NATIONAL QUALIFICATIONS

Mathematics
Higher Mini-Prelim Examination 2010/2011

Assessing Unit 3 + circle + revision from Units 1 & 2

Paper 1
Time allowed - 50 mins

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 10 (20 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (19 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Mini Prelim 2010/2011 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

Sample Question

A line has equation $y = 4x - 1$.

If the point $(k,7)$ lies on this line, the value of k is

A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).

Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

SECTION A

1. If $f(x) = (2x-1)^4$ then $f'(1)$ equals

A 4

B 1

C 2

D 8

2. The **maximum** value of the function $g(x) = 3\sin x + 2\cos x$ is

A $\sqrt{13}$

B 5

C 0

D 2

3. The radius of the circle with equation $x^2 + y^2 + 4x - 2y = 4$ is

A 2

B 3

C 1

D $\sqrt{24}$

4. If k is a constant of integration then $\int \sin 4x \, dx$ is

A $-\cos 4x + k$

B $4\cos 4x + k$

C $-\frac{1}{4}\cos 4x + k$

D $\frac{1}{4}\cos 4x + k$

5. The value of $\log_{\sqrt{2}} 4$ is

A 2

B $4\sqrt{2}$

C $\frac{1}{4}$

D 4

6. Given that the vectors $\begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} p \\ -2 \\ 3 \end{pmatrix}$ are perpendicular, the value of p is

A 0

B 8

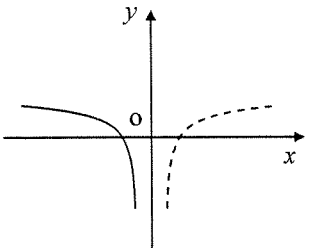
C 4

D -6

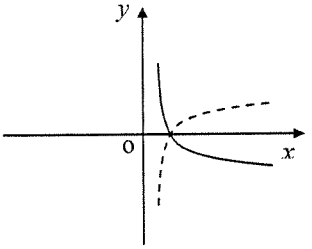
7. Part of the graph of $y = \log_{10} x$ is shown in each diagram below as a broken line.

Which diagram also shows, as a full line, part of the graph of $y = \log_{10} \frac{1}{x}$?

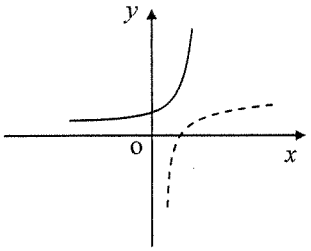
A



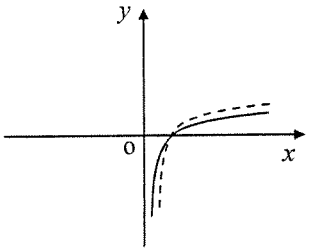
B



C



D



8. $\boldsymbol{a} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ g \end{pmatrix}$ is a **unit** vector. Which of the following could be the value of g?

A $\frac{1}{2}$

B 1

C -1

D $\frac{1}{\sqrt{2}}$

9. The equation of the circle, centre (3, -5) with the y-axis as a tangent is

A $(x+3)^2+(y-5)^2=9$

B $(x-3)^2+(y+5)^2=25$

C $(x-3)^2+(y+5)^2=9$

D $(x+3)^2+(y-5)^2=34$

10. If $\sin\alpha=\frac{3}{5}$ and $\sin\beta=\frac{5}{13}$, where α,β are acute angles, the value of $\sin(\alpha+\beta)$ is

A $\frac{64}{65}$

B $\frac{56}{65}$

C $\frac{33}{65}$

D $\frac{64}{65}$

[END OF SECTION A]

SECTION B

ALL questions should be attempted

11. Given that $(x+1)$ and $(x-3)$ are both factors of $2x^3 - 5x^2 + ax + b$, find a and b. **4**

12. A function is defined as $f(x) = 6\cos^2 \frac{1}{2}x^\circ + \sqrt{3}\sin x^\circ$.

(a) By using the fact that $\cos^2 x^\circ = \frac{1}{2}(\cos 2x^\circ + 1)$ show clearly that this function can be expressed in the form

$$f(x) = 3\cos x^\circ + \sqrt{3}\sin x^\circ + 3.$$ **3**

(b) Express $3\cos x^\circ + \sqrt{3}\sin x^\circ + 3$ in the form $k\cos(x-\alpha)^\circ + 3$ where $0 < \alpha < 360$ and $k > 0$. **3**

(c) Hence solve the equation $f(x) = 0$ for $200 < x < 360$. **4**

13. (a) Given that $y = \sqrt{3}(\sin^2 x - \cos 2x)$, show clearly that

$$\frac{dy}{dx} = \sqrt{3}(3\sin 2x).$$ **3**

(b) Hence find the gradient of the tangent to the curve $y = \sqrt{3}(\sin^2 x - \cos 2x)$ at the point where $x = \frac{\pi}{6}$. **2**

[END OF SECTION B]

[END OF QUESTION PAPER]