

Higher Mini Prelim - Unit 3 – Paper 1

2010/2011 (Answers + Marking Scheme)

Section A - Answers

1	D	2	A	3	B	4	C	5	D
6	B	7	B	8	D	9	C	10	B

Section B - Marking Scheme – 19 marks

2 marks each (20 marks)
Total 39 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
11	ans: $a = -4; b = 3$ (4 marks)	<ul style="list-style-type: none"> •¹ uses synthetic division to find one equation •² uses synthetic division to find other eq. •³ knows to use system of equations •⁴ solves for a and b <ul style="list-style-type: none"> •¹ $b - a = 7$ •² $b + 3a = -9$ •³ evidence •⁴ $a = -4; b = 3$
12(a)	ans: proof (3 marks)	<ul style="list-style-type: none"> •¹ applies given info to new function •² knows to substitute in function •³ simplifies to required form <ul style="list-style-type: none"> •¹ $\cos^2 \frac{1}{2}x^\circ = \frac{1}{2}(\cos x^\circ + 1)$ •² $6[\frac{1}{2}(\cos x^\circ + 1)] + \sqrt{3} \sin x^\circ$ •³ $3(\cos x + 1) + \sqrt{3} \sin x^\circ; 3 \cos x + 3 + \sqrt{3} \sin x^\circ$
(b)	ans: $\sqrt{12}\cos(x - 30)^\circ + 3$ (3 marks)	<ul style="list-style-type: none"> •¹ finds k •² finds $\tan \alpha$ •³ finds α <ul style="list-style-type: none"> •¹ $k = \sqrt{9+3} = \sqrt{12}$ •² $\tan \alpha = \frac{\sqrt{3}}{3}$ •³ $\alpha = 30^\circ$
(c)	ans: 240° (4 marks)	<ul style="list-style-type: none"> •¹ equates to 0 •² simplifies •³ finds values •⁴ discards <ul style="list-style-type: none"> •¹ $\sqrt{12}\cos(x - 30)^\circ + 3 = 0$ •² $\cos(x - 30)^\circ = -\frac{3}{\sqrt{12}}$ •³ $x = 180, 240^\circ$ •⁴ 240°
13(a)	ans: proof (3 marks)	<ul style="list-style-type: none"> •¹ differentiates first term in brackets •² differentiates second term in brackets •³ contracts $2 \sin x \cos x$ and simplifies <ul style="list-style-type: none"> •¹ $\sqrt{3}(2 \sin x \cos x).....$ •²$2 \sin 2x$) •³ $\sqrt{3}(\sin 2x + 2 \sin 2x) = \sqrt{3}(3 \sin 2x)$
(b)	ans: 9/2 (2 marks)	<ul style="list-style-type: none"> •¹ subs into derivative •² evaluates <ul style="list-style-type: none"> •¹ $\sqrt{3}(3 \sin 2(\frac{\pi}{6}))$; $\sqrt{3}(3 \sin \frac{\pi}{3})$ •² $\sqrt{3} \times 3 \times \frac{\sqrt{3}}{2} = \frac{9}{2}$

Total 38 marks

	Give 1 mark for each • ans: (11, - 10, 2) (2 marks)	Illustration(s) for awarding each mark
1(a)	<ul style="list-style-type: none"> •¹ valid method •² answer 	<ul style="list-style-type: none"> •¹ evidence of using stepping out/section formula •² (11, - 10, 2)
(b)	ans: proof (4 marks) <ul style="list-style-type: none"> •¹ finds \vec{DA} •² finds \vec{DC} •³ finds $\vec{DA} \cdot \vec{DC}$ •⁴ conclusion 	<ul style="list-style-type: none"> •¹ $\vec{DA} = \begin{pmatrix} -10 \\ 10 \\ -5 \end{pmatrix}$ •² $\vec{DC} = \begin{pmatrix} -7 \\ -6 \\ 2 \end{pmatrix}$ •³ $\vec{DA} \cdot \vec{DC} = 70 - 60 - 10 = 0$ •⁴ since $\vec{DA} \cdot \vec{DC} = 0$; $\angle ADC$ is right angled
(c)	ans: 133.7° (3 marks) <ul style="list-style-type: none"> •¹ finds $\vec{BA} \cdot \vec{BC}$ •² knows how to find $\cos ABC$ •³ calculates the angle 	<ul style="list-style-type: none"> •¹ $\vec{BA} = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} -1 \\ -12 \\ 5 \end{pmatrix}$ $\vec{BA} \cdot \vec{BC} = -54$ •² $\cos ABC = \frac{-54}{6 \times \sqrt{170}}$ •³ $ABC = 133.7^\circ$
2(a)	ans: $P = 4t^{\frac{1}{2}}$ (3 marks) <ul style="list-style-type: none"> •¹ knows form of equation, takes logs, expands •² finds b •³ writes relationship 	<ul style="list-style-type: none"> •¹ $y = ax^b$; $\log_2 P = \frac{1}{2} \log_2 t + 2$ •² $b = \frac{1}{2}$; $\log_2 a = 2$; $a = 2^2$; $a = 4$ •³ $P = 4t^{\frac{1}{2}}$
(b)	ans: proof (3 marks) <ul style="list-style-type: none"> •¹ subs into expression •² starts to simplify •³ completes simplification to answer 	<ul style="list-style-type: none"> •¹ $4t^{\frac{1}{2}} = \sqrt{8} + 4$ •² $t^{\frac{1}{2}} = \frac{1}{4}(\sqrt{8} + 4)$; $t = [\frac{1}{4}(\sqrt{8} + 4)]^2$ •³ $t = \frac{1}{16}(8 + 8\sqrt{8} + 16)$; $t = \frac{1}{16}(24 + 16\sqrt{2})$ $t = \frac{24}{16} + \sqrt{2}$; $t = \frac{3}{2} + \frac{2\sqrt{2}}{2}$; $t = \frac{1}{2}(3 + 2\sqrt{2})$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	ans: $p = 6$ (2 marks) • ¹ subs into equation of circle • ² solves for p	$\bullet^1 p^2 + 144 - 192 + 12 = 0$ $\bullet^2 p^2 = 36; p = 6$
(b)	ans: $(0, -8)$ (1 mark) • ¹ states centre of circle	$\bullet^1 (0, -8)$
(c)	ans: $2y = 3x - 42$ (4 marks) • ¹ finds gradient of ST • ² finds gradient of tangent • ³ subs into equation of straight line • ⁴ finds coords of point R	$\bullet^1 m_{ST} = -\frac{2}{3}$ $\bullet^2 m_{tan} = \frac{3}{2}$ $\bullet^3 y + 12 = \frac{3}{2}(x - 6)$ [or equivalent] $\bullet^4 3x - 42 = 0; x = 14 \quad (14, 0)$
(d)	ans: $(x - 7)^2 + (y + 4)^2 = 65$ (3 marks) • ¹ finds midpoint of SR (centre of circle) • ² finds radius • ³ subs into equation of circle	\bullet^1 centre of circle $(7, -4)$ $\bullet^2 \sqrt{65}$ $\bullet^3 (x - 7)^2 + (y + 4)^2 = 65$
4 (a)	ans $k = -0.0241$ (3 marks) • ¹ establishes equation • ² takes ln of both sides • ³ solves for k	$\bullet^1 e^{28.8k} = \frac{1}{2}$ $\bullet^2 \ln e^{28.8k} = \ln \frac{1}{2}$ $\bullet^3 k = -0.0241$
(b)	ans 30% (3 marks) • ¹ sets up equation • ² evaluates • ³ interprets answer	$\bullet^1 \frac{y}{y_o} = e^{-0.0241 \times 50}$ or equivalent $\bullet^2 0.29969.....$ $\bullet^3 30\%$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	ans: $x = 1$ and $x = 3$ (3 marks)	<ul style="list-style-type: none"> •¹ knows condition for intersection •² establishes quadratic equation •³ solves for x <ul style="list-style-type: none"> •¹ $\sqrt{4x - 3} = x$ •² $x^2 - 4x + 3 = 0$ •³ $x = 1, 3$
(b)	ans: $\frac{1}{3} \text{unit}^2$ (4 marks)	<ul style="list-style-type: none"> •¹ sets up the integral for area •² integrates correctly •³ substitutes limits in correctly •⁴ evaluates <ul style="list-style-type: none"> •¹ $\int_1^3 ((4x - 3)^{\frac{1}{2}} - x) dx$ •² $\left[\frac{1}{6} \sqrt{(4x - 3)^3} - \frac{1}{2} x^2 \right]_1^3$ •³ $\left(\frac{1}{6} \sqrt{(4 \times 3 - 3)^3} - \frac{1}{2} (3)^2 \right) - \left(\frac{1}{6} \sqrt{(4 \times 1 - 3)^3} - \frac{1}{2} (1)^2 \right)$ •⁴ $\frac{1}{3} \text{unit}^2$

Total 38 marks