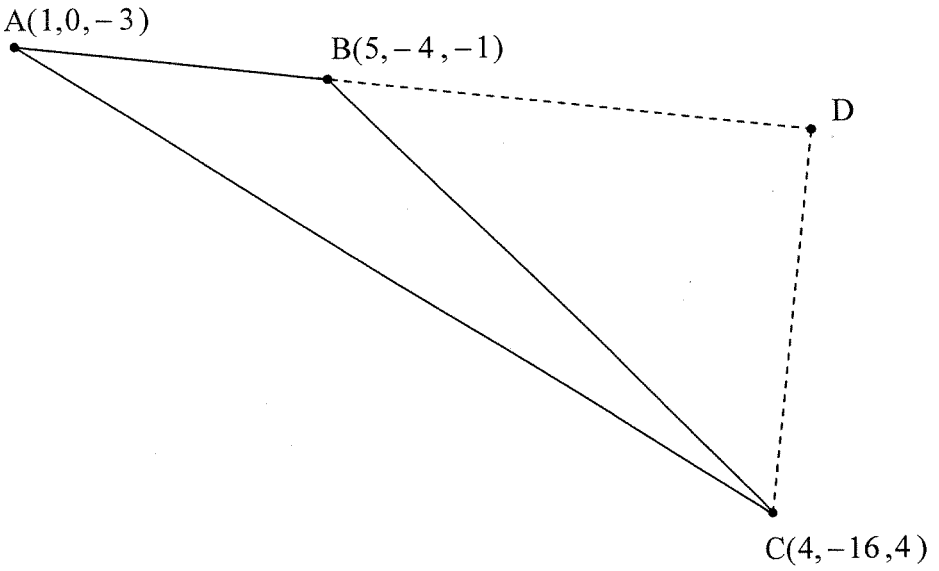
Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

PAPER 2

1. Triangle ABC has vertices A$(1,0,-3)$, B$(5,-4,-1)$ and C$(4,-16,4)$ respectively.

A, B and D are collinear such that $\frac{AB}{BD} = \frac{2}{3}$.

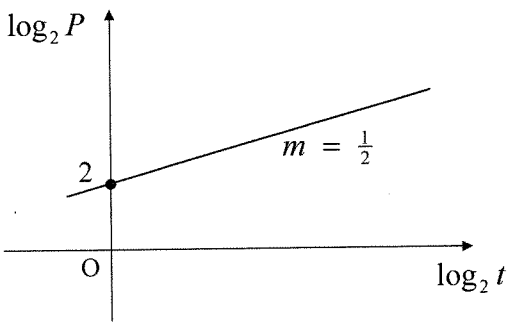


(a) Find the coordinates of D. **2**

(b) Hence show clearly that angle ADC is a right angle. **4**

(c) Calculate the size of angle ABC **3**

2. The diagram, which is not drawn to scale, shows part of a graph of $\log_2 P$ against $\log_2 t$. The straight line has a gradient of $\frac{1}{2}$ and passes through the point (0,2).

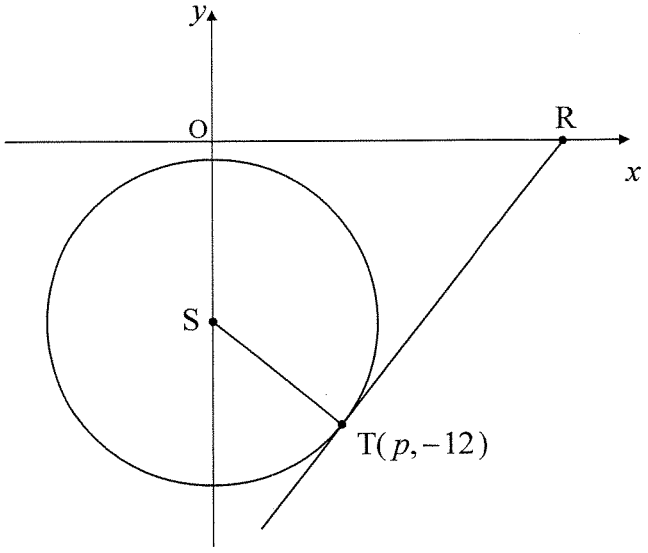


(a) Find an equation connecting t and P. **3**

(b) Hence show clearly that when $P = \sqrt{8} + 4$, t takes the value $\frac{1}{2}(3 + 2\sqrt{2})$ **3**

3. The circle, centre S, has as its equation $x^2 + y^2 + 16y + 12 = 0$.

T$(p, -12)$ is a point of tangency.



(a) Find the value of p, the x-coordinate of T. **2**

(b) Write down the coordinates of S, the centre of the circle. **1**

(c) Find the equation of the tangent through T and hence state the coordinates of R. **4**

(d) Establish the equation of the circle which passes through the points S, T and R. **3**

4. The radioactive element Strontium-90 can be used in production of Nuclear Power. It decays according to a law of the form $y = y_o e^{kt}$ where y is the amount of radioactive substance present at time t years and y_o is the initial amount of radioactive substance.

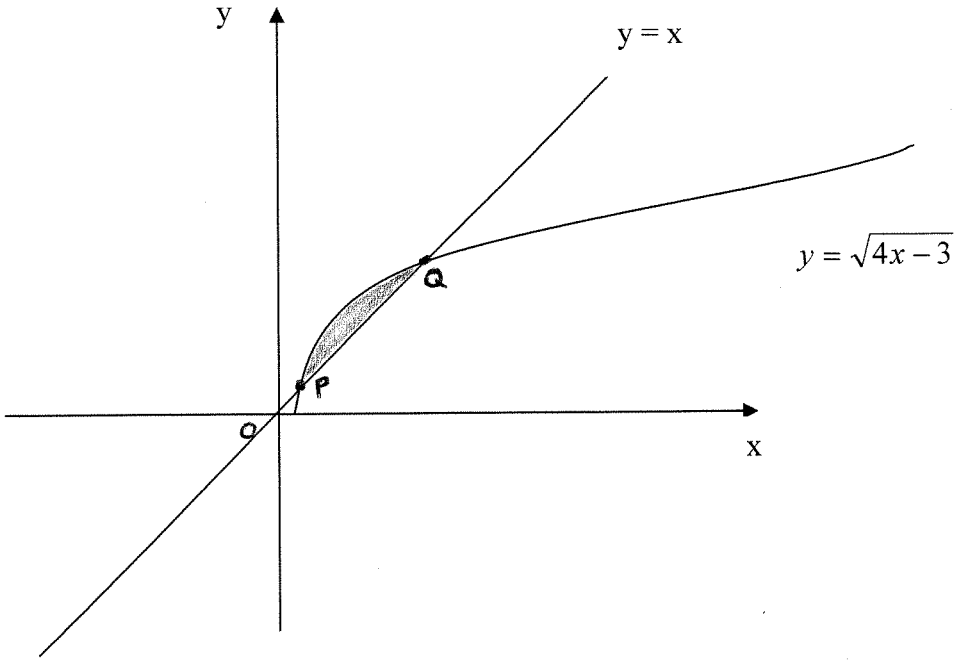
(a) The half-life of Strontium (i.e. the time taken for half the radioactive substance to decay) is 28.8 years. Find the value of k correct to 3 significant figures. **3**

(b) What percentage of Strontium in a sample will still be present after 50 years? **3**

5. The diagram below shows sections of the graphs of $y = \sqrt{4x-3}$ and $y = x$

(a) Find the x-coordinates of the points P and Q **3**

(b) Calculate the shaded area. **4**



[END OF QUESTION PAPER]