

Mathematics 1 (Advanced Higher) - Practice Unit Assessment 1

Outcome 1

1. Expand $(a + 2)^5$. (2)
2. Express $\frac{4x + 3}{(x - 1)(x + 6)}$ in partial fractions. (3)

Outcome 2

3. Differentiate the following functions with respect to x .
- (a) $f(x) = 4x^6 \ln x$ (2)
- (b) $f(x) = \frac{7x + 4}{2x - 3}$ (2)
- (c) $f(x) = \exp(\tan x)$ (2)

Outcome 3

4. Find (a) $\int e^{4x} dx$ (b) $\int \frac{5x^4}{x^5 + 1} dx$ (2, 2)
5. By using the substitution $u = 1 - \cos x$ find $\int (1 - \cos x) \sin x dx$. (3)

Outcome 4

$$f(x) = \frac{x^2 + 4x + 7}{x + 1}, x \neq -1, x \in \mathcal{R}.$$

- (a) Write down the equation of the vertical asymptote of the graph of $y = f(x)$. (1)
- (b) Show that the graph has a non-vertical asymptote and find its equation. (2)
- (c) Sketch the graph of $y = f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. (6)

Outcome 5

7. Use Gaussian elimination to solve the following system of equations.

$$\begin{aligned} 2x + y + z &= -7 \\ x + 2y + z &= -8 \\ x + y + 2z &= -9 \end{aligned} \quad (5)$$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 1

	Give one mark for each •	Illustrations for awarding each mark
Out 1 1.	ans: $a^5 + 10a^4 + 40a^3 + 80a^2 + 80a + 32$ 2 marks <ul style="list-style-type: none"> • all powers in expansion • all coefficients in expansion 	<ul style="list-style-type: none"> • all powers • all coefficients Binomial coefficients are acceptable.
2.	ans: $\frac{1}{x-1} + \frac{3}{x+6}$ 3 marks <ul style="list-style-type: none"> • method (using A/B etc.) • first value • second value 	<ul style="list-style-type: none"> • $\frac{4x+3}{(x-1)(x+6)} = \frac{A}{x-1} + \frac{B}{x+6}$ • A = 1 • B = 3 Accept A = 1, B = 3 with no working
Out 2 3(a)	ans: $f'(x) = 24x^5 \ln(x) + 4x^5$ 2 marks <ul style="list-style-type: none"> • know the product rule • differentiate terms 	<ul style="list-style-type: none"> • $(4x^6)' \ln x + 4x^6(\ln x)'$ • $24x^5 \ln x + 4x^6 \frac{1}{x}$
3(b)	ans: $f'(x) = -\frac{29}{(2x-3)^2}$ 2 marks <ul style="list-style-type: none"> • know the quotient rule • differentiate 	<ul style="list-style-type: none"> • $\frac{(7x+4)'(2x-3) - (7x+4)(2x-3)'}{(2x-3)^2}$ • $\frac{7(2x-3) - (7x+4)2}{(2x-3)^2}$
3(c)	ans: $f'(x) = \sec^2 x \exp(\tan x)$ 2 marks <ul style="list-style-type: none"> • chain rule factor • exponential 	<ul style="list-style-type: none"> • $\sec^2 x$ • $\exp(\tan x)$

Marking Scheme - Mathematics 1 (Advanced Higher) - Practice Unit Assessment 1 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 3 4(a)	ans: $\frac{1}{4}e^{4x} + c$ 2 marks <ul style="list-style-type: none"> • exponential • multiplier 	<ul style="list-style-type: none"> • e^{4x} • $\frac{1}{4}$
4(b)	ans: $\ln x^5 + 1 + c$ 2 marks <ul style="list-style-type: none"> • $\ln()$ being involved • integrate 	<ul style="list-style-type: none"> • $\ln()$ • $\ln(x^5 + 1)$ No marks to be deducted if modulus signs or constant of integration are omitted
5.	ans: $\frac{1}{2}(1 - \cos x)^2 + c$ 3 marks <ul style="list-style-type: none"> • represent as integral in u • integrate • express in terms of x 	<ul style="list-style-type: none"> • $\int u du$ • $\frac{1}{2}u^2$ • $\frac{1}{2}(1 - \cos x)^2$
Out 4 6(a)	ans: $x = -1$ 1 mark <ul style="list-style-type: none"> • state equation of vertical asymptote 	<ul style="list-style-type: none"> • $x = -1$
6(b)	ans: $y = x + 3$ 2 marks <ul style="list-style-type: none"> • restatement of the function • state equation of non-vertical asymptote 	<ul style="list-style-type: none"> • $x + 3 + \frac{4}{x + 1}$ • $y = x + 3$

Marking Scheme – Mathematics 1 (Advanced Higher) – Practice Unit Assessment 1 (Cont'd)

	Give one mark for each •	Illustrations for awarding each mark
Out 4 6(c)	ans: (0, 7) (1, 6) (-3, -2), justification and sketch 6 marks <ul style="list-style-type: none"> • differentiate • Set $f'(x) = 0$ • both pairs of coordinates of turning points • justification of nature • intercept and attempt to solve $f(x) = 0$ • sketch of graph (marks are not deducted if candidates do not annotate graph fully) 	<ul style="list-style-type: none"> • $f'(x) = 1 - \frac{4}{(x+1)^2}$ • $f'(x) = 0$ • (1, 6) and (-3, -2) • $f''(x) = \frac{8}{(x+1)^3}$ • $f''(1) > 0$ so local minimum • $f''(-3) < 0$ so local maximum • (0, 7) and $x^2 + 4x + 7 = 0$ •
Out 5 7.	ans: $x = -1, y = -2, z = -3$ 5 marks <ul style="list-style-type: none"> • method • first modified system • second modified system • method of back-substitution • values of x, y and z 	<ul style="list-style-type: none"> • $\begin{bmatrix} 2 & 1 & 1 & -7 \\ 1 & 2 & 1 & -8 \\ 1 & 1 & 2 & -9 \end{bmatrix}$ • $\begin{bmatrix} 2 & 1 & 1 & -7 \\ 0 & 3/2 & 1/2 & -9/2 \\ 0 & 1/2 & 3/2 & -11/2 \end{bmatrix}$ • $\begin{bmatrix} 2 & 1 & 1 & -7 \\ 0 & 3/2 & 1/2 & -9/2 \\ 0 & 0 & 4/3 & -4 \end{bmatrix}$ • $z = -3$ • $y = -2, x = -1$