

Mathematics 1 (Advanced Higher) - Practice Unit Assessment 2

Outcome 1

1. Expand $(w + z)^4$. (2)
2. Express $\frac{3x - 16}{(x + 3)(x - 2)}$ in partial fractions. (3)

Outcome 2

3. Differentiate the following functions with respect to x .
- (a) $f(x) = \ln(x^2 + 5)$ (2)
- (b) $f(x) = \frac{x^2 + 7}{2x + 1}$ (2)
- (c) $f(x) = e^{2x} \sin 3x$ (2)

Outcome 3

4. Find (a) $\int 6e^{3x} dx$ (b) $\int \frac{4x}{2x^2 + 3} dx$ (2, 2)
5. By using the substitution $u = \sin x$ find $\int \sin^{3/2} x \cos x dx$. (3)

Outcome 4

6. $f(x) = \frac{2x^2 + 7x + 3}{(x - 3)(x + 1)}$, $x \neq -1, 3$, $x \in \mathcal{R}$.
- (a) Write down the equations of the vertical asymptotes of the graph of $y = f(x)$. (2)
- (b) Show that the graph has a horizontal asymptote and find its equation. (2)
- (c) Sketch the graph of $y = f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. (6)

Outcome 5

7. Use Gaussian elimination to solve the following system of equations.

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + z &= 4 \\ 4x + 9y + z &= 16 \end{aligned}$$

(5)

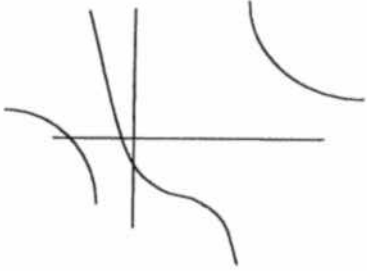
Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 2

	Give one mark for each •	Illustrations for awarding each mark
Out 1 1.	ans: $w^4 + 4w^3z + 6w^2z^2 + 4wz^3 + z^4$ 2 marks <ul style="list-style-type: none"> • all powers in expansion • all coefficients in expansion 	<ul style="list-style-type: none"> • all powers • all coefficients Binomial coefficients are acceptable.
2.	ans: $\frac{5}{x+3} - \frac{2}{x-2}$ 3 marks <ul style="list-style-type: none"> • method (using A/B etc.) • first value • second value 	<ul style="list-style-type: none"> • $\frac{3x-16}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$ • A = 5 • B = -2 Accept A = 5, B = -2 with no working
Out 2 3(a)	ans: $f'(x) = \frac{2x}{x^2+5}$ 2 marks <ul style="list-style-type: none"> • know how to differentiate $\ln()$ • chain rule factor 	<ul style="list-style-type: none"> • $\frac{1}{x^2+5}$ • $2x$
3(b)	ans: $f'(x) = \frac{2x^2 + 2x - 14}{(2x+1)^2}$ 2 marks <ul style="list-style-type: none"> • know the quotient rule • differentiate 	<ul style="list-style-type: none"> • $\frac{(x^2+7)'(2x+1) - (x^2+7)(2x+1)'}{(2x+1)^2}$ • $\frac{2x(2x+1) - (x^2+7)2}{(2x+1)^2}$
3(c)	ans: $f'(x) = 3e^{2x} \cos 3x + 2e^{2x} \sin 3x$ 2 marks <ul style="list-style-type: none"> • know the product rule • differentiate terms 	<ul style="list-style-type: none"> • $e^{2x}(\sin 3x)' + (e^{2x})' \sin 3x$ • $e^{2x}(3\cos 3x) + (2e^{2x}) \sin 3x$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 2 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 3 4(a)	ans: $2e^{3x} + c$ 2 marks <ul style="list-style-type: none"> • exponential • multiplier 	<ul style="list-style-type: none"> • e^{3x} • $6 \times \frac{1}{3}$
4(b)	ans: $\ln(2x^2 + 3) + c$ 2 marks <ul style="list-style-type: none"> • $\ln()$ being involved • integrate 	<ul style="list-style-type: none"> • $\ln()$ • $\ln(2x^2 + 3)$ No marks to be deducted if modulus signs or constant of integration are omitted
5.	ans: $\frac{2}{5} \sin^{\frac{3}{2}} x + c$ 3 marks <ul style="list-style-type: none"> • represent as integral in u • integrate • express in terms of $\sin x$ 	<ul style="list-style-type: none"> • $\int u^{\frac{3}{2}} du$ • $\frac{2}{5} u^{\frac{3}{2}}$ • $\frac{2}{5} \sin^{\frac{3}{2}} x$
Out 4 6(a)	ans: $x = 3, x = -1$ 2 marks <ul style="list-style-type: none"> • state equations of vertical asymptotes 	<ul style="list-style-type: none"> • $x = 3, x = -1$
(b)	ans: $y = 2$ 2 marks <ul style="list-style-type: none"> • restatement of the function • state equation of non-vertical asymptote 	<ul style="list-style-type: none"> • $2 + \frac{11x + 9}{x^2 - 2x - 3}$ • $y = 2$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 2 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 4 6(c)	ans: $(-1/2, 0)$ $(-3, 0)$ $(0, -1)$ and sketch 6 marks <ul style="list-style-type: none"> differentiate Set $f'(x) = 0$ show there are no stationary points x - intercepts solve $f(x) = 0$ sketch of graph (marks are not deducted if candidates do not annotate graph fully) 	<ul style="list-style-type: none"> $f'(x) = \frac{-11x^2 - 18x - 15}{(x^2 - 2x - 3)^2}$ $f'(x) = 0$ $b^2 - 4ac < 0$ for $-11x^2 - 18x - 15$ $x = -\frac{1}{2}$ and $x = -3$ $(0, -1)$ 
Out 5 7.	ans: $x = -3, y = 3, z = 1$ 5 marks <ul style="list-style-type: none"> method first modified system second modified system method of back-substitution values of x, y and z 	<ul style="list-style-type: none"> $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 4 \\ 4 & 9 & 1 & 16 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 5 & -3 & 12 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix}$ $z = 1$ $y = 3, x = -3$