

Mathematics 1 (Advanced Higher) - Practice Unit Assessment 3

Outcome 1

1. Expand $(2a + b)^3$. (2)
2. Express $\frac{7x-5}{(x-1)^2}$ in partial fractions. (3)

Outcome 2

3. Differentiate
- (a) $f(x) = 7x^4 \sin 2x$ (2)
- (b) $f(x) = \frac{4x-1}{x+6}$ (2)
- (c) $f(x) = \ln(5x-2)$ (2)

Outcome 3

4. Find (a) $\int 2e^{\frac{1}{2}x} dx$ (b) $\int \frac{4x}{x^2-7} dx$ (2, 2)
5. By using the substitution $u = x^2 + 4x + 5$ find $\int \frac{x+2}{x^2+4x+5} dx$. (3)

Outcome 4

6. $f(x) = \frac{x^2 + 6x + 12}{x + 2}$, $x \neq -2$, $x \in \mathbb{R}$.
- (a) Write down the equation of the vertical asymptote of the graph of $y = f(x)$. (1)
- (b) Show that the graph has a non-vertical asymptote and find its equation. (2)
- (c) Sketch the graph of $y = f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. (6)

Outcome 5

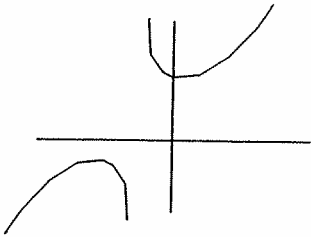
7. Use Gaussian elimination to solve the following system of equations.

$$\begin{aligned} x + y + z &= 6 \\ 3x - 2y - 3z &= -10 \\ 2x + y - 3z &= -5 \end{aligned} \quad (5)$$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 3

	Give one mark for each •	Illustrations for awarding each mark
Out 1 1.	<p>ans: $8a^3 + 12a^2b + 6ab^2 + b^3$</p> <p>2 marks</p> <ul style="list-style-type: none"> • all powers in expansion • all coefficients in expansion 	<ul style="list-style-type: none"> • all powers • all coefficients <p>Binomial coefficients are acceptable.</p>
2.	<p>ans: $\frac{7}{x-1} + \frac{2}{(x-1)^2}$</p> <p>3 marks</p> <ul style="list-style-type: none"> • method (using A/B etc.) • first value • second value 	<ul style="list-style-type: none"> • $\frac{7x-5}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$ • A = 7 • B = 2 <p>Accept A = 7, B = 2 with no working</p>
Out 2 3(a)	<p>ans: $f'(x) = 28x^3 \sin 2x + 14x^4 \cos 2x$</p> <p>2 marks</p> <ul style="list-style-type: none"> • know the product rule • differentiate terms 	<ul style="list-style-type: none"> • $(7x^4)' \sin 2x + 7x^4(\sin 2x)'$ • $28x^3 \sin 2x + 7x^4(2 \cos 2x)$
3(b)	<p>ans: $f'(x) = \frac{25}{(x+6)^2}$</p> <p>2 marks</p> <ul style="list-style-type: none"> • know the quotient rule • differentiate 	<ul style="list-style-type: none"> • $\frac{(4x-1)'(x+6) - (4x-1)(x+6)'}{(x+6)^2}$ • $\frac{4(x+6) - (4x-1)1}{(x+6)^2}$
3(c)	<p>ans: $f'(x) = \frac{5}{5x-2}$</p> <p>2 marks</p> <ul style="list-style-type: none"> • know how to differentiate $\ln(\)$ • chain rule factor 	<ul style="list-style-type: none"> • $\frac{1}{5x-2}$ • 5

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 3 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 4 6(c)	ans: (0, 6) (-4, -2), justification and sketch 6 marks <ul style="list-style-type: none"> differentiate Set $f'(x) = 0$ both pairs of coordinates of turning points justification of nature intercept and attempt to solve $f(x) = 0$ sketch of graph (marks are not deducted if candidates do not annotate graph fully) 	<ul style="list-style-type: none"> $f'(x) = 1 - \frac{4}{(x+2)^2}$ $f'(x) = 0$ (0, 6) and (-4, -2) $f''(x) = \frac{8}{(x+2)^3}$ $f''(0) > 0$ so local minimum $f''(-4) < 0$ so local maximum (0, 6) and $x^2 + 6x + 12 = 0$ 
Out 5 7.	ans: $x = 1, y = 2, z = 3$ 5 marks <ul style="list-style-type: none"> method first modified system second modified system method of back-substitution values of x, y and z 	<ul style="list-style-type: none"> $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & -2 & -3 & -10 \\ 2 & 1 & -3 & -5 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -5 & -6 & -28 \\ 0 & -1 & -5 & 17 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -5 & -6 & -28 \\ 0 & 0 & -19/5 & -57/5 \end{bmatrix}$ $z = 3$ $y = 2, x = 1$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 3 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 3 4(a)	ans: $4e^{\frac{1}{2}x} + c$ 2 marks <ul style="list-style-type: none"> • exponential • multiplier 	<ul style="list-style-type: none"> • $e^{\frac{1}{2}x}$ • $2 \div \frac{1}{2}$
4(b)	ans: $2 \ln x^2 - 7 + c$ 3 marks <ul style="list-style-type: none"> • $\ln()$ being involved • integrate • multiplier 	<ul style="list-style-type: none"> • $\ln()$ • $\ln(x^2 - 7)$ • 2 <p>No marks to be deducted if modulus signs or constant of integration are omitted</p>
5.	ans: $\frac{1}{2} \ln(x^2 + 4x + 5) + c$ 3 marks <ul style="list-style-type: none"> • represent as integral in u • integrate • express in terms of $\ln x$ 	<ul style="list-style-type: none"> • $\frac{1}{2} \int \frac{1}{u} du$ • $\frac{1}{2} \ln u$ • $\frac{1}{2} \ln(x^2 + 4x + 5)$
Out 4 6(a)	ans: $x = -2$ 1 mark <ul style="list-style-type: none"> • state equation of vertical asymptote 	<ul style="list-style-type: none"> • $x = -2$
6(b)	ans: $y = x + 4$ 2 marks <ul style="list-style-type: none"> • restatement of the function • state equation of non-vertical asymptote 	<ul style="list-style-type: none"> • $x + 4 + \frac{2}{x + 2}$ • $y = x + 4$